Charles Waistell.

THE

UNIVERSAL

MEASURER,

AND

MECHANIC.

In THREE PARTS.

A WORK equally uſeful to the GENTLEMAN, TRADESMAN, and MECHANIC.

With eleven NEAT COPPER-PLATES.

By A. FLETCHER *Philomath*,

LONDON:

Printed for J. RICHARDSON, in *Pater-noſter Row*.
MDCCLXII.

GENERAL CONTENTS.

PART FIRST.

Of practical geometry, shewing how to construct, reduce, divide, inscribe, circumscribe, &c. figures in plain and solid geometry; plain trigonometry, wrought geometrically, arithmetically, and instrumentally; heights and distances; levelling; surveying of mines; with a great many practical problems.

PART SECOND.

The theory of geometry; mensurations; conic sections, and mechanics; demonstrated in an easy, new, and universal method, independant of fluxions. To which are prefixed, the elements of algebra; infinite series; progressions; summing of series; logarithms; &c.

PART THIRD.

Decimals; feet and inches; sliding-rules; mensurations of plains, and solids of all kinds, both by the pen and sliding-rule; surveying, plotting, and dividing of lands; gauging, inching, ullaging, &c. with a large and curious collection of questions and solutions relating to measuring; gauging, mechanics, as pendulums, pumps, barometers, mills, engines, wheel-carriages, strength of walls, beams, &c. gunnery; circular motion, proving the true figure of the earth, &c.

36

ADVERTISEMENT.

The AUTHOR is now preparing for the PRESS,

A

TREATISE

OF

OPTICS.

Wherein will be ſhewn, the properties of LIGHT, with its REFLECTIONS, REFRACTIONS, HEAT, COLD, COLOURS, MICROSCOPES, TELESCOPES, BURNING-GLASSES, MULTIPLYING-GLASSES, MAGIC LANTHERNS, &c.

With conſiderable IMPROVEMENTS.

The whole illuſtrated with COPPER-PLATES.

It will be publiſhed as ſoon as the ſucceſs of the preſent Undertaking is determined.

BROUGHTON, December, 1761.

ADVERTISEMENT.

The AUTHOR continues, as uſual, to teach all the Branches of the MATHEMATICS, according to the beſt and lateſt Improvements.

THE

PREFACE.

SINCE trade and business must necessarily engage the care and attention of the greater part of mankind; it is no wonder that the classics and study of dead languages, should be less cultivated than the mathematics, which have gained the preference, and of late years been much improved: tho' (like all other sciences) the mathematics are but in their progressive state towards perfection; certainly then, every endeavour to improve them, or to render the study thereof easy and pleasant, deserves encouragement.

Measuring *and* mechanics *are branches of mathematics, which come so often in use, that few men who live in the world, but have occasion for their assistance one time or other. The gentlemen may be agreeably employed in measuring the works of the several artificers he may have occasion to employ, in surveying, setting out, and dividing lands; and thereby not only prevent frauds, but be enabled to form a better judgment of its value.*

The mechanic *is nearly concerned to employ some of his leisure hours in acquiring a competent knowledge of the principles, whereon his labour depends; that he may be capable not only to set a just value on his workmanship, but to contrive the nearest way to work; and to improve the invention of his implements, for expedition as well as ease.*

The favourable reception this book has met with, encouraged me to employ my utmost care and attention in improving this second

edition, in order to make it more compleat; with the addition of mechanics: *a subject of the greatest service in the affairs of life.*

It would ill become the author, to say more in behalf of these sheets; let it suffice, therefore, to give a brief account of this work, which is divided into three parts.

The first is a compleat body of practical geometry, with the demonstration of every difficult problem.

Problem 1st to 40th the construction of figures in plane geometry; conic sections; and architecture; with the method of fixing butments to bridges. From prob. 40 *to* 52, *the rules of proportion; extraction of the square and cube roots geometrically. From* 52 *to* 68 *problems concerning the circle. From* 68 *to* 87, *circumscription, and inscription of figures. From* 87 *to* 92, *reduction of figures, and plots. From* 92 *to* 119, *division of figures all manner of ways. From* 119 *to* 128, *plane trigonometry, right and oblique angled, solved both geometrically, and arithmetically. From* 128 *to* 142, *heights and distances; with the description and use of the instruments for that purpose. From* 142 *to* 145, *levelling, and surveying of mines. From* 145 *to* 152, *curious questions in navigation, mechanics,* &c. *From* 152 *to* 164, *the most useful problems in solid geometry; the construction of the five regular bodies; how to take dimensions of solids, as casks, frustums,* &c.

PART 2d, *is altogether theory; containing from problem* 164 *to* 178, *algebraic definitions, characters; addition, subtraction, multiplication and division, of whole quantities; as also of fractions, and surds; involution, evolution, proportion, progression; infinite series; solution of equations, by converging, and infinite series, reversions of series; an easy way to sum up any series; with many new investigations, logarithms,* &c. *Prob.* 178. *The chief theorems in Euclid, with many new theorems and their demonstrations,*

Prob. 179, *of sines and tangents; with the axioms of trigonometry. From* 179 *to* 182, *of conic sections. From* 182 *to* 191, *the theory of mensurations; with the demonstration of theorems, for measuring, and gauging, all kinds of planes, and solids, both by particular rules, and by one general method, as also by a new and universal series; with theorems for knowing the forms of casks, inching, ullaging,* &c. *Prob.* 191, *a general method of maxima, and minima. From* 191 *to* 201, *the theory of* mechanics *shewing the general laws of motion simple and compound, elastic and non-elastic, of gravity, vibrations of pendulums, and musical strings; mechanic powers, wheel carriages, centers of gravity, oscillation, percussion and gyration, pressure and strength of walls, beams,* &c. *motion of projectiles, hydrostatics, pneumatics, hydraulics,* &c. *Prob.* 201, *the principles of fluxions and fluents, with their application to many parts of mechanics.*

Part 3d. *Divided into three sections. Sect.* 1*st*, *Decimals made plain and easy, with several useful contractions, and the extraction of roots. This section (after common arithmetic) should be learned, then practical geometry. By observing this method, you will be better enabled to study mensurations to advantage. Sect.* 2*d. Construction, description, and use of* Coggleshal's *sliding-rule. Sect.* 3. *Multiplication of feet and inches; called cross multiplication. Sect.* 4. *Superficial measure of planes and of solids; with the methods of taking dimensions with, or without inches. Sect.* 5. *The several artificers work. Sect.* 6. *How to measure all sorts of solids. Sect.* 7. *Surveying, plotting and dividing of lands,* &c. *Sect.* 8. *Gauging in all its parts; with the description and use of instruments for that purpose. Each example in these three sections is wrought by pen and sliding-rule. Sect.* 9. *Questions with their solutions.* 1. *From quest.* 1 *to* 40, *exercises in mensurations, surveying, and gauging.* 2. *From* 40 *to* 50, *mensurations of solids, by their centers of gravity.* 3. *From* 50 *to* 85, *shewing the nature and use of the mechanic powers.* 4. *From* 85

to 115, *of wheel carriages, strength and stress of walls, beams,* &c. 5, *From* 115 *to* 140, *pendulums, sounds, musical instruments.* 6. *From* 140 *to* 166, *the maxima of bodies moving in and by fluids; the forces of moving bodies, and the resistance they meet with; the perfections of mills, engines,* &c. 7. *From* 166 *to* 192, *the specific gravities of bodies, with a table; the admirable properties of the air, weather-glasses, pumps, and fire-engines.* 8. *From* 192 *to* 231, *the practice of gunnery in resisting and non-resisting mediums.* 9. *From* 231 *to* 261, *of friction, sails of ships, spouting-fluids, waves, whirling bodies, &c.* 10. *From* 261 *to* 294, *laws of central forces, circular motion, true figure of the earth; gravities, densities,* &c. *of the planets and tides.*

To conclude, I have been careful to omit no article that seemed useful or necessary, and as cautious to avoid things perplexing or superfluous: and to make the whole more familiar and easy to the learner, I have kept as much as possible within the limits of algebra; altho' by this work may be acquired, a clear and distinct notion of fluxions and fluents.

THE UNIVERSAL MEASURER AND MECHANIC.

PART FIRST.

A Complete Body of Practical *GEOMETRY.*

POSTULATES.

1. IT may be taken for granted, that a right line may be drawn between any two given points.

2. That a given right line may be produced at pleasure.

3. That from a given point a line may be drawn either parallel to another, or in such a manner as to make any given angle with any other given line.

4. That a circle or any arch thereof may be described upon a plane with a pair of compasses opened to any extent.

5. That a line may be described by the motion of a point.

6. That a space, plane, or area, may be described by a line moving parallel to itself, or about any point as a center.

7. That a solid may be described by a plane moving parallel to its self, or about any part thereof as an axis.

It is to be obſerved, that in working geometrically, all points and lines ſhould be as ſmall as poſſible, (the former being without dimenſions, and the latter only one dimenſion, viz. length) and the figures will be more exact. Alſo, when you begin to work any problem, obſerve carefully what things are given, and with theſe work exactly as the reading directs, making all lines of illuſtration pricked or dotted, and all lines given or required, black; be careful to draw all the lines as directed whether they are in the figure or not.

PROBLEM I.

To draw a line C D *parallel to a given line* A B, *and at the diſtance of the line* e f *from it.* Fig. 1.

1. With the given line e f in the compaſſes, and one foot on any two different points in A B, as at E and F, ſtrike two arches n and s.

2. Lay a ruler to touch the tops of theſe two arches, and draw the line C D, which is the parallel required.

Note. Black lines or arches commonly repreſent things given and required, and thoſe pricked or dotted, ſhew how the problem is worked. In geometry, as in arithmetic, there is always ſomething given and required: and here obſerve, the things given you may prick down as you pleaſe, then work by the directions for thoſe required.

PROBLEM II.

To draw a line G H *parallel to a given line* I K, *and to paſs thro' a given point* P. Fig. 2.

1. With one foot in any part of the given line I K, as at Q, ſweep a half circle to paſs through P, 2. make the arch I M = to the arch P K 3. thro' M and P draw the line G H, and its done. See Problem 42.

PROBLEM III.

To biſect or divide a given line A B *into two equal parts.* Fig. 3.

1. With any radius greater than half the ſaid line, and one foot firſt on one end and then on the other, ſtrike two arches, croſſing each other in C and D, 2. thro' the points C and D draw the line C E D; then will A E = E B.

PROBLEM IV.

To erect a perpendicular BD, *upon the end* B, *of any given line* AB. Fig. 4.

1. Lay a ruler along AB, and produce it to C, then with one foot on B ſweep the pricked half circle, cutting ABC in n and m, 2. upon n and m ſtrike two arches croſſing each other in D, draw DB and its done.

PROBLEM V.

To raiſe a perpendicular DB *upon any given point* B, *in a given line* A C. Fig. 4.

1. As in the laſt problem, take any two points n and m, equally diſtant from B, upon which as centers, ſtrike two arches croſſing each other in D, thro' D and B draw DB, and its done.

PROBLEM VI.

From a given point P *upon a given line* AB, *to let fall a perpendicular.* Fig. 5.

1. With one foot in the given point P, ſtrike an arch to cut the given line AB in two points n and m, 2. upon n and m with any radius ſtrike two arches croſſing each other in C, 3. thro' P and C draw PCE, which is the ⊥ required.

PROBLEM VII.

Given any angle BAC, *to make another angle* b a c *equal to it.* Fig. 6.

1. Draw a line a c, 2. with any radius and one foot on A, the given angular point, ſtrike an arch BC to cut each ſide, with the ſame radius on a, ſtrike the arch b c, take the arch BC in your compaſſes and lay it from c to b, 3. thro' a and b draw a b, and its done.

PROBLEM VIII.

To divide a given angle BAC, *into two equal angles*, DAC =DAB. Fig. 7.

1. With any radius and one foot on A, ſweep the arch n m, 2. with any radius upon n and m ſtrike two arches croſſing each other in D, draw AD, and its done.

Note. When three letters B A C express an ∠, the middle letter A stands at the angular point.

PROBLEM IX.

How to make a line of Chords. Fig. 8.

1. Draw a line A m, which make radius, and upon A sweep the arch m 90. 2. From m upon this arch, lay the radius A m to 60. 3. Take half the arch m 60 and lay it from 60 to 90. 4. Divide the arch 60, 90, into three = parts, which will also divide m 60 into six = parts. 5. Set one foot of your compasses in m, and strike arches from each of these parts to the line A m, and its done.

Note. Each of these equal parts in the arch should be divided into ten more.

PROBLEM X.

To make any acute angle; suppose one of 35° Fig. 10.

1. Draw a line A C, with the chord of 60° upon A sweep an arch C 40. 2. From the same chords take 35°, and lay it from C upon the arch, thro' which draw the line A B; so is the angle B A C one of 35°.

PROBLEM XI.

Upon the point A, *and with the line* A C, *to make a right angle,* i. e. *one of* 90°. Fig. 10.

☞ This is the same as erecting a ⊥.

1. Upon A with the chord of 60° strike the arch C 90°, from the same scale of chords take 90°, and lay it upon the arch from C to 90. 2. Thro' A and 90 draw A D; so is the angle D A C one of 90°, as required.

PROBLEM XII.

Upon the point A, *and with the line* A B, *to make an obtuse* ∠, B A C, *suppose of* 95°. Fig. 9.

1. Upon A with the chord of 60°, sweep the arch B C, then, because the arch B C is to be above 90°, which is the whole length of the line of chords, you may take any two chords whose sum is 95°, as 60° and 35°, and lay them upon the arch B C one after the other, from B to C, then draw A C, and its done.

PROBLEM XIII.

Upon any given line A B, *to make an equilateral triangle.* Fig. 11.

1. Make the given line radius, and upon A ſweep the arch B C, alſo upon B ſweep the arch A C, croſſing each other in C. 2. Join A C and B with lines, and you'll have A B C, the triangle required.

PROBLEM XIV.

Given three lines A B, A D, A C, *to make a triangle.* Fig. 12.

1. If A B be one of the given lines, take either of the other two in your compaſſes as A D, and with one foot on B ſtrike an arch, then with A E in your compaſſes and one foot on A, ſtrike another arch croſſing the former arch in C. 2. Join C A and C B, and its done.

NOTE. One of the given lines muſt be ſhorter than the ſum of the other two, or it cannot poſſibly be a triangle.

PROBLEM XV.

Required the greateſt triangle that it is poſſible to make with a given baſe A B, *and ſum of the other two ſides* D E. Fig. 13.

1. This muſt be when the perpendicular C P is the longeſt poſſible; therefore with F E = F D = half D E in your compaſſes, and one foot ſeverally on A and B, ſweep two arches croſſing each other in C. 2. Join A C and C B, and its done. For it is plain, if D E = C A + C B be a thread or cord faſtened to two pins A B, and a needle put in C, the double thereof and moved any way, keeping the cord at a conſtant ſtretch, the ⊥ C P by ſuch motion will be ſhortened, whence the iſcoſceles triangle A B C is that required.

PROBLEM XVI.

Given the diagonal A B, *and the four ſides* A C, A D, A E, A F, *of a trapezia* A I G H, *to make the trapezia.* Fig. 14.

1. Make A G = A B. 2. With any of the given lines in your compaſſes as A C, and one foot on either A or G, ſuppoſe in A, ſtrike an arch H, alſo with ſome other of the lines, as A E, and one foot in G, ſweep an arch, croſſing the laſt arch in H. 3. Join H A and H G, and

one of the triangles is made; then with the other two lines A D and A F, form the triangle A I G, below the diagonal A G, and its done. It is but making two triangles by problem 14.

PROBLEM XVII.

Given any right lined figure a b c d e, *to make another equal and similar to it.* Fig. 15.

1. Divide the given figure a b c d e into Δs, by drawing the diagonals a c and a d, 2. by problem 14, with the three lines a c, a b, and b c, make the Δ A B C, also upon A C, with the two lines a d and d c, make the Δ A D C, lastly, upon A D, with the two lines a e and d e, make the Δ A E D, and its done.

PROBLEM XVIII.

Given any right lined figure a b c d, *to make another figure* A B C D *similar to it, whose sides may be twice as large, or in any other proportion.* Fig. 16.

1. This may be done by the last problem, by doubling every line as you lay it down: or thus, by problem 7. make the ∠ B A D = ∠ b a d, continuing A D, until it be = twice a d, then upon D make an ∠ A D C = twice ∠ a d c, making D C = twice d c, then make an ∠ D C B upon C = ∠ d c b, making C B = twice c b, which in this case will meet A B in B, making A B = twice a b, and the ∠ A B C = ∠ a b c, if it is truly drawn.

PROBLEM XIX.

Upon any given line A B *to make a square* A B C D. Fig. 17.

1. By problem 5, upon B one end of the given line, raise a ⊥ B D, making it equal to A B, 2. with A B in your compasses and one foot severally on D and A, strike two arches crossing each other in C, 3. join C D and C A, and its done.

PROBLEM XX.

Given two lines A B *and* B C, *to make a right angled parallelogram, or rectangle* A B C D. Fig. 18.

1. By problem 5, upon A raise the ⊥ A D, making it = B C, 2. with A B in your compasses and one foot on D, sweep an arch, 3. with B C in your compasses and one foot on B, cross the last arch in C, join D C and B C, and its done.

NOTE. If the dimenſions of figures be given in numbers, they may be made by a ſcale of equal parts, or a diagonal ſcale, the ſame way as if the dimenſions were given in lines; as in ſome of the following figures.

PROBLEM XXI.

To divide a given line A B *into any number of equal parts, ſuppoſe into five.* Fig. 19.

1. At each end of the given line, by problem 7, make an *L*, viz. the *L* B A 5 = *L* A B 5, 2. take any ſmall diſtance in your compaſſes, and run it along the line A 5, dividing it into 5 = parts, run the ſame diſtance from B to 5, dividing B 5 into 5 = parts, 3. thro' theſe diviſions draw lines, as 1-4, 2-3, &c. and they will divide A B as required. See theorem 1.

PROBLEM XXII.

How to make a ſcale of equal parts, as A B. Fig. 20.

1. Having drawn a line A B, take any diſtance in your compaſſes and run it along the line 11 times, as from A to o, from o to 10, &c. to 100, 2. divide the diſtance A o into 10 = parts, which ſerves the whole ſcale for units; ſo that e 20 is 25, the diſtance e 70 is 75, &c.

PROBLEM XXIII.

How to conſtruct and uſe a diagonal ſcale. Fig. 22.

1. A diagonal ſcale is divided into 10 large equal diviſions, as the diſtance e q. or e 100, is one of theſe diviſions, and theſe ſignify hundreds, the diviſions 10, 20, 30, &c. between e and 100, along the ſide of the end diviſion, denote tens, and thoſe on the end, as 1, 2, 3, &c. ſignify units, being all equal diviſions. Now to find any number upon this ſcale, ſuppoſe 1 chain 43 links, or 143 of any equal parts, as inches, feet, yards, &c. firſt ſet one foot on x which ſtands over againſt 40, and under 3, is the place of 43, and extend the other foot along that line to y, ſo have you 143 in your compaſſes; in like manner, the diſtance w u is 185, t u 85, &c. which will become eaſy with practice.

PROBLEM XXIV.

To make a rhombus A B C D, *whose height* B P *may be* 20, *and each side* A B = B D, *&c.* 30. Fig. 23.

1. From the scale of equal parts take 30, and lay it from A to B. 2. Make AB radius, and with one foot on A, sweep the arch C, and on B sweep the arch D, from the same scale take 20, with which, by problem 1, in any two different points on A B, sweep two arches over whose tops draw C D, cutting the first two arches in C and D. 3. Join B D and C A, and its done.

PROBLEM XXV.

To make a rhomboides AB C D, *whose length* A B = C D *may be* 143, *side or end* A D = B C 85, *and height* D P 80. Fig. 21.

1. From the diagonal scale, with the length 143 and height 80, by problem 20, make a rectangle D C E P, producing E P towards A. 2. From the same scale, with the breadth 85, and one foot on C and D severally, the other will cross E P in A and B, join C B and D A, and its done.

PROBLEM XXVI.

To make a rhomboides A B C D *equal to a rectangle* D C E P, *whose acute angles shall be equal to a given angle* z. Fig. 21.

1. Make A B = P E, and upon A and B, by problem 7, make the ∠s E B C, and P A D, each = the given angle z, drawing A D and B C, till they are each = the given breadth 85. 2. Join D C and its done. But if the height 80 were given, and not the breadth 85, you must continue A D and B C till they meet D C, drawn parallel to A B at the distance of 80 from it. By problem 1. For demonstration see theorem 1, and 2.

In some of these figures you'll find more lines, arches and letters than is mentioned in the reading for the problem or figure, such figures serve more problems than one: these lines, &c. need not be drawn or regarded, till you come to the problem that refers to them: but I would advise a learner that is desirous to understand geometry, to draw a figure for each problem.

PROBLEM XXVII.

To make any regular polygon, suppose a pentagon. Fig. 24.

1. Describe a circle and cross it with a diameter A 5, which divide into as many equal parts as the polygon hath sides, so a pentagon having five sides, the diameter must be divided into 5 equal parts. 2. With the whole diameter A 5 in your compasses, and one foot on A and 5 severally, sweep two arches crossing each other in D. 3. Lay a ruler to D and the second division on A 5 and draw a line D E, cutting the circle in E. 4 Join A E, which will be a side of the required pentagon, and will divide the periphery of the circle into 5 equal parts as required.

PROBLEM XXVIII.

Upon a given line C D *to make any regular polygon, suppose a pentagon.* Fig. 25.

1. As directed in the last problem, find A E, the side of a pentagon of any size, then upon each end C and D of the given line C D, makes an $\angle$ $=$ to the $\angle$ E A 5, in figure 24, and at Q where the lines meet, is the center of a circle, which being described with the radius Q D $=$ Q C, the given side C D will divide it into 5 equal parts as required.

PROBLEM XXIX.

To make an oval or ellipsis, TISG. Fig. 26.

1. Describe two equal circles, passing thro' each others centers B and C, and crossing each other in a and e. 2. With the diameter S C or B T of either circle, and one foot on e sweep the arch I, also with one foot on a, sweep the arch G, and its done.

PROBLEM XXX.

Another way to make an oval or ellipsis. Fig. 26.

1. Upon any plane where you would make an oval, strike in two pins, suppose at B and C, and to these pins fasten the ends of a thread, as B P C. 2. With a pin extend the thread as far as possible, and by moving the pin about, you may describe the periphery of an ellipsis.

PROBLEM XXXI.

Given the axis e u *and ordinate* A B, *of a common parabola* A u B, *to draw the parabola.* Fig. 27.

1. A B being ⊥ to u e, produce u e towards C. 2. Upon u e describe a circle u A C B, to pass thro' the three points u, A, B, cutting u e (produced) in C. 3. Upon u e, describe as many more circles as you please, each passing thro' u, and cutting the axis u e in a, b, d, &c. 4. Take C e and lay it from A to n, from b to m, from d to o, &c. 5. Thro' these points n, m, o, &c. draw lines or chords each parallel to AB, and where these chords cut their respective circles, are the points thro' which the parabolic curve must pass, and may be drawn with a steady hand. See theorem 60.

PROBLEM XXXII.

Given the transverse axis u T *of an hyperbola* A B u, *and* u K = T H *the distance of the focus from the ends* u *and* T *of the axis, to draw the hyperbola.* Fig. 28.

1. Produce u K any length as to C for the abscissa. 2. Divide u C into as many parts (= or not =) as you please, as in the points m, n, a, &c. 3. With the radii T m, T n, T a, &c. and one foot on H, strike the arches e e, e e, e e, &c. 4. With the radii u m, u n, u a, &c. and one foot on K, cross the former arches, with the arches oo, oo, &c. 5. Thro' these intersections draw the curve A C B. —— Note. The more intersections you make use of the better, in both these figures, as they will not only help you in the drawing, but will also render them more exact. See theorem 62.

PROBLEM XXXIII.

Upon a given line A C *to make a gothic arch* A T C. Fig. 29.

1. Divide the base or ordinate A C, into 3, 4, or 5 = parts, suppose into 3 = parts, C D = D E = E A, then with A D two of these parts, and one foot in D and E severally, sweep two arches A B and G C, crossing each other in T, and its done; A T C being a gothic arch of the third point; because A C was divided into 3 = parts, but if you divide A C into 4 or 5, &c. = parts, the arch is said to be of the 4th, 5th, &c. points.

PROBLEM XXXIV.

Upon a given line or base A C, *to make a low arch.* Fig. 30.

1. Upon A C by problem 13, make an equilateral triangle A C F. 2. Upon F, with the radius F A = F C, ſweep the arch A E C; now if the ⊥ F B be produced, it will biſect the arch in E, whereof E B being about $\frac{1}{8}$ of A C, will give the height or thickneſs of the ſtreight arch for ſquaring or cutting the ſtones, &c. and by the figure may be ſeen the different forms of the ſtones, &c.

PROBLEM XXXV.

To deſcribe a Catenaria T I S *whoſe height* C I *may be* 55 *feet, and its breadth* T S 200 *feet.* Fig. 31.

1. The geometrical conſtruction of this curve being tedious, it will be eaſieſt done thus; on an upright wall or plane lay the diſtance 200, from any ſcale of equal parts, parallel to the horizon, as from T to S, there ſtrike in two pins, upon which ſuſpend a flexible line, (a chain of very ſmall links is beſt) as T G S, till its middle at G be 55 of the ſame = parts below T S, ſo will T G S form the catenaria required.

PROBLEM XXXVI.

To take the plan of any curve, arch, bended hedge, &c. and lay it down on paper. Fig. 31.

1. Meaſure ſtreight from S one end of the curve S G T, towards T the other end, ſetting down the diſtances S a, S a, &c. where you meaſure perpendicularly from S T to the curve, ſet down alſo theſe ⊥s a e, a e, &c. called offsets, and always be careful to meaſure the offsets from the ranging line T S to the extremities of the ſaid curve, which done you may have a true plan by drawing a line T S, upon which from a ſcale of = parts, lay down all the diſtances S a, S a, &c. upon which points erect the ⊥s a n, a n, &c. and from the ſame ſcale lay on theſe offsets their reſpective diſtances, as from a to n, &c. thro' theſe points draw the curve.

PROBLEM XXXVII.

The arches used in architecture, are as follow;

1. Semi-circular arches, being half-circles. 2. Scheme arches are less than a semi-circle, and commonly contain 70 or 90°. 3 Elliptical arches, are such as consist of a semi-ellipsis, formerly used over chimneys for mantle trees. 4 Gothic arches. 5. Streight arches, are sometimes over windows, doors, galleries, cielings, &c. because they take up but little room. 6. These are the arches which were formerly used in vaults, bridges, and other buildings, amongst which the semi-circular arch was most esteemed both for beauty and strength. But the strongest arch possible is the catenaria; whose strength is such, that a number of globes, or a heavy flexible line, being put into that form, will not fall, but support themselves; this appears from the formation of that curve, (Fig. 31) those above the line TS, must support with the same force, as they gravitate below it.

PROBLEM XXXVIII.

To find the push or butment of an arch A E M. Fig. 32.

1. Let E be the middle of the underside of the bridge, &c A E M. 2. Join A E, and let A D be ⊥ to the horizon A M. 3 Upon A, with the radius A E, draw the quadrant B E D, so is A B = A E = AD. 4. Join DB, which cuts A E in I. 5 From I upon A M let fall the ⊥ I L. Lastly, make A C = A L, then A C will be the length of the butment required, fit to hold the push of the arch A E, being suspended upon the point A as a center. For if A E, (the chord of the arch A E) become A B parallel to the horizon it is plain, it must have its own length in the same parallel on the other side of A to ballance it with, which suppose 20; but when the same A E is raised into A D ⊥ to the horizon, upon the fixed point A, it will then require nothing to ballance it with. Whence it follows, that if 20 ballance A E when in a parallel position, and 0 when in a ⊥ one, that half the sum of 20 and 0, will ballance it when in a mean between A B and A D, i. e. when it makes an ∠ with the horizon of 45°.

PROBLEM XXXIX.

Given two lines B A *and* B C, *to find such a third line* A C, *as that the triangle* A B C *may be the greatest possible.* Fig. 33.

1. With the two given lines make a right ∠ C B A, and join the ends C and A, so is C A the line sought, and A B C the △. For if B D = B C, the △s A B C and A B D, stand upon the same base A B, and therefore are (by theorem 33) as their heights. It is plain by the figure A B C, that BC has the greatest height, and consequently it is the greatest triangle possible. By the same method it may be proved, that if any number of lines be given to find another line so as to make the greatest figure possible, the line required must be the diameter of a circle, and the given lines so connected, as to be inscribed in the half periphery.

PROBLEM XL.

Given the hypothenuse A B *of a right angled* △, *to find the legs, so that the* △ *may be the greatest possible.* Fig. 34.

1. Upon the given hypothenuse A B, sweep a semi-circle whose middle is at C, join C A and C B and its done, A B C being the triangle sought, and A C = B C the two legs. For, as in the last problem the △ A B C, is greater than the △ A B D.

Note All right-lined figures inscribed in circles, are the greatest that can possibly be made: for a circle being the greatest figure under the same periphery, it follows, that the nearer any figure approaches to it, the greater it will be.

T e RULES of PROPORTION,

Extraction of the Square and Cube Roots, Geometrically:

IN TWELVE PROBLEMS.

PROBLEM XLI.

To divide a line A B *into such proportion, as the line* C u *is to the line* D u. Fig. 35.

1. Upon either end of the given line A B, as on A, make an ∠ by drawing A w. 2. Make A F = C u, by taking C u in your compasses and laying it from A to F; also, lay D u from F to w, join w B, and parallel to w B, thro' F, by problem 2, draw F G and its done. For A F is to F w, as C u is to D u. And if these lines be laid down from a diagonal scale, as marked in the figure, you'll find G B = 58,7 and A G = 134,3 from the same scale. See theorem 9.

PROBLEM XLII.

Given two lines A B *and* D u, *to find a third line in proportion to them.* Fig. 36.

1. Make an angle D A C; lay D u from A to E, and A B from E to D, join E B. 2. Thro' D, and parallel to E B, by problem 2, draw D C, or a parallel may be drawn by making = ∠s thus, upon E sweep an arch to cut the two containing sides E A and E B of the ∠ A E B, with the same radius describe an arch upon D, take the arch described upon E, and lay it upon that described on D, from the line E D, and where it cuts draw the line D C, which (per theorem) will be parallel to E B, whence you'll have B C for the line required. For as A E (= D u 70) is to A B 124, so is E D (= A B 124) to B C 219,6. See theorem, and problem 45.

PROBLEM XLIII.

Given three lines, A B, C u, *and* D u, *to find a fourth proportional.* Fig. 37.

1. With any two of the given lines make an ∠ B A E, making A B = A B, A E = D u, and E D = C u, join B E, and thro' D parallel to B E (by the last problem, or by problem 7) draw D C, cutting A B produced in C; so is B C the fourth proportional line required. For, (by theorem 9,) as A E (D u) is to A B, so is E D (C u) to B C.

PROBLEM XLIV.

If 180 *Labourers do a piece of Work in* 115 *days, in what time will* 106 *labourers do it.* Fig. 37.

1. Make any ∠ C A D, and from a diagonal scale lay 106 from A to B, 115 from A to E, and 180 from E to D, join E B, and thro' D, parallel to E B, draw D C cutting A B C in C, so is C B = 186, from the same scale, as required.

By these two last problems you may work any proportion by lines or scales of equal parts, whether direct or inverse.

PROBLEM XLV.

Between two lines E D 36, *and* C D 100, *to find a mean proportional.* Fig. 38.

1. Having made one right line E C of the two given ones D E and D C, upon E C as a diameter, deſcribe a ſemicircle E P C. 2. Upon D, where the two lines meet, raiſe the ⊥ D P, cutting the arch in P, ſo is D P the mean geometrical proportion required. For as C D : D P : : D P : D E, by theorem 14; whence C D × D E = □ D P = 3600, ſo D P = 60, as will alſo be found if you lay D P on the ſame ſcale of equal parts that you took D E and D C from.

Note. By this figure you may alſo work problem 42.

PROBLEM XLVI.

To extract the ſquare root of any number, ſuppoſe 3600, *by lines, or a diagonal ſcale.*

RULE. A geometrical mean proportional between any two lines or numbers, is the ſquare root of their product.

EXAMPLE. 100 × 36 = 3600, and a mean proportional between 100 and 36 is by the laſt problem found = 60, whence 60 is the ſquare root of 3600 Alſo, 10 × 360 = 3600, and a mean proportion between 10 and 360, will alſo be found = 60. Likewiſe 40 × 90 = 3600, and 30 × 120 = 3600, and a mean proportional between 40 and 90, or between 30 and 120, will alſo be found 60, = the ſquare root of 3600.

PROBLEM XLVII.

Between two given lines A *and* B, *to find two mean proportional lines.* Fig. 39.

1. Make a right ∠ K C H, drawing the ſides C H and C K at large. 2. Lay the line B from C to E, and the line A from C to D, and join E D, find its middle, which is at F, then with F E = F D, upon F deſcribe the ſemicircle E G D. 3. With the leſſer line B in your compaſſes and one foot on D, croſs the ſemicircle in G. 4. Upon the point G, move a ruler till it cut the lines C H and C K in H and K, at an equal diſtance from F, ſo ſhall E H and D K be the lines ſought. For as E C (B) : D K : : D K to E H, and as E C : D K : : E H : C D, (A). 13. E. 6.

PROBLEM XLVIII.

To extract the cube root geometrically. Fig. 39.

RULE. The cube root of any number is the first of two mean proportionals between unity and that number. Or, which is the same, and easier to do, Take any number which is less than the cube root of the given number, square this number, and divide the given number thereby; then the first of two mean proportionals, between this quotient and the abovesaid taken number, will be the cube root of the given number.

EXAMPLE. What's the cube root of 67584? Take 32, which squared is 1024, then $67584 \div 1024 = 66$; so the first of two mean proportionals between 32 and 66, will be the cube root of 67584; therefore, if from the diagonal scale you make $CE = 32$, and $CD = 66$, and work as in the last problem, you'll have $DK = 40\frac{3}{4}$ nearly, from the same scale, for the cube root of 67584, or the first of two mean proportionals between 32 and 66. By this, and problem 46, with a good diagonal scale, you may readily find the cube and square root of a number.

PROBLEM XLIX.

To find out two lines EF *and* FC, *which shall have such proportion to each other, as the square of a given line* A *hath to the square of another given line* B. Fig. 40.

1. Make a right $\angle$ EDC, (i. e. CD $\perp$ to DE) then lay the line B from D to C, and A from D to E, join CE. 2. From D upon CE, let fall the $\perp$ DF, and its done. For, by theorem 15, as $\square$A : $\square$B, :: EF : FC.

PROBLEM L.

To divide a line CD *in power, as the line* A *is to the line* B. Fig. 41.

1. By problem 41, divide CD into such proportion as A to B, i. e. as B : A :: CE : ED. 2. Upon CD, as a diameter, describe the semicircle CFD. 3. Upon E raise the $\perp$ EF, cutting the semicircle in F,

3. Join F C and F D, which are the two lines required. For, by theorem 16, as B : A : : $\square$ C F : $\square$ D F, and as A + B : $\square$ C D : : B : $\square$ C F, and : : A : $\square$ F D.

PROBLEM LI.

To enlarge any line C E *in power, according to any proportion; ſuppoſe as the line* A *to the line* B. Fig. 41.

1. By problem 41, make it as A : B : : C E : C D; i. e. make C B = A, and C A = B, join E B, and thro' A parallel to E B, draw A D, cutting C E produced in D. 2. Upon C D deſcribe the ſemicircle C F D. 3. Upon E erect the $\perp$ E F, cutting the ſemicircle in F, join C F for the line required. For, by theorem 15, as A : B : : $\square$ C E : $\square$ C F.

PROBLEM LII.

To cut a line A D *in extreme and mean proportion.* Fig. 42.

1. Upon A raiſe the $\perp$ A F, making it = A D. 2. Biſect A F in G and join G D. 3. produce F A, making G I = G D. 4. Make A C = A I, and its done. For as A D : A C : : A C : C D. 30. E. 6.

Demonstration. Put a = A D = A F, then A G = $\frac{1}{2}$ a by conſtruction, and put e = A I = A C, then a − e = C D, and the laſt proportion will be, a : e : : e : a − e, therefore, e e = a a − a e, or, a a = e e + a e, which quadratic equation ſolved (by article 81, part 2.) gives $e = \sqrt{\frac{5}{4}aa} - \frac{1}{2}a = a \times : \frac{\sqrt{5}-1}{2}$; now, $\square$ G D = $\square$ A G + $\square$ A D = $\frac{1}{4}$a a + a a = $\frac{5}{4}$a a, and G D − G A = A C = e, i. e. $\sqrt{\frac{5}{4}aa.} - \frac{1}{2}a = e$ as before; ſo the figure is truly conſtructed.

Note. Becauſe the ſquare root of 5 cannot be exactly had, it ſhews that no number can be exactly divided in extreme and mean proportion, but any line may be ſo divided geometrically, as above.

The chief PROBLEMS belonging to the CIRCLE are fifteen, as follow:

PROBLEM LIII.

Given any Circle A B C D, *to find* E *its center.* Fig. 43.

1. Any where in the circle draw a chord O O, which bisect with the line B D. 2. Bisect B D with the line A C, so is E, the crossing of these two lines, the center of the circle; and the lines themselves are two diameters, at right angles to each other.

PROBLEM LIV.

Given any arch of a circle G C, *to find its tangent* D C, *sine* G B, *secant* A D, *&c.* Fig. 44.

1. Make A E ⊥ to A C, so is the arch E G C a quadrant. 2. Thro' A the center of the given arch, and over G its top, draw the line A D. 3. Draw C D ⊥ to A C, or parallel to A E; also G B parallel to A E, and E F and G I both parallel to A C, so is C D the tangent, D A the secant, G B the sine, G I = A B the co-sine, B C the versed sine, I E the co-versed sine, A F the co-secant, and E F the co-tangent of the arch G C, each of which may be measured on the diagonal scale that A C was taken from. In like manner, because one of these arches E G and G C is the complement of the other, therefore, E F is the tangent, and C D the co-tangent of the arch E G, &c.

PROBLEM LV.

To describe a circle thro' any three points, as the three angular points of a triangle A B C. Fig. 45. *Or, to find a point* D *equally distant from the three points* A B C, *not lying in a streight line.*

1. By problem 3, bisect the distance between any two of the three points, as A and B; again bisect the distance between the third point and either of these, as B and C, and at D where these two bisecting lines meet,

is the center of the circle required, which ſweep with the radius D A, and its done.

PROBLEM LVI.

Given any arch of a circle as A B C, *to find the whole circle.* Fig. 45.

1. Take any three points in the given ſegment, as A B C, and by the laſt problem find D, the center of a circle to paſs thro' theſe three points. 2. With the radius D A = D B = D C, deſcribe the circle.

If the chord A C, and verſed ſine or height D B (fig. 46.) were given, to find the diameter of the whole circle, it is the ſame, for the circle required muſt paſs thro' the three points A B C.

To do this arithmetically.——Let A C = 120, and D B = 36, then, as E D 36 : D P 60 : : D P 60 : D C (fig. 38.) 100, which added to E D 36 gives 136 = E C, for the whole diameter.

PROBLEM LVII.

Given D B *the diameter, and* I B *the verſed ſine of a ſegment* O B O, *to find* O I *half the chord of that ſegment.* Fig. 43.

This is but to find I O a mean proportional between D I and B I, as is done by problem 45. Theſe two problems are of great uſe in meaſuring ſegments of circles, globes or ſpheres.

PROBLEM LVIII.

To find the length of any circular arch as A B C. Fig. 46.

1. Divide the chord A C into four = parts, and ſet one of theſe parts on the arch from C to G. 2. From G to E the third of theſe = parts, draw G E, which is = half the length A B C nearly.

PROBLEM LIX.

To find the length of the aboveſaid arch A B C *nearly by arithmetic.* Fig. 46.

1. Multiply A B the chord of half the ſegment by 8, and from that product take A C the chord of the whole ſegment. 2. One third of the remainder is nearly equal to the length of the arch.

60 = A B.
8

———

480
100 = A C.

———

3)380
126.66 the anſwer.

Note. This method gives the length of the arch ſomewhat too little, and the greater the arch the greater the error. There can be no rule given for finding the length of an arch exactly, till ſome method be found for ſquaring the circle truly; yet this way of finding an arch, will ſerve in common practice.

If you know what degrees are contained in any ſegment's arch, you may find the length of the arch truly by this

RULE. As the periphery of any circle in degrees is to its periphery in equal parts, ſo is any arch in degrees and decimal parts of a degree, to the ſame arch in equal parts.

EXAMPLE. Suppoſe the circumference of a circle to be 71, and the arch to contain $52^\circ\ 15'$, or 52.25°, what is the length of that arch?

As $360^\circ : 71 :: 52.25^\circ : 10.304$ the anſwer.

PROBLEM LX.

From any given circle A O F C, *to take away a ſegment* C F O A, *containing an* ∠ C F A = *to a given* ∠ z. Fig. 47.

1. Draw the line B A L ⊥ to the diameter A F. 2. Make the ∠ C A B = the given ∠ z. drawing A C till it cut the circle in C, and it is done. For the ∠ C A B taken from 90° leaves the ∠ F A C, and F A C taken from 90° leaves the ∠ A F C, (becauſe the ∠ A C F = 90°) conſequently the ∠ C F A = ∠ C A B.

PROBLEM LXI.

Upon a given line B F, *to make a ſegment of a circle* F L O B, *in which the* ∠ F L B *ſhall be* = *to a given* ∠ A B F. Fig. 48.

1. Upon the angular point B, erect the ⊥ B L. 2. Upon F make the ∠ B F I = the compliment of the given ∠, viz. = ∠ F B L, cutting B L in I, ſo is I the center of the ſegment required, which deſcribe, join F L, and it is done.

PROBLEM LXII.

Given any circle A *to find how many circles of equal magnitude will touch it and one another.* Fig. 49.

1. Becauſe the three ſides of an equilateral △ are equal, if with half of any of theſe ſides in your compaſſes, and one foot on each angular point, you ſweep a circle, it is plain theſe three circles will touch one another, as the circles A B C, deſcribed upon the angular points of the equilateral △ A B C; now, becauſe any circle may be divided into ſix equilateral △s, therefore divide the periphery of the given circle A into ſix = parts. 2. Thro' A its center, and each of theſe parts, draw lines A B, A C, &c. each = twice its radius beyond its periphery, which will be the diameters of the circles required, in number ſix.

PROBLEM LXIII.

To find D E *the radius of a circle = to the ſum of three or more given circles, whoſe ſemi-diameters are* A B, A C, *and* A D. Fig. 50.

1. Make a right ∠ D A E. 2. Lay A B from A to B, and A C, from A to C, join B C, ſo is C B the ſemi-diameter of a circle equal to the two given ones A B and A C. 3. Lay B C from A to E, and A D from A to D, join E D and its done. See theorem 13.

PROBLEM LXIV.

To find A B *the ſemidiameter of a circle, = to the difference between two given circles, whoſe ſemidiameters are* A C *and* C B. Fig. 50.

1. Make a right ∠ E A D, in which make A C = A C. 2. With B C in your compaſſes and one foot in C, croſs the ⊥ A E in B; ſo is B A the ſemi-diameter required. See theorem 13.

PROBLEM LXV.

To draw a tangent T P, *to any aſſigned point* P, *in a given curve* A P D. Fig. 51.

1. A tangent to any point in a circle, is at right angles with a diameter, from that point; therefore find C the center of a circle (upon A B the

axis of the given curve) to touch the given point P. 2. Join P C, and ⊥ to P C draw P T, which is the tangent to the point P, both in the curve and circle.

PROBLEM LXVI.

To draw a line C P *perpendicular to any given curve or arch,* A P D. Fig. 51.

This is already done in the laſt problem. For C P is ⊥ to the tangent T P, and conſequently to the curve in the point P.

PROBLEM LXVII.

To make a ſpiral upon a given line E e. Fig. 52.

1. Divide the given line E e into = parts e d, d c, c b, b B, B C, &c. two of which neareſt the center, biſect in the point A. 2. Upon A, with the radiuſes A B, A C, A D, &c. deſcribe the ſeveral ſemicircles above the line e E. 3. With the radiuſes g b, g c, g d, &c. and one foot in g, ſweep the ſemicircles below the line e E, and it is done.

CIRCUMSCRIPTION and INSCRIPTION of FIGURES, in twelve Problems.

DEFINITION. A figure is ſaid to be inſcribed in another figure, when all the angles of the former are in the periphery of the latter. Or, to inſcribe a figure A within a figure B, is to cut the greateſt figure A that can be out of the figure B.

PROBLEM LXVIII.

In a given circle A E B D *to inſcribe a ſquare.* Fig. 53.

1. Draw the diameter D E, and biſect it with the diameter A B. 2. Join the points A E, E B, B D, and D A, with right lines, and it is done.

Note. By this method you may make a ſquare.

PROBLEM LXIX.

About a given circle to circumſcribe a ſquare, a b d e. Fig. 53.

1. Croſs the circle with two diameters D E and A B. 2. Take the radius D C = C A, &c. of the given circle, and one foot in A, D, B, E, ſeverally, deſcribe the pricked arches, croſſing in the points a, b, d, e, which points join with right lines, and its done.

PROBLEM LXX.

Out of a given circle to cut the greateſt rhomboides poſſible, whoſe length may be to its breadth, as the line b *is to the line* a, *and each of its acute* $\angle$*s = to a given* $\angle$ z. Fig. 54.

1. Draw the diameter A C, and produce it if neceſſary. 2. Lay the line a from the center D to C, and make the $\angle$ E C P = the given $\angle$ z, making C P = to the line b. 3. Draw D P cutting the circle in u. 4. Draw u z parallel to C P. 5. Produce u z till z n = z u. 6. Make D y = D z, and with z u in your compaſſes and one foot in y, croſs the circle in a. 7. Join Y a, and produce it till y e = y a, join a n and e u, and it is done. For the Δs D z u and D C P are alike; therefore as a : b : : D C : C P : : D z : z u : : a n : a u, by taking a n = z y = 2 D z, and a u = 2 z u.

PROBLEM LXXI.

Out of a given circle to cut the greateſt rectangle poſſible, whoſe length may be to its breadth as 3 *to* 2, *or* 30 *to* 20. Fig. 55.

1. Draw the diameter A B, produced if neceſſary. 2. Take 30 from a ſcale of = parts and lay it from the center C to a. 3. Upon a, erect the $\perp$ a n, making it = 20 from the ſame ſcale. 4. Draw C n cutting the circle in b. 5. Make A e, B q, and B f each = A b, and join f q, q e, e b, and b f, and its done. For e b being parallel to n a, the Δs n C a, and b C o are alike; and therefore, as in the laſt problem, as 20 : 30 : : n a : a C : : b o : o C : : e b : e q.

PROBLEM LXXII.

Within a triangle A B C, *to inſcribe a circle.* Fig. 56.

1. By problem 8, biſect any two of the $\angle$s, ſuppoſe the $\angle$s B and C, and where the biſecting lines meet, as at D, is the center of the circle.

PROBLEM LXXIII.

Given D R *the radius of a circle, or the circle itself* L G F, *to be cut into a triangle, whose sides may be in a given ratio, as the lines* a, b, c. Fig. 57.

1. With the three given lines a, b, c, or any other three lines in the same ratio, make a △, a b d; and by problem 55, find E the center of its circumscribing circle. 2. Upon E, with the given radius D R, describe the circle, and thro' the angular points a, b, d, draw the semidiameters E a, E b, and E d, which join with right lines, and you'll have F L G for the △ required.

Note. By this method, if any figure be inscribed in a circle, you may inscribe another similar figure in any other circle, whether greater or less, by describing one circle upon the center of another, and drawing semi-diameters thro' every ∠, &c.

PROBLEM LXXIV.

To inscribe in a circle F L G, *a* △ F G L, *having* = ∠*s with a given* △, a, b, d. Fig. 57.

1. Draw the line G A touching the circle in any point G. 2. Upon G, make the ∠ F G A = ∠ a b d, or any ∠ of the given △ drawing G F till it cut the circle in F. 2. Upon F make the ∠ G F L = ∠ b a d, or any other of the given ∠s; join G L, and G L F is the △ required. 2. E. 4.

PROBLEM LXXV.

To circumscribe about a given circle F G H, *a* △ L m n, *having* = ∠*s with those in a given* △ A B C. Fig. 58.

1. Extend the side A C, making the two external ∠s D C B and E A B. 2. At E the center of the circle, make the ∠ F E H = ∠ D C B, and the ∠ H E G = ∠ E A B. 3. Join these points F, G, H, with right lines to touch the circle, and you'll have L m n for the triangle required. 3. E. 4. These two problems may be done also by problem 73.

PROBLEM LXXVI.

To inscribe and circumscribe a circle about any regular figure, suppose a hexagon. Fig. 59.

1. Bisect any two sides E E and E D, of the given polygon, and at C, where these bisection lines meet, is the center of the polygon. 2. From C draw the line C E, so is C d the radius of the inscribed circle, and C E that of the circumscribing circle. 41. E. 4.

PROBLEM LXXVII.

To inscribe and circumscribe about any circle any regular figure, suppose a hexagon. Fig. 59.

1. By problem 27, in the given circle inscribe the figure. 2. Bisect the sides of this inscribed figure with the lines C d, C d, &c. and where these bisecting lines touch the circle's periphery as in e, e, &c. draw lines parallel to E E, E D, &c, which will cut one another, and form the figure required.

PROBLEM LXXVIII.

In a given circle H I K, *to describe any number of equal circles, suppose three.* Fig. 60.

1. Divide the given circle's periphery into as many = parts as are the number of inscribed circles, H, I, K, and from the center C, draw lines C H, C I, &c. to each of these parts. 2. Draw a line I K between any two of these = parts. 3. On C K produced, lay K L = $\frac{1}{2}$ I K. 4. Parallel to I L (having joined I L) draw K F, cutting C I in F, so is F I the radius of one of the circles required, = E H = G K. Otherwise, by problem 73, make a regular polygon of as many sides as you want circles, upon each ∠ of this polygon sweep an = circle to touch each other, and the center of the polygon will be the center of the circumscribed circle; then it will be, as the radius of this circle is to the radius of one of its inscribed ones, so is the given circle's radius, to the radius of one of the required ones.

PROBLEM LXXIX.

About a □ A B C D, *to describe a* △ e f g, *having* = ∠*s with a given* △ E F G. Fig. 61.

1. Upon any side A B of the given □, make an ∠ A B f = ∠ G, and the ∠ B A f = ∠ E, continuing f B and f A till they meet C D produced in g and e, and it is done; for A B is parallel to C D, the ∠ g = ∠ f B A = G, and the ∠ e = ∠ f A B = ∠ E, &c.

PROBLEM LXXX.

About any △ A B C *to describe a* □. Fig. 62.

1. Produce A B, or any side of the given △, and from the third angular point C, (by problem 4) let fall the ⊥ C D, making D E = A D. 2. With the radius A D or D E and one foot on A and E severally, describe two arches crossing each other in F, join E F and A F, so is A D E F the □ required.

PROBLEM LXXXI.

Within any △ E P F, *to inscribe a* □ L L S K. Fig. 63.

1. By problem 6, let fall the ⊥ P O. 2. Parallel to P O draw F Q, making it = the base E F. 3. Join O Q cutting P F in S, from S draw S K parallel to P O, and S L parallel to E F, also, L L parallel to P O, and it is done. For by similar △s, as F O : E F (F Q) : : K O (e S) : S K, and, F O : F E : : e S : L S, therefore, K S = S L, consequently, K L L S (being a rectangle) is a □.

PROBLEM LXXXII.

To augment a □ A B C D *to any proportion, suppose to an octuple proportion.* Fig. 64.

1. Produce A B and A D, any two sides of the given □, as also the diagonal A C. 2. Make A L and A H each = A C, and complete the □ A H E L, which is double the □ given A B C D. 3. Make A M and A I each = A E, and complete the □ A I F M, which is double the □ A H E L, or = 4 times □ A B C D. 4. Make A K and A N each = A F, and complete the □ A K G N, and it is done. For, by theorem 13, the □ A K G N is 8 times the □ A B C D. To diminish a □ is but just the converse work, as is evident by the figure. What is said here of squares, will also serve in all like figures, as is plain by theorem 36.

PROBLEM LXXXIII.

Within any △ A B C, *to inscribe a rectangle, whose length may be to its breadth as* E *to* F. Fig. 65.

1. By problem 43, make it as E : C A : : F : C F, then make C Q = C F and parallel to the ⊥ B O. 2. Join O Q cutting C B in a, draw a d and b c, each parallel to B O, and join a b, so is a b c d the rec-

tangle required; for, (by similar triangles, as in the last problem) as O C : O d : : C F (C Q) : d a, and as O C : O d (a e) : : A C : b a, so by equality of ratios, C F : A C : : d a : b a; but by construction, as E : F : : A C : C F, therefore, as E : F : : b a : da. Q. E. D.

PROBLEM LXXXIV.

Required the greatest rectangle that can be cut out of any given △ A B C. Fig. 65.

1. Bisect B O the ⊥ with the line a b. 2. Parallel to B O draw a d and b c, so is a b c d the rectangle required. By theorem 142.

PROBLEM LXXXV.

Within any regular polygon A q n o m, *to inscribe an equilateral* △ A B D. Fig. 66.

1. Thro' A, (or any ∠ of the polygon,) and C the center of the circumscribing circle, draw the line A C G. 2. With the radius C A, upon A, describe the arch C E, whose middle is at F, so draw A F, cutting the side of the polygon in B. 3. With A B in your compasses, and one foot on A, cross q n in D, join A D and D B, and it is done; for join A E which will be the side of a hexagon, also = A C by construction, now ∠ C A E, the ∠ of a hexagon is double the ∠ C A F, that of an equilateral △, and by construction, arch C F = F E, Q, E, I, or, the ∠ C A E being = 60°, the ∠ C A F = ∠ C A D will be = 30°, be the polygon what it will.

PROBLEM LXXXVI.

In a given rectangle A B E F, *to inscribe a* △ A P B, *whose two sides* A P *and* P B *may be as* a *to* e. Fig. 67.

1. By problem 41, make it as a : e : : A C : C B, (i. e. lay a from A to a, and e from a to e, join e B, and thro' a draw a C parallel to e B) also make it as A C — B C : B C : : A C : C e. 2. With the radius C e describe the semicircle C R D, and draw the radius O R ⊥ to C D, on which lay A E = B F from O to n. 3. Thro' n and parallel to C D, draw the chord P P, cutting the semicircle in two points P and P, (which shews that the problem admits of two different solutions) join P A and P B, and it is done. See theorem 20.

REDUCTION of FIGURES

Of one form to another, keeping their areas or magnitudes equal. In ſeven problems.

PROBLEM LXXXVII.

To reduce a rectangle A B C D *to a* □ C E F G. Fig. 68.

If you conſider what two things multiplied together will give the area of any figure, then a mean geometrical proportion between theſe two things is the ſide of a □ = in area to that figure; ſo produce C D a ſide of the given rectangle, till C L = A D, or C B its breadth, upon L D as a diameter, deſcribe a ſemicircle cutting C B produced in G, ſo is C G (by problem 45,) a ſide of the □ required. In this manner you may reduce any number of rectangles to a □, for it is but reducing every ſingle ▭ &c. to a □, and then adding all theſe □s together, as taught in problem 63.

PROBLEM LXXXVIII.

To reduce any trapezia A B C D *to a* △ A B C. Fig. 69.

1. Draw the diagonal A C, and parallel thereunto draw B E, cutting D C produced in E, join A E, and it is done: for the △s B A C and E A C both ſtand upon A C, and have each an ∠ in the parallel B E, and ſo (by theorem 31) are equal.

PROBLEM LXXXIX.

From a given point P *in any right lined figure* A B C D E, *to reduce it to a* △ P H G. Fig. 70. *See the laſt problem.*

1. Join B D, and parallel thereunto thro' C draw a e, and join a B, ſo is the △ B C D reduced to the △ B a D, for they both ſtand upon B D, and have each an ∠ in the parallel a e. 2. Join P B and parallel to it draw a H, ſo is the △ B P A = △ P H B, for they both ſtand upon B P, and have each an ∠ in the parallel a H; whence the △ P H B is = the ſpace O D a P. 3. Join A P, and draw E G parallel to

A P, then join E G, so is the △ P G A = △ P E A, for they both stand upon A P, and have each an ∠ in the parallel E G, so P H G is the △ required.

PROBLEM XC.

Given any number of different △s, suppose two A B C *and* C D E, *to make one* △ N F O = *to them all, whose* ⊥ *or height* F m *shall be* = *to a given line* a b. Fig. 71.

1. On the same line N E, join A C and C E, or any two sides of the given △s. 2. By problem 1, at the distance of a b draw G Q parallel to N E, cutting the sides B A and B C, of the △ A B C, in I and F. 3. Join F A, and parallel thereunto draw B N, cutting C A extended in N, so is the △ F N C = △ A B C, (having joined F N). 4. Produce C D till it cut F Q in Q, join Q E, and parallel thereunto draw D O, cutting A E in O, then join O Q, so is the △ O Q C = △ E D C. 5 Join Q N or F O, and it is done. For draw the ⊥s B n and F m, now because B N is parallel to F A, the △s N B C and A F C are similar (having the same ∠ at C;) therefore, as B N : N C : : F m : A C, ergo, N C × F m = A C × B n, i. e. △ A B C = △ N F C; after the same manner it may be proved, that the △ C D E = △ C Q O, also, (by theorem 31.) = △ F O C.

PROBLEM XCI.

To find the sum and difference of any right lined figure A B C D E. Fig. 70. *and a given parallelogram* A B C D. Fig. 72.

1, By problem 89, the irregular figure A B C D E, is reduced to the △ G P H, whose base G H, bisect with the line Q M, and draw R G parallel to Q M. 2. Thro P and parallel to G H, draw R T, so is the △ G P H, reduced to the rectangle G B Q M. 3. Make the ∠ T M N = ∠ B or C of the given ▭ A B C D, and parallel to M T draw G S, so is the rectangle G R Q M = ▭ S G M T, extend G H on both sides, making M N = A B, or C D, and make M U = A C or B D, thro' U draw F Z parallel to G H, draw Z N parallel to U M, so is the ▭ M U Z N = the given one A B C D, then (by problem 90,) draw U G, and parallel thereto draw T L, cutting H G extended in L, parallel to U M draw L F; lastly, lay A B from L to w and from F to

t, and its done. For, (by problem 90,) ▭ L F U M = ▭ G S T M, therefore the ſum required is the ▭ L F Z N, and the difference is the ▭ F t w L; whence, (by problem 8,) you may reduce any irregular plot to a △; and (by problem 90,) any number of △s or ▭s to one △ or ▭, and by this problem find the ſum of ſuch ∠s and ▭s, and ſo perform menſurations geometrically.

PROBLEM XCII.

To reduce any plot A B C D E, *according to any proportion, as* a *to* b.

Fig. 73.

1. By problem 50, take D E, or any of the ſides, and make it as b + a : b : : □ D E : □ e E, and draw the diagonals E C, E B, &c. 2. Thro' e and parallel to D C, draw e f, cutting E C in f, parallel to B C and B A draw f g and g h, and its done. For by theorem 15, as a : b : : A B C D E : e f g h E.

PROBLEM XCIII.

To enlarge a plot e f g h E *according to any proportion, as* b *to* a.

Fig. 73.

1. By problem 51, enlarge any ſide E e in power as b to a, i. e. make it as b : b + a : : □ E e : E D, draw the diagonals E f, E g, &c. at large, 2. Thro' D and parallel to e f draw D C, cutting E f in C, thro' C and parallel to f g draw C B, cutting E g in B, &c. and its done. For as b : a : : plot e f g h E : plot A B C D E, by theorem 16.

DIVISION of FIGURES or PLOTS.

In twenty-six PROBLEMS.

This depends chiefly on Theorems 14, 15, 16, 31.

PROBLEM XCIV.

To divide a circle E F D G *into any proportion, suppose as the line* D e *is to the line* e A, *by another circle* a d e b *described upon* C *the same center.* Fig. 74.

1. Having drawn the diameters D E and F G at right angles to each other, then (by problem 41.) divide the semi-diameter C D, as D e : e A : : D n : n C, and upon n E as a diameter, describe the semi-circle n a E, cutting F G in a. 2. Upon C with the radius C a, describe the circle a d e b, and its done; for, as D e : e A : : the area of the ring D d F a e b G E : to the area of the circle a d e b; for D e + e a is = or proportional to the area of the circle E F D G, and e A to the circle a d e b by construction, so by theorem 16, as D e + e A : □ D C, or □ D E : : e A : □ d C or □ d b, for C a is a mean proportional between C D and C n, or = C D divided, as by problem 50, for let a = C D = C e, and e = C n, figure 74, then : a + e : × e = (E.C × C n) a e + e e = □ a C; also, let a = D e, and e = E e, figure 73, then a × e = (D e × e E) a e = □ e e, and a e + a a = (□ e e + □ e E) □ E a e, whence it is the same whether you find a mean proportional between a whole line C D and C n one of its segments, or divide the said whole line in power, as by problem 50.

PROBLEM XCV.

If E F D G *be a round table, whose diameter* E D *or* F G, *is* 100 *inches, let it be required to divide it into two unequal parts,* O E G O *and* O F O, *as* 10 *to* 1, *by a chord line* O O. Fig. 74.

1. This is easiest done by the table of areas of segments, thus, the diameter of the circle being 100, its area will be 7854, then as 11

(viz. 10 + 1) : 1 : : 7854 : 714, the area of O F O; but the greatest area in the table of segments is 0,7854, (viz. the area of a circle whose diameter is unity) the 11th part of which is ,0714, the nearest to which in the table under segment area, is ,071741, and the versed sine against this ,071741 is 0,147; then say as 1 : 0,147 : : 100 : 14,7 = F z, the height or versed sine of O F O, which taken from F G 100, leaves G z 85,3 the versed sine of O G O, as required.

PROBLEM XCVI.

To divide a □ A G f e *into any possible proportion, as* a *to* b, *by a line drawn from any point, as at* P. Fig. 75.

1. It is plain by the figure, that the line of division must cut the sides e A and f G, so thro' g and n the middles of the other two sides, draw the line n g, which must divide the given □ A G f e into 2 = parts, which, by problem 41, make as a : b : : n L : L g. 2. Thro' L and the given point P draw U L P, and it will be as a : b : : U A E G P : f e U P as required. For if t w be parallel to e f or A G, then because as a : b : : n L : L G; and all □s or △s standing upon the same or equal bases e f t w, A G, are as their heights L g and L n, therefore, as a : b : : n L : L G : : □ A G w t : □ w f e t; and because L t = L w, and e A, g n and f G are all parallel, it follows that △ L P w = △ L u t, consequently the line u P may turn upon L as a center, and will still keep the □ A G f e divided in the same proportion as long as it cuts the sides e A and G f, but if it vary so as to cut a side G f and an end A G, the proportion cannot hold; for then the △ L P w will not be = △ L u t, and this you are to understand in the following problems where mention is made of possible proportion.

PROBLEM XCVII.

To divide a plot A B C D E, *having two opposite sides* A B *and* C D *parallel, into any possible proportion as* a *to* b, *by a line drawn from a point* P *any where given, and passing thro' these two parallel sides.* Fig. 75.

1. By problem 89, the figure is reduced to the trapezia G A B D, thro' the middle of whose sides G A and B D, draw the line n g, which by problem 41, make as a : b : : n L : L g. 2. Thro' L and the given

point P, draw the line U L P, and its done. For, as a : b : : U A E Ç P (=U A n G P) : D B U P(=U P f e :) for draw f e parallel to A G, so the ∠s B g e and D g f are equal, as by the last problem.

PROBLEM XCVIII.

To divide a △ A B C *into any proportion as* a *to* b, *by a line drawn from an* ∠ a *to the opposite side* B C. Fig. 76.

1. By problem 41, make it as a : b : : C D : D B, join A D, and it is done. For the △s C A D and C A B and D A B, all standing on one line B C, and meeting in one point A, have the same height, and so, by therem 31, are as these lines on which they stand, i. e. as a : b : △ C A D ; △ D A B.

PROBLEM XCIX.

To divide a plot A B C D *into any proportion that is possible as* a : b, *by a line* n A *drawn from an* ∠ A *to its opposite side* D C. Fig. 77.

1. From the ∠ A, by problem 89, reduce the plot to a △ A E D, whose base D E divide, by problem 4, as a : b : : E n to n D. 2. Join n A, and its done. For, by the last problem, as A : b : : △ A n E (= A n C B) : △ A n D.

PROBLEM C.

To divide a △ A B C *into any proportion as* a : b, *by a line drawn from a given point* E *in one of its sides* B C. Fig. 78.

1. From the given point E to its opposite ∠ A, draw the line E A, then, by problem 41, divide B C, i. e. as a : b : : B D : D C. 2. Thro' D and parallel to E A, draw D N, join E N, and it is done, i. e. as B E N A : E N C, : : a to B. For join A D, and it will be, by problem 98, as a : b : : △ B A D : △ C A D; and by the figure, △ D A N + △ N D C = △ D A C; also, △ C D N + △ N D E, but △ D A N = △ E D N, for they both stand upon N D, and have each an ∠ (A and E) in the parallel A E, consequently △ E N C = ∠ D A C.

See this done by a different method, by problem 104.

PROBLEM CI.

From any assigned point P *in the side of any plot or right-lined figure* A B C D E, *to divide the same into any proportion that is possible, as* a *is to* b. Fig. 79.

1. By problem 89, the figure is reduced to the △ G P H, whose base G H, make, by problem 41, as a : b : : HS : S G. 2. Join PS, and it is done. That is, as a : b : : SPDCB (SPH) : PEAS(PGS) by problem 98. For, △ P E = △ P A G, by problem 88 and by the figure, △ P G A + △ P A S = P G S = P E A S. Q. E. D.

PROBLEM CII.

To divide a ∠ A B C *into any proportion as* a *to* b, *by a line* F u, *making given* ∠s B F U *and* B U F, *with the sides of the* △ *which it cuts, or (which is the same thing) to be parallel to a line* D *any where given.* Fig. 80.

1. By problem 41, make one of the sides, which the divisional line FU is to cut, as a to b, viz. as a : b : : B g : g C, join A g, and thro' A parallel to D draw A e; then, by problem 50, make it as e B : □ e B : : B g : □ B F, viz. B f, as by the figure. Lastly, thro' F and parallel to D or e A, draw F u, and it is done. That is, as a : b : : B U F : U A C F. For the △s B A G and B A e, having the same height, are as their bases B g and e B, and by the problem △ B u F must be = △ B A g; therefore, by theorem 15, as e B (△ e B A) : □ e B : : B g (△ B g A) = △ B U F) : □ B F. Q. E. D.

Note. If the line of division F U were to be the shortest possible, (a : b continuing the same) then, by problem 15, it must cut the sides at = ∠s, viz. ∠ B F U = ∠ B U F; also, if F U were to be parallel to any side A C, it is but drawing A e, so as to make ∠ B e A = ∠ B C A, viz. A e would become A C.

PROBLEM CIII.

To divide a plot A B C D E *into any proportion that is possible, as* a *to* b, *by a line* z d *drawn parallel to any given line* R. Fig. 81.

1. It appears by the figure that the line z d of separation must cut the sides A E and C D, therefore produce those sides till they meet in H, and then the figure reduced to a △ is G A F, whose base G F, make as a : b : : G a to F a, join A a, and draw A Q parallel to the given line R. 2. Make it, (by problem 50,) as H Q : □ H Q : : H a : □ H d, so thro' d and parallel to R, or A Q draw z d, and it is done. That is, as a : b : : z A B C d : d D E z. For, as in the last problem, the △s H a A and H Q A, are as their bases H a and H Q; and z d is parallel to A Q. Therefore, by theorsm 15, as H Q : □ H Q : : H a (△ H a A = △ H D Z) : □ H d. In the same manner, you may divide a plot by the shortest line possible, or by a line drawn parallel to one of its sides. See the last problem.

PROBLEM CIV.

To divide a △ A B C *into any proportion as* a *to* b, *by a line* P M *drawn from a point* P *without the* △. Fig. 82.

1. Thro' the given point P, and parallel to A C draw P G, cutting B A extended in G. 2. By problem 41, make it as a : b : : A H : H B; also, by problem 43, make it as G P : A C : : A H : H I. 3. By problem 45, find A L a mean proportional between A G and A I, which A L lay from H to M, join P M and its done. That is, as a : b : : A M D : M D C B. For, by problem 100, figure 78, it is evident, that, as C E : C D : : C A : C N, so by joining N E that problem is done; whence, if the point R were in the line A C, as at D, it would be as A D : A C : : A H : a fourth term, which laid from A towards B, will give the point required. But, P is the given point, and the △s A M D and G M P are similar, because of their two parallel sides A D and G P; so, by theorem 15, as △ G M P : △ A M D : : □ G M : □ A M. Q. E. D.

PROBLEM CV.

To divide a plot, A B C D E F, *according to any proportion that is possible, as* a *to* b, *by a line* S P, *drawn from a given point without the plot, as at* P. Fig. 83.

1. It appears by the figure, that the divisional line must cut the sides A F and C D, so produce these sides till they meet in K, then from the ∠ C opposite to the given point P, the figure, by problem 89, is reduced to the △ M C N, whose base M N, make as a : b : : M O : O N. 2. Thro' P, and parallel to M N, draw P L, cutting C D extended in L, then make it, as L P : K O : : K C : K X; find K q a mean proportional between L K and K X. 3. Always divide or find R the middle between K and X, and join R q, which R q set from R to S, and draw S P, so it will be as a : b : : A B C S P : P S D E F as required. See the last problem.

PROBLEM CVI.

To divide a △ A B C *into any proportion, as a to* b, *by a line* R Q *drawn from a given point within the* △ *as* P. Fig. 84.

1. The dividing line being to cut A C and B C, divide either of these lines, as A C, by problem 41, thus, as a : b : : A a : a C, and thro' the given point P, draw M L parallel to A C, and upon A C let fall the ⊥s U P and e B, then make it as U P : e B : : $\frac{1}{2}$ C a : C N. 2. Thro' N and parallel to B C draw N M meeting L M in M, from the □ of M P take the □ of L P, and a side of that □ which remains is = Q N, so thro Q and P draw the line of separation Q P R, and it will be as a : b : : A Q B R : C R Q. For by construction, U P : e B : : $\frac{1}{2}$ C a : C N, ergo U P × N C = e B × $\frac{1}{2}$ a C, i. e. △ N L C = $\frac{1}{2}$ △ C Q R = Q R L N; so that the line Q R is to pass thro' P in such a manner as to make the space Q R L N = △ N L C, &c.

Note. P may be so far within the △ and the proportion so unequal, that this, and problem 119 may each be impossible.

PROBLEM CVII.

To divide a plot A B C D E F *into any proportion that is possible, as* a : b, *by a line* W U *drawn thro' a given point* P, *within the plot.* Fig. 85.

1. It appears that the line of division will cut the sides A F and B C, so produce these two sides till they meet in K. 2. By problem 89, the plot is reduced to the △ L C M; whose base L M make as a : b : : L N : N M, then, as a ⊥ from P to A F : a ⊥ from C to L M : : $\frac{1}{2}$ K N : K O. 3, Thro' O, and parallel to K C draw R O, also thro' P and parallel to A F draw q R meeting R O in R. 4. From the □ of P R take the □ of q P, and a side of that □ which remains is = O U, so thro' U and P draw the line U P W, and its done; that is, as a : b : : A B W U : U W C D E F. See the last problem.

PROBLEM CVIII.

To divide a □ A B C D *into four right angled* △*s and a* □ *in the middle, whose areas may be all equal, viz. each = a fifth part of the given* □. Fig. 86.

1. Make D E = C F, each = two fifths of A D a side of the given □, and draw C F which will be parallel to D C and A B. 2. Upon C D as a diameter describe the semicircle cutting E F in G, join G D and G C, so is G D C one of the △s required. The other three △s may be found in the same manner; or thus, which is easier, draw B G ⊥ to G C and A H ⊥ to B I, and G D produced will cut A H perpendicularly in L, so is D G C, C I B, B H A, and A L D the four △s, and G I H L the □ as required. For G P being = $\frac{2}{5}$ D A, it is plain, by theorem 24, the △ D G C is = $\frac{1}{5}$ of the □ A B C D, which with the other three = △s, make $\frac{4}{5}$ of the said □, so consequently there must remain $\frac{1}{5}$ for the little □ G I H L. Q. E. D.

PROBLEM CIX.

To divide a right angled △ A B C *into four right angled ones* C D F, B F D, B D E, *and* A D E, = *to each other, and similar to the given one* A B C. Fig. 87.

1. From D the middle of the hypothenuse draw D B, also D E and D F, parallel to B C and A B, and it is done. For, because D E is parallel to B C, and D F to A B, and D C = D A, therefore, E A = E B = D F, and F B = F C = E D. Q. E. D.

PROBLEM CX.

To find a point O *in a given* △ A B C, *from which if three lines be drawn to the three* △*s* A, B, C, *they shall divide the whole* △ A B C *into three other* △*s* A O B, B O C, *and* C O A, *whose areas shall be as* m, n, *and* u *respectively.* Fig. 88.

1. In A C and A B produced if neceffary, make C E and A F each = n + n + u, joining E B and C F, make C e = m, A d = n, and draw e b and d f parallel to E B and C F. 2. Thro' b and f draw b Q and f P parallel to A C and A B, and at O where thefe two lines interfect each other is the point required. For draw b H and B D ⊥s to A C, then the △s C B E and C b e, as alfo C B D and C b H are fimilar; therefore, it will be as m (C e) : m + n X u (C E) :: C e : C B :: b H : B D : △ A O C : △ A B C, by theorem 31, for the △s A O C and A B C have one bafe A C; in the fame manner it may be proved, that as n : m + n + u :: △ A O B : △ A B C, confequently, as u : m + n + u :: △ B O C : △ A B C. Q. E. D.

PROBLEM CXI.

By a line from an ∠ A *in a given* △ A B C, *to cut off a part towards the* ∠ B *that shall be = to a given* □, *whose side is* a b. Fig. 89.

1. Take A h = ½ the neareft diftance between the ∠ A and its oppofite fide B C, (viz. ½ a ⊥,) and by problem 42, make it as A h (C b) : a b (C d) :: a b : d q, which d q lay from B to P, join A P, and it is done. For the △ A P B is = the area of a □ whofe fide is = a b, becaufe, by conftruction, A h (½ A d) × B P = a b × a b. This may be done differently by the following problem.

PROBLEM CXII.

By a line A G *drawn from a given* ∠ A, *in a plot* A B C D E, *to cut off a part towards the* ∠ B = *to a given rectangle* a b a b. Fig. 90.

1. If the line of divifion cut the fide C D, then extend C D and A E till they meet, but the △ A B C is greater than the □ a b a b, fo A G

must cut B C, whence, if by problem 87, you reduce the rectangle a b a b to a □, the work will be the very same as in the last problem. Or thus, thro' A and parallel to B C draw L S, and upon C B raise the ⊥ B L, then, by problem 90, make the rectangle B L S O = to the given one a b a b, make O G = O B and draw A G, so is the △ A B G = □ B L S O = □ a b a b as required; for □ B L S O and △ A B G have the same height B L, and B G the base of the △ is double B O the base of the rectangle. Q. E. D.

PROBLEM CXIII.

From a given point P *in the side of a* △ A B C *to cut off a part* B G P = *to a given rectangle* b c f e. Fig. 91.

1. Thro' P and parallel to its opposite side A B draw L S. 2. Upon B raise the ⊥ B L meeting S L in L. 3. Upon B L, by problem 90, make the rectangle B L S O = the given rectangle b c f e, or make it as B L : e b : : b c : L S = B O, which B O lay from O to G, join P G and it is done, as is evident by the last problem.

PROBLEM CXIV.

To cut off from a plot A B C D E F *a part* A B P G F, = *to a given rectangle* a b c d, *by a line* P G *passing thro' a given point* P *in one of its sides* B C Fig. 92.

1. Because the dividing line is to cut E F and B C, produce these sides till they meet which is at H. 2. By problem 91, add the figure H B A F to the given rectangle a b c d, and you'll have the □ b e f c, to be cut off from the △ H C E thro' P, so work as in the last problem, and you'll get A B P G F = the given □ a b c d, or the △ H P G = the □ b e f c.

PROBLEM CXV.

To cut off from a △ A B C *a part* A n P = *to a given* □ A b c D, *by a line* n P *making an* ∠ n P A *with the side* A C *of the* △ *which it cuts* = *to a given* ∠ E b A. Fig. 93.

1. From E upon A C let fall the ⊥ E F, and make b Q = 2 E F. 2. Upon A Q as a diameter, describe the semicircle cutting c b in m, thro' m and parallel to A C draw m n meeting A B in n. Lastly, thro' n and parallel to E b draw n P and it is done. For the △s A E b and A n P are similar, because E b in parallel to n P, and ∠ A common; so as E F : A B : : n S (m b) : A P. But by problem 45, □ m b = A b × b Q = A b × 2 E F, by construction; whence, as E F : : A b

$$: : \sqrt{Ab \times 2EF} \; . . \; \frac{Ab \times \sqrt{:} \, Ab \times 2EF :}{EF} = AP, \text{ and } AP \times \tfrac{1}{2}$$

$$nS = AP \times \tfrac{1}{2} \sqrt{:} \, Ab \times 2EF = \frac{Ab \times \sqrt{:} \, AB \times 2EF :}{EF} \times \tfrac{1}{2} \sqrt{:}$$

$$Ab \times 2\,EF : = \frac{Ab \times Ab \times 2EF}{2EF} = \square\, Ab = \text{area } \triangle\, AnP, \text{ which}$$

is also = the given □ A b c D. Q. E. D.

PROBLEM CXVI.

From a given plot z T B D C N, *to cut off a part* n T z N P = *to a given rectangle* A e u b, *by a line* z P *making given* ∠*s with the sides* C N *and* B F, *viz.* ∠ N P n = ∠ E b F. Fig. 94.

1. Let the sides B T and C N, by which the dividing line is terminated, be produced till they meet in A; then it is plain, that if to the given rectangle A e u b, by problem 91, you add the space A T z N, and of that sum make a □ A b c D, the work will be the very same as in the last problem, and the directions for it will serve for this also.

Note. In this manner you may cut off a part by a line drawn parallel to any of the sides suppose D B, it is out to draw E b parallel to D B, &c. or also by the shortest line possible. This is but to make A E = A b and then join G B, and let fall the ⊥ E F, &c.

PROBLEM CXVII.

To cut off from a △ B C, *a part* E A D = *to a given* □ *whose side is* a b, *by a line* P D *drawn from a point* P *without the* △. Fig. 95.

1. Thro' P and parallel to A B, a side of the given Δ opposite to P, draw Q P meeting the side A C in R, draw A Q $\perp$ to A B, and upon it make the $\square$ A Q S O = the given $\square$. 2. With the radius A G = 2 A O — R P, and one foot on A, describe the arch G, with the radius O A and one foot on O, cross it with another arch in G, draw A G produced to F, making G F = R P, join F O, and make O D = F O; thro' D and P draw D P and its done. For, draw R H parallel to E D, upon O with the radius O A describe the circle, and extend F O to m, making the diameter n O m. Then, by construction, G F = R P, A F = A T and F O = O D, whence A D = m F and D T = n F, and, by theorem 11, A D $\times$ D T = m F $\times$ n F = A F $\times$ F G = A T $\times$ R P; whence, A D $\times$ D T = A T $\times$ D H, because D H = R P, (for D H is parallel to R P, and R H to P D) now if to each of these = $\square$s be added the $\square$ A D $\times$ A T we shall have $AD \times \overline{DT + AT}$ $= AT \times \overline{DH + AD}$, or $\square$ A D = A T $\times$ A H; now, let b = R r = A Q = O S, a = A O = O T, r = R P = D H, then because the Δs E A D and R A D are similar, it will be as 2 a + r (A H) : b (R r) : : $\sqrt{:4aa+2ar}:$ ($\sqrt{:AT \times AH}$) : $\dfrac{b\sqrt{:4aa+2ar:}}{2a+r}$ = OE,

whence $\frac{1}{2}$ O E $\times$ A D = $\dfrac{b\sqrt{:4aa+2ra:}}{4a+2r} \times \sqrt{:4aa+2ar:}$

$= \dfrac{b \times :4aa+2ar}{4a+2r}$ = b a = area Δ A D E, which is also = area $\square$ A Q S O. Q. E. D.

PROBLEM CXVIII.

To cut off from a given plot I H L C E B, *a part* L E D I H *= to a given rectangle* A B a O, *by a line* E D *drawn from a given point* P *without the figure.* Fig. 96.

1. Becauſe the dividing line ſeems to cut the ſides C L and B I, produce thoſe ſides till they meet in A. 2. Thro' P and parallel to A B draw R Q meeting the ⊥ A Q in Q and the ſide A C in R. Now it is plain by the figure, that if to the given rectangle A B a O you add the ſpace A L H I, and of that ſum upon A Q make the rectangle A Q S O, the work will be the ſame as in the laſt problem, and performed by the ſame directions and letters.

PROBLEM CXIX.

From a given point P *within a* △ A B C, *to cut off a part* A E D, = *to a given rectangle* A Q S O. Fig. 97.

1. Thro' the aſſigned point P and parallel to A B, draw a line Q S meeting A C in R. 2. Upon A B erect the ⊥ A Q meeting S Q in Q. 3. Upon A Q make the □ A Q S O = to the given one A Q S O. 4. With the radius A G = A O + 2 R P and one foot on A deſcribe the arch G, and with the radius O A upon O croſs it with another arch in G, join A G, make A F = A O, and thro' O and F draw O F, make O D = F O, ſo thro' D and P draw D E and its done. For, upon O, with the radius O A deſcribe the circle, and extend F O till it become the diameter n o m, parallel to E D thro' R draw R H, ſo is H D = R P, becauſe Q S is parallel to A B; alſo A D = m F, becauſe A F = A O, F G = 2 R P and D O = F O by conſtruction, whence, D T = n F. Now, by theorem 10, A F × F G = n F × m F = A D × D T = A O × 2 R P = (2 A O) T A × R P, whence A D × D T = A T × H D the very ſame as in problem 117, this being the ſame ſtep in the demonſtration there.

Alſo, as problem 118 is done by problem 117, it will be as eaſy by this problem to cut off a part from a plot by a line drawn thro' a point within the plot, &c.

N. B. Theſe problems for cutting areas, and the foregoing ones for dividing areas in proportion, may both be wrought one way; for example, if it were required to divide 54 in ſuch proportion as 5 to 1, firſt, 5 + 1 = 6, then ſay, as 6 : 54 : : 1 : 9 or 5 : 45, ſo that to divide 54 as 5 to 1, is the ſame as to cut 54 into 2 parts, 45 the greater and 9 the leſs, be the figure what it will. Alſo if you would cut 9 acres

from some figure or plot whose area is 54 acres and choose to do it by proportion, then $54 - 9 = 45$, so that proportion will be as $9 : 45$ or lower as 1 to 5.

Thus have I explained two ways of performing those difficult problems; but the easiest and most practicable method for dividing, &c. such surfaces, you'll find in surveying, yet the foregoing methods are geometrically true, and may serve as good excercise for young students in geometry.

PLANE TRIGONOMETRY,

In Nine PROBLEMS.

Shewing the Rules, Axioms, &c. with the Geometrical, Instrumental, and Logarithmetical Solutions of all the Cases of Right and Oblique Triangles.

PROBLEM CXX.

DEFINITIONS.

1. Trigonometry implies the measuring of $\triangle$s, and is here understood to point out the relation or proportion that is between the sides and $\angle$s of a $\triangle$, from the respective data in the various cases of right and oblique angled $\triangle$s.

2. Every $\triangle$ consists of 6 parts, viz. 3 sides and 3 $\angle$s, any three of which, if one side be known, the rest may be found; but from the 3 $\angle$s only no determined $\triangle$ can be obtained, for the problem admits of infinite answers, since any side may be supposed at pleasure, and with it and the $\angle$s may sides be found corresponding; the same reasoning holds

when only two of the ſix parts are given. In taking heights, diſtances, &c. your only buſineſs will be to ſecure three parts of a △, and one of them muſt be a ſide, to find the reſt, or elſe all your labour will be to no purpoſe.

3. A diagonal ſcale is a ſcale of = parts diagonally divided, from which larger numbers may be taken than from a ſcale of = parts only. Thoſe ſcales are for meaſuring and laying down right lines, as the ſides of △s, &c. but the ∠s are meaſured and laid down from a ſcale of chords.

4. All the parts of the ſame figure whether ſides, ⊥s, baſes or diagonals, muſt be taken from the ſame ſcale of = parts, otherwiſe your work will be faulty; but an ∠ may be meaſured from any ſcale of chords.

5. If your numbers to lay down be too large or too ſmall for your diagonal ſcale, they may be diminiſhed by diviſion or augmented by multiplication thus, 25205 divided by 100 makes 252.05, ſo you may take 252 and a little more from your diagonal ſcale; then ſuppoſe a ſide of the ſame figure make $750\frac{1}{2}$ on your ſcale you muſt multiply by 100, the reaſon of which is evident, viz. becauſe you divided by 100 in laying down, ſo that your $750\frac{1}{2}$ becomes 75050; the contrary holds if your numbers be too little; this alſo muſt be obſerved on Gunter's ſcale or line of numbers on all ſcales, rules, &c. ſo that if you multiply or divide any right-lined part or number, in order to increaſe or diminiſh that part or number, you muſt do ſo with all the reſt. Six ∠s will not admit of ſuch multiplication or diviſion, ſo that whether they are great or ſmall they muſt be projected or meaſured in their original ſtate. But the radius of the ſcale of chords may be increaſed and diminiſhed at pleaſure. And as 60° is the radius of a circle whoſe periphery is 360°, you muſt always with the chord of 60 and one foot on the angular point, deſcribe an arch to cut the two ſides ſubtending an ∠, produced if neceſſary, and then this arch or ſubtenſe meaſured on the ſcale of chords you took the radius from ſhews the quantity of that ∠ if not above 90°.00, but if more you muſt meaſure it at twice, and take

the two meaſurements in one ſum. This meaſure of the arch is to be underſtood when an ∠ is mentioned. See problems 10, 11, 12.

7. You muſt meaſure all ∠s or project from the ſame ſcale of chords you took the radius 60° from. Hence it appears that the ſides of an ∠ have no relation or connection with the quantity of that ∠; for let the ſides of an ∠ be ever ſo long or ſhort the meaſure of the ∠ is ſtill the ſame invariable quantity.

8. A plane ∠ is made by the meeting of two right lines called its ſides, and if its meaſure be 90° it is called a right ∠, if leſs than 90° it is called an acute ∠, if more than 90° it is called an obtuſe ∠; but the two laſt are generally called obtuſe ∠s.

9. A plane △ is made by the interſection of three right lines called its ſides, and if any two of theſe ſides make an ∠ of 90° it is called a right angled △, and theſe two ſides are together called legs, the longer of which is commonly called the baſe, and the other the ⊥ or cathetus, and the longeſt ſide which is always oppoſite to the right ∠ is called the hypothenuſe.

10. If each ∠ of a △ be leſs than 90° it is an acute angled △, but if one ∠ be more than 90° it is an obtuſe angled △.

11, If the 3 ſides of a △ be = the △ is equilateral, if two ſides are equal it is called iſoſcelar, but if all the ſides are unequal it is a ſcalene △.

12. Since, by theorem 4, the ſum of the 3 ∠s of every plane △ is 180°, it follows, that if any two of theſe ∠s be given the third ∠ is found by ſubtracting the ſum of thoſe that are given from 180°; and in a right angled △, becauſe the right ∠ is 90°, if one of the acute ∠s be given, the other is found by ſubtracting the given one from 90°.

13. Complement is what an arch or ∠ wants of 90°, and ſuplement is what an arch or ∠ wants of 180°.

14. In right angled plane △s, are 7 caſes, and in oblique angled ones 5, all ſolved arithmetically by theſe 4 axioms.

Axiom I.

In all right angled plane $\triangle$s, as the greater leg : the leſſer leg : : radius : the tangent of the leſs acute $\angle$, or as the leſſer leg : the greater : : radius : tangent of the greater $\angle$, (ſee theorem 47) this is when the legs are given to find an $\angle$, or a leg and an $\angle$ to find the other leg.

Axiom II.

In all plane $\triangle$s, as any ſide : the ſine of its oppoſite $\angle$: : any other ſide : the ſine of its oppoſite $\angle$. And the contrary. See theorem 48. This is when oppoſite ſides and $\angle$s are given and required. And hence we learn, that the greateſt ſide ſubtends or oppoſite to the greateſt $\angle$, the leaſt ſide to the leaſt $\angle$, $=$ ſides to $=$ $\angle$s, and the contrary. Theſe two axioms will ſolve all the 7 caſes of right angled plane $\triangle$s, and by them alſo may all the 5 caſes in oblique angled $\triangle$s be ſolved, by dividing the oblique angled $\triangle$ into two right ones with a $\perp$.

Axiom III.

As the ſum of any two ſides : their difference : : the co-tangent of $\frac{1}{2}$ their included $\angle$, viz. $\frac{1}{2}$ the contained $\angle$ or $\frac{1}{2}$ the $\angle$ made by theſe two ſides, or : : the tangent of $\frac{1}{2}$ the ſum of the other two $\angle$s : tangent of $\frac{1}{2}$ their difference ; then $\frac{1}{2}$ the ſum of the unknown $\angle$s $+$ $\frac{1}{2}$ their difference is the greater $\angle$, and $\frac{1}{2}$ the ſaid ſum $-$ $\frac{1}{2}$ the ſaid difference is the leſs $\angle$. See theorem 50. This is when two ſides and the $\angle$ made by them are given to find the other two $\angle$s.

Axiom IV.

When the three ſides are given, as the longeſt ſide : the ſum of the other two : : the difference of theſe two ſides : the difference of the ſegments of the baſe or a fourth number, which taken from the greateſt ſide or baſe, the $\perp$ falls on the middle of the remainder, (ſee theorem 49) $\frac{1}{2}$ of which is the leſs ſegment of the baſe, and this $\frac{1}{2}$ added to the 4th number gives the greater ſegment of the baſe, and then the $\angle$s are found by axiom 2.

Or an ∠ may be found in this cafe without letting fall a ⊥, thus, take each fide feparately from the ½ fum of all the fides, but firft that fide oppofite to the ∠ you want to find, noting their remainder; then, as the product of half the fum of the fides and firft remainder : the product of the other two remainders : : □ of radius : the □ of the tangent of ½ the ∠ required. Thus when any three parts of a △ are given, to find a fourth part confider to which axiom it belongs, by which you'll be able to form the proportion; where obferve, that if fines or tangents be mentioned in the proportion, you muft ufe the numbers belonging to fuch fines or tangents, but thofe terms which mention neither are natural numbers.

15. On the fcale for this purpofe are three lines the length of the fcale, the firft marked at the end Num. is a line of numbers, and is the fame with that on the flider in the common fliding rule, the next marked Sin. is a line of fines marked 10, 20, 30, &c. to 90 at the end; then the line of tangents ending at 45 marked Tan. this line is alfo numbered back again to 80° or 89°. The fame lines are fet on fliding fcales which are made to work with a flider inftead of compaffes. This line of tangents both increafes and decreafes from 45 the end towards the left hand, fo as any number of degrees increafing is their compliment to 90 decreafing.

Note. In all fcales you muft reckon the divifions the fame way that the numbers or figures are reckoned. In extending on this line between any two tangents both greater or lefs than 45, the compaffes cannot fall off the fcale, but if one is greater and the other lefs than 45, the compaffes will go beyond the line; in fuch cafes let the foot beyond 45 reft, and bring up the other to 45, there reft it, and turn the other foot upon the line within 45, and it points the anfwer. This can only happen in the ufe of axiom the third, for in axiom firft one turn is always radius 45.

16. The radius of the line of fines is 90, radius of tangents 45, each at the lines end.

17. On this fide of the fcale are other lines, as T R being the tangent line divided into 8 points, with halves and quarters of the feaman's

compaſs, to be uſed by axiom 1; when the ∠s are required or given in points of the compaſs, called rhumbs. S R a line of ſines divided into rhumbs, to be uſed by axiom ſecond, when the ∠s are in rhumbs. V S verſed ſines, of uſe in ſpherical trigonometry. Mer. meridional parts, which with E P joining it, a line of = parts, is uſed in Mercator's ſailing. Theſe lines are all conſtructed by the rules given in part third, for the ſliding gunter, &c. viz. only laying down the logarithms of natural ſines, tangents, &c. from the ſame ſcale the line of numbers is from.

18. On the other ſide of this ſcale are the ſcales for projection; and is a ſcale of 24 inches or two feet, two diagonal ſcales, one double the other; Rum. a ſcale of rhumbs; Cho. a ſcale of chords made by problem 9; Sin. a ſcale of ſines, with Sec. a ſcale of ſecants following, both beginning at one point Sin. Tan. and S T a ſcale of tangents and ſemi or half tangents, theſe may be made by problem 54; for if G C be an arch of 35°, then G B is ſine of 35°, A D the ſecant, C D the tangent of 35°, or ſemi-tangent of 70°. Fig. 44.

19. Rules of proportion on Gunter's ſcale, extend from the firſt term to the ſecond or third of the ſame denomination, and that extent will reach from the third or ſecond term of another kind to the anſwer or fourth term of the ſame kind, viz. extend from ∠ to ∠ whether ſine or tangent, or from ſide to ſide, ſo will your extent thus taken reach in the firſt caſe from ſide to ſide, and in the latter from ∠ to ∠.

20. Rules of proportion on the ſliding ſcale called ſliding Gunter. Set the firſt on the ſlide either of the two means on the rule, then againſt the other mean on the rule ſtands the fourth term on the rule. Or, ſet the firſt term on the rule to either of the means on the ſlider, then againſt the other mean on the rule ſtands the fourth term or anſwer on the ſlider.

21. Rules of proportion by the logarithms, add the logarithms of the two means, viz. of the ſecond and third terms together, from that ſum take the logarithm of the firſt term, and the remainder is the logarithm of

the fourth term. Or as you write the logarithm of the firſt term out of the table of logarithms, begin at the left hand ſide, and take the reſidue of each figure to 9, and the laſt to 10, this is called the complement arithmetical marked C. Ar. which with the logarithms of the two means being added into one ſum, rejecting 10 in the index, (or number pricked off to the left hand ſide) gives the logarithm of the fourth term without ſubtraction.

22. In the following problems you have the proportions all ſet down which you may work as you pleaſe, by ſcale and compaſſes, ſliding ſcale, or by logarithms; which laſt is moſt exact, becauſe the logarithmetical tables give the anſwer to moſt places of figures.

PROBLEM CXXI.

Caſes firſt and ſecond. In a right angled plane △ A B C, *there is given the two acute* ∠*s* B 33°.45′ *and* C 56°.15′, *with the leg* A B 121.394, *to find the hypothenuſe* B C, *and the other leg* A C. Fig. 98.

1. Make the line A B = 121.394 from a diagonal ſcale. 2. Upon A, by problem 11, make the right ∠ n A m. 3. Upon B, by problem 10, the ∠ d B a = 33° 45′, theſe laſt two lines meeting in C, form the △ required. Then the leg A C laid on the ſame diagonal ſcale is found = 81.1, and B C = 146.

Arithmetically, by Axiom 1.

As tang. ∠C 56°.15′	10.1751074	or by taking the com. ar.	9.8248926
: tangent radius 45°	10.0000000		10.0000000
: : A B 121.394	2.0841692		2.0841692
: A C 81.113	1.9090618		1.9090618

Becauſe the log. index is 10, you muſt take it from 19, and reject 20 in the ſum.

In finding the logarithmetical ſines and tangents, if the degrees be under 45, find it at the top of the table, and the minutes in the little

column on the left hand side; but if the degrees be above 45, find them at the bottom of the table and the minutes in the little column on the right hand side: so above 56° and against 15′ in the tangent column stands 10.1751074 the logarithmetical tangent of 56° 15′ and in the side column stands 9.9198464, the logarithmetical sine of 56°.15. If you have the log. sine given to find the degrees and minutes, it is the reverse of the former, for with the given log you enter its column whether sine or tangent, and having found the degrees, you take the minutes which come nearest. The log. of a number is had by finding your number under minutes in the table, and against it stands the logarithm required; always observing that the index must be 1 less than the number of places of whole numbers in the given number.

If the logarithm of a number be given to find the number, it is just the converse work; so 1.9090918 being given I look for it amongst the logarithms, and finds .9090744 to be the nearest, and against it under numbers stands 8111, so the answer will be 81.11, because the given index is 1, &c. for others.

By scale and compasses. The extent from 56°.15′ to 45° in the tangents reaches in the numbers from 121.394 to a little above 81.

By the sliding scale. Set tangent 56°.15′ to tangent 45°, then against number 121.394 stands better than number 81, for the answer.

Note. Log. radius is always 10.000 &c. Lastly, B C is had by axiom 2d, and so may A C too; thus, as sine ∠ C 56°,15′ : radius sine 90° ∠ A : : A B 121.394 to B C 146; and as sine ∠ C 56°,15′ : sine ∠ B 33°,45′ : : B A 122.394 : A C 81,11.

PROBLEM CXXII.

Case 3, *Given the two acute* ∠*s* B 33°,45′ *and* C 56°,15, *with the hypothenuse* B C 136, *to find the legs* A B *and* A C. Fig. 98.

1. Draw the line B a, and on B, by problem 10, make the ∠ d B a = 33°,45′. 2. From a diagonal ſcale take 146 and lay from B to C, then at C make the ∠ B C A = 56°,15′; or, from C upon B a let fall the ⊥ C A, and the figure is made; then A B and A C laid on the ſame diagonal ſcale are found 121 and 81. Arithmetically, by axiom 2d, as ſine ∠ A 90° : ſine ∠ B 33°,45′ : : BC 146 : A C 81,11, and ſo is ſine ∠ C 56°,15′ : AB 121,39.

PROBLEM CXXIII.

Caſes 4th and 6th. Given the legs A B 121,4 *and* A C 80, *to find the oblique* ∠*s* B *and* C, *and the hypothenuſe* B C. Fig. 96.

1. By a diagonal ſcale make B A = 121,4. 2. Upon A, by problem 11, make the ∠ B A C = 90°, and from the ſame diagonal ſcale take 80, and lay from A to C, join B C and the figure is protracted. Then B C laid on the diagonal ſcale is found = 145. For the ∠s, with the chord of 60° and one foot in either B or C ſuppoſe C, deſcribe an arch d m, which laid on the ſame chords gives 56°,30′ for the ∠ C, whoſe complement is 33°,30′ for the ∠ B. Arithmetically, by axiom 1, as B A 121,4 : A C 80 : : tangent 45° : tangent ∠ B 33°,30′. And by axiom 2, as ſine ∠ B 33°,30′ : ſine ∠ A 90° : : A C 80 : B C 145 nearly.

PROBLEM CXXIV.

Caſes 5th and 7th. Given the hypothenuſe B C 146, *and a leg* B A 121,39, *to find the other leg* A C *and* ∠*s* B *and* C. Fig. 99.

1. Draw the line B A, on which from A to B by a diagonal ſcale lay 121,39. 2. Upon A, by problem 11, make a right ∠ B A D. 3. From the ſame diagonal ſcale take 146, and with one foot in B deſcribe an arch e a croſſing A D in C, join B C and the figure is made. Then with the chord of 60° and one foot in either B or C, ſuppoſe B, deſcribe an arch d n, which laid on the ſame chords gives 33°,45′ for the ∠ B, the complement to which is 56°15′ the ∠ C, and the leg A C laid on the

diagonal ſcale is found $=$ 81,1. Arithmetically, by axiom 2, as B C 146 : A B 121,39 : : radius (ſine ∠ A 90°) : ſine ∠ C 56°,15′; and as radius : ſine ∠ B 33°,45′ : : B C 146 : A C 81,11.

PROBLEM CXXV.

Caſe 1ſt. Of oblique angled plain △s given two ∠s A 62°,30′ *and* B 37°,30′, *and, by definition 9th, the third* ∠ C *is alſo known to be* $=$ 80°, *with a ſide* A C 350, *to find either of the other, ſuppoſe* B C. Fig. 100.

1. By a diagonal ſcale make A C $=$ 350. 2. Upon A, by problem 10, make an ∠ B A C $=$ 80°, and the two lines A B and C B meeting in B form the △ required. Then B C and A B laid on the ſame diagonal ſcale are found $=$ 510 and 566. Arithmetically, by axiom 2, as ſine ∠ B : ſine ∠ A 62°,30′ : : A C 350 : B C 50 9,976; alſo, as ſine ∠ B 37°,30′ : ſine ∠ C 80° : : A C 350 : A B 566,2.

PROBLEM CXXVI.

Given the two ſides B C 509,976, *and* A C 350, *with the* ∠ B 37°,30′ *oppoſite to one of them* A C, *to find one of the other* ∠*s* A *or* C. Fig 100.

1. By the diagonal ſcale make B C $=$ 509,976. 2. Upon B, by problem 10, make the ∠ E B C $=$ 37°,30′. 3. With 350 from the diagonal ſcale and one foot in C deſcribe an arch to cut E B in the points A and F, which ſhews that this caſe is doubtful, whether both the unknown ∠s be acute as A and C, or the one F C B acute and the other C F B obtuſe; for from the things given either B C F or B C A may be the △ required, by joining C A and C F, ſo let A B C be the △ required, then with the chord of 60 and one foot on C deſcribe the arch m n, which laid on the ſame chords gives 80° for the ∠ A C B; then, by definition 9, ∠ C 80° + ∠ B 37°,30′ gives 117°,30′, which taken from 180 leaves ∠ A 62,30′ $=$ the arch a e being ſtruck upon A with the chord of 60. Arithmetically, by axiom 2, as A C $=$ C F 350 : C B 509,976 : : ſine ∠ B 37,30′ : ſine ∠ A 62,30′, if both the ∠s be acute; but if C F B be the ∠ required, then this 62°,30′ taken from 180° leaves 117°,30′ for the ∠ C F B.

Note. When you are to take the ſine or tangent of any ∠ above 90, you muſt firſt take ſuch an ∠ from 180, and take the ſine or tangent of the remainder. For it is plain by figure 44, that B G is the ſine of the arch H E G as well as of the arch C G, and that theſe two arches together make a ſemicircle. See it otherwiſe proved in theorem 48.

PROBLEM CXXVII.

Caſes 3d and 4th. Given the two ſides A C 350, *and* C B 510, *with the* ∠ C 80° *comprehended between them, to find the other two* ∠s A *and* B, *and the third ſide* A B. Fig. 100.

1. By problem 10, make an ∠ n C m = 80°, extending the ſides C n and C m, till by the diagonal ſcale C A = 340 and C B = 510, join A B and the △ is made. 2. With the chord of 60 and one foot in either A or B, ſuppoſe in A, deſcribe the arch e a, which laid on the ſame chords gives 60°,30′ for the ∠ A, and by definition 9, you'll find it = 566. Arithmetically, by axiom 3. firſt 350 added to and taken from 510 gives 860 and 260 for the ſum and difference of the two ſides, and 80 taken from 180 (definition 9) leaves 100 half of which is 50°, the half contained ∠s complement or half ſum of the oppoſite ∠s A and B; Then, as 860 : 260 : : tang. 50 : tang. 12,30′ half the difference of the ſaid ∠s, ſo 50° + 12,30′ = 62,30′ the greater ∠, and 50 — 12,30′ = 37,30′ the leſſer ∠; laſtly, as ſine 62,30′ : ſine 80 : : C B 510 : A B 566,3. By axiom 2.

PROBLEM CXXVIII.

Caſe 5. *Given the three ſides* A B 213,5, A C 107,5, and B C 250,2, *to find the* ∠s. Fig. 101.

1, To lay down this figure is already taught in problem 14, and to meaſure the ∠s you muſt have recourſe to the foregoing problems or the general rule laid down in definition 6 and problem 120; therefore to do it arithmetically by axiom 4, firſt the baſe is B C = 250,2, then 213,5 + 107,5 = 321 the ſum, and 213,5 — 107,5 = 106 the difference of the other two ſides; then as 250,2 : 321 : : 106 : 136 nearly, the fourth number which is = B F, now B C 250,2 — B F 136=

F C 114, half of which is 57 = C G = F G, and F G 57 + F B 136 = B G 193; then in either of the right angled Δs A B G or A C G there is given the hypothenuſe A B or A C, and a leg B G or C G, ſo by axiom 2, as A B 213,5 : B G 193 : : radius : ſine ∠ G A B 64,45′, the complement to which is 25,15′ = ∠ A B G. Again, as A C 107,5 : ſine ∠ B 25,15′ : : A B 213,5 : ſine ∠ C 57,55′, and : : B C 250,2 : ſine ∠ A 96,50′, or having any two of theſe three ∠s the third is found by definition 12.

The Application of

TRIGONOMETRY

To the meaſuring Heights and Diſtances; with the Deſcription and Uſe of the common Inſtruments for that purpoſe.

PROBLEM CXXIX.

DEFINITIONS continued from Problem 120.

23. Of the Chain. Mr. Gunter's chain of 100 = links each 7,92 inches is now the only chain in uſe among ſurveyors, whoſe length is 22 yards or 66 feet or 4 poles, perches, rods, &c. and this diviſion of the chain into = parts, makes links the decimal parts of a chain, conſequently chains and links work together in all reſpects as decimal fractions.

24. At firſt ſetting forward to meaſure, to prevent miſtakes and for the ſake of expedition, let the perſon going before with the end of the chain, have ten ſmall ſticks, each about two feet long cut to a point at one end

to ſtick down into the ground with more eaſe at the end of every chain, which muſt be carefully taken up by the perſon following with the other end of the chain, and thus proceed till all the ſticks are taken up by the perſon following the chain, if the line be ſo long, being always careful to move in a ſtreight line from mark to mark. If the diſtance be not ſurveyed when all the ſticks are out, let them be returned to the leading aſſiſtant, and ſo proceed till the whole diſtance is meaſured, obſerving to ſet down how often the ſticks have been changed, that no error be made in that caſe.

24. At the end of every 10 links there is commonly a piece of braſs a ring or ſome other mark for the more ready reckoning the links.

26. Chains and links may be ſet down thus, 7ch. 40l. or 7,40 ch. or 7,4 ch, or 740 links, which laſt is beſt ſignifying 7 chains and 40 links or 740 links.

27. If the chain be too long for uſe in meaſuring ſmall parcels, you may take dimenſions with the half chain, and ſet down half the number, as for example, ſuppoſe you have 11 ſticks and 25 odd links, it muſt be ſet down 5 chains 75 links or 575 links, and then in plotting or finding the area it will be the ſame as if you had meaſured with the whole chain.

28. For taking angles in the field are various inſtruments contrived, but all that can be propoſed from them is to meaſure an *L* included between two lines or hedges that may be tak en as ſtreight lines, the chief of which are, the plain table for ſmall incloſures, the theodolite, circumferentor, and ſemicircle for champaign grounds, the three laſt are nearly of one form, being only a circle or ſemicircle divided into degrees and minutes, if the inſtrument be large enough to admit of ſuch ſubdiviſions.

29. The plain table is a board or rather three pieces put together, orming a parallelogram fit to hold a ſheet of paper, with a frame round it to keep the table together and to hold the paper faſt, with a looſe index with ſights to lie upon the table &c. its uſe is chiefly to take the plot of a field. See ſurveying.

30. The ſemicircle or theodolite is half a circle or two quadrants whoſe limb is divided into degrees, &c. as by problem 9, beginning on the limb at one end of the diameter and ending in 180 at the other, and beginning again at 181 and ending at 360, ſo that as the index goes off at one end at 180, it comes on the other at 181, conſequently it anſwers all the purpoſes of a whole circle divided into 360°, in taking angles it muſt be placed flat with its face horizontal, on which the index turns on the center, on the index are two upright pins or ſights to ſpy the object thro' in the time of obſervation.

31 There is a line and plumet put thro' the central hole when altitudes are taken therewith, and the object is ſeen thro' two ſights fixed in the diameter, the limb hanging downwards, the line and plumet cuts the angle of the altitude; if you count the degrees from the middle of the limb but the co-altitude, if the degrees are numbred from the corner next your eye, the ſame is true in a quadrant which is half a ſemicircle with a line and plumet ſuſpended at the center of the arch a (in fig. 102) the ſights upon the edge as a B, being directed to the object C, the line hanging at liberty will be parallel to C D, if C D is upright whence its plain (by fig. 102) that the $\angle$ u a B = $\angle$ B C D and the arch e u = $\angle$ A B C, as by laſt definition.

32. Circumferentor is a circle upon a card divided into degrees &c. and fixed on a board or in a box, a needle touched upon a loadſtone moving flat upon its center, this is uſed in ſurveying mines &c.

34. Protractor a ſmall inſtrument in form of a half circle, for laying down $\angle$s readier than a ſcale of chords, thus lay its diameter along the line on your paper with its center on the angular point, then againſt the degrees of the $\angle$ on the protractors limb or arch make a mark on the paper, thro' which and the angular point draw a line and the $\angle$ is made.

33. Level, fill a cylinderic glaſs tube with water excepting a ſmall ſpace which cloſe at both ends, and when the tube is laid in a level

horizontal situation the vacuity will shew its self in the middle, but will vary there-from on the least inclination.

34. Some note down the dimensions of a survey into a field book divided into columns, but I would recommend the following, draw an eye draught of the plot as near as you can, then when you have measured any line, angle or offset, &c. write down its length or quantity, along the side or in the angle corresponding in your eye draught; so shall you have a rough plan to make a true one by, which is both easier and safer to plan by than any table by what form or method soever it be contrived.

PROBLEM CXXX.

To find the altitude C D *of any object, as a tower, &c. by a quadrant* B e a, *being an inaccessible altitude.* Fig. 102.

1. The method of observing by a quadrant is plain by the figure compared with definition 31; thus, at any convenient place B, view C the top of your object along the edge B a of the quadrant, and suppose the plumet a u hanging at liberty to cut 52,5′ from e, viz. arch e u = 52,5′ which, by definition 31, is the measure of the ∠ C B A. 2. Measure in a streight line from your eye at B to the tower, which suppose B A = 85 feet, and your observation is finished.

To lay this down. You have given the leg A B = 85, and the ∠s B 52,30′ and C its complement, so the figure is laid down by problem 121, and the required height is C A laid on the diagonal scale is found = 105,8, to which add A D suppose 5 feet the height of your eye above the level of the bottom D F (as you must always do in such cases) and it gives C D = 110,8 feet. Arithmetically, by axiom 2, as sine ∠C 37,30′ : sine ∠B 52,30′ : : B A 85 : C A 105,8 feet.

PROBLEM CXXXI.

To find A B *the height of any object* D H C, *as a steeple, &c. and come no nearer its bottom than* S, *being an inaccessible altitude.* Fig. 103.

1. Upon S observe B the top of the object, along the edge of a quadrant, or by a semicircle, the plumet u a hanging at liberty, as by the last problem, and suppose the ∠ B S A to be found $=51,30'$. 2. Move back any distance in a line with S A, suppose to F 75 yards, and upon F make observation to B as before, supposing the ∠ B F A to be $26,30'$. 3. Now to lay this down. Make S F $=$ 75 from a diagonal scale, and upon S and F, by problem 10, make the ∠s B S A $=51,30'$ and B F A $=26,30'$ and at B where these two lines meet let fall the ⊥ B A cutting F S produced in A, and by the diagonal scale you'll find the height B A $=$ 62 yards the answer; also by the same scale you may find S A &c. Arithmetically, In the oblique △ B F S is given S F $=$ 75, ∠ B F S $=26,30'$, and ∠ B S F $=$ (complement ∠ B S A $51,30'$ to 180) $128,30'$, whence ∠ B F S $=25°$ (definition 12) so, by axiom 2, as sine ∠ B F S $25°$: sine ∠ BFS $26,30'$: : FS 75 yards : side F B 79,18, then as sine ∠ B A F 90 : sine ∠ A F B $26,30'$: : F B 79,18 : A B 61,97 the height required.

Note. If from a point F you observe a point B, it is plain that you see it in a streight line F B, in like manner a point B is seen in a streight line from a point S, whence it is plain, that these two lines must meet in the point B, so if you measure the distance S F 75 yards, feet, or chains, &c. and lay 75 from any small $=$ parts on paper from S to F to represent these 75 yards, &c, and draw the lines F B and S B to make the same ∠s (F and S) as you took by observation, it is evident, these two lines F B and S B must meet in B, forming a true plan of your observations, in which plan all the unknown lines, &c. will be just so many of the said $=$ parts as they would be yards, &c. in the thing itself. This holds true in all heights, distances, &c. whatever, which being considered, these things may be easily done.

PROBLEM CXXXII.

To find the heights C z *and* B A $=$ e z *of two inaccessible objects standing upon one another.* Fig. 103.

1. Upon any place where both points B and C may be seen, as at F, observe these two points with your instrument as before directed, and let the ∠s C F A $= 44^\circ$, and B F A $= 26,30'$, 2. Move directly forwards to the bottom of the object any distance, suppose from F to S 75 chains there observe B and C as before, and let the ∠s C S A be $=$ $67,50'$ and B S A $= 51,30'$. To lay down this, draw a line F S A at pleasure, make F S $= 75$ from a diagonal scale or scale of $=$ parts, and upon F and S, by problem 10, make the ∠s as you found them by observation, and C and B the meeting of these two lines will be the tops of the objects, from which let fall the ⊥s B A and C e, then by the same scale of $=$ parts you'll find A B $= 62$ and C z $= 50$ chains the heights required. Arithmetically, as by the last problem, you found A B, so find C e, and the difference is C z.

PROBLEM CXXXIII.

To find m F *the perpendicular height of a hill.* Fig. 104.

1. As in problem 131, chuse any two convenient places on the same plane or level as n and s which let be 100 chains distance, and from each place s and n observe T the top of the mountain, and let the ∠s be s 30° and n 42°, by which the figure may be laid down exactly as in problem 131, and by the diagonal scale you'll find m T $= 160$ chains the required height; the arithmetical solution is also the very same with that in problem 131.

PROBLEM CXXXIV.

To find AB *the height of a tower, &c. by making two observations on the side of a hill some distance from it.* Fig. 105.

1. On any part D of the hill's side observe a point E, where you intend your second station to be, and let the ∠ B D E $= 13^\circ$ made between looking at E and at B the bottom of the tower, and the ∠ A D E $= 48^\circ$, being the degrees cut between observing E and A the top of the tower. 2. At D view some other mark beyond E, as F in the same line D E, measure the distance D E which suppose 87 yards, then let

the degrees cut between looking at F and B be 18°, and between F and A 68° when you ſtand at E. 3. To lay this down, draw the line D E F at pleaſure, make D E = 87, on D make the ∠s B D F = 13° A D F 48°; alſo, upon E make the ∠s B E F = 18°, A E F = 68°, and at A and B where theſe lines meet will be the top and bottom of the tower, which diſtance A B is found = 155 yards by the diagonal ſcale. The arithmetical calculation may eaſily be deduced from what is done in problem 131.

PROBLEM CXXXV.

To find A B *the height of any tree, tower, &c. by a ſquare* a b E, *or two ſtreight ſticks faſtened together at right* ∠s. Fig. 106.

1. Let the ſticks be each of the ſame length, and faſtened ſo as to form a true ſquare, their length may be about four or five inches; this done, ſet an end E to your eye holding the ſide E b level, move back or forward till over the top of the other ſtick you can juſt ſee the top A of your object, and at the ſame time obſerve ſome point B towards the foot of it along E b, then meaſure the diſtance E B and it will be = to A B. For the Δs E b a and E B A are ſimilar, and E b is = b a, ſo E B will be = B A.

PROBLEM CXXXVI.

To find A B *the height of any object by the ſhade of the ſun.* Fig. 106.

1. Near the bottom of the object ſet up a ſtreight ſtick a b parallel to the ſaid object, then it is plain, if B E be the length of the object's ſhadow, that b E will be the length of the ſtick's ſhadow; therefore, as E b the ſtaff's ſhadow is to b a the ſtaff's length, ſo is E B the length of the object's ſhadow to B A its height. For b a being parallel to B A, the Δs E a b and E A B are ſimilar.

The 5 following problems belong to

DISTANCES.

PROBLEM CXXXVII.

To find the diſtance of any object C, *from a place* A. Fig: 107.

1 Having A for one ſtation, or place of obſervation, you muſt make choice of another which let be B, and the diſtance A B = 145 feet, place your theodolite at A with its face parallel to the horizon and that end of the diameter where the degrees begin towards B, then along the ſaid diameter, or thro' the ſights on it view B, there ſcrew the inſtrument faſt, and turn the index till through the ſights thereon you ſee the object C, and ſuppoſe the index then cut 78°, (viz.) to move over the arch a e. 3. This done come to B, and obſerve your firſt ſtation A along the diameter, then turn the index to C, and as before let the ∠ A B C be 44° and you have done your obſervation. To lay down which is the very ſame with problem 125, for in the △ A B C is given the ſide A B 145 and the ∠s C A B 78°, and A B C 44° which laid down (as by fig.) you'll find by the diagonal ſcale A C = 120 the anſwer. Ar. as ſine ∠ B C A 58° : ſine ∠ A B C 44° : : A B 145 : A C 120 feet.

PROBLEM CXXXVIII.

To find the diſtances of ſeveral places ſuppoſe two C *and* D, *from any propoſed ſtation* A, *by any inſtrument that can take an* ∠. Fig. 107.

1. You may work this problem like the laſt only becauſe here are two objects, you muſt take two ∠s at A which let be D A B = 52°, and D A C = 26°. 2. Meaſure from A to your ſecond ſtation B which ſuppoſe 145 yards, there take two ∠s again (viz.) ∠ A B C = 44° and ∠ C B D = 38°, then to lay this down on paper, make A B = 145

from a diagonal ſcale, (by problem 10) upon B make the ∠s A B C = 44° and C B D = 38°, alſo, upon A make the ∠s D A B = 52° drawing A D till it cut B D in D, and ∠ D A C = 26° continuing A C till it cut B C in C, ſo is D and C the required object, and by the ſame diagonal ſcale will be found A C 120, and A D = 200 yards the diſtance between A and the two objects C and D; alſo, by the ſame ſcale you may find B C and B D their diſtances from B and C D their diſtance aſunder. The arithmetical calculation, is eaſy from the laſt problem.

PROBLEM CXXXIX.

To meaſure any level field a b c d, *which can be all ſeen from two different ſtations, and not come near it.* Fig. 108.

1. This is but to take two convenient ſtations F and S, and on each of them make obſervations to every ∠ or corner in the field &c. (by laſt problem) thus placing your theodolite at F, look for S and every corner, and let the degrees cut by the index between looking at S and d be 26°, between d and c 34°, c and b 25° b and a 16°. 2. Meaſure the diſtance F S which let be 120 chains, and placing your inſtrument on S look for F and every corner of the field, and let the degrees cut by the index between looking at F and at a, b, c, d, be 40, 18, 24, 36°. To plan this, draw a line F S = 120, upon F make all the ∠s as you took them at your firſt ſtation, and upon S as you took them at the ſecond, drawing S a till it meet F a, S b till it meet F b, S c till it meet F c, and S d till it meet F d. Join theſe points a, b, c, d, and you'll have a true plan of the required field, which by the ſame ſcale you took S F from, you may find the diſtance of every corner a, b, c, d, from each ſtation F and S, as alſo their diſtance from each other, and alſo meaſure a diagonal and two ⊥s, and ſo find its area.

Note. In the ſame manner you may take the plan of a field at two ſtations within the field, and ſtill mind to take an ∠ between each ſtation and the next corner when you are upon the other ſtation, for its plain the figure could not be laid down if you wanted the ∠ F S a and

d F S; alſo, let your ſtations be as far diſtant as you conveniently can, then the points a, b, c, &c. where the lines meet will not be ſo acute, for in ſuch ſmall ∠s it is not eaſy to ſee where the lines interſect one another, and conſequently the plan cannot be true.

PROBLEM CXL.

To find the diſtance of ſome place as A *by three ſticks.* Fig. 36.

1. Set up a ſtreight ſtick any where as at C, from which move in a right line towards the object A any diſtance ſuppoſe to B 200 yards, there ſet up another ſtick, and move from B any diſtance in a right line acroſs E B as at E 100 yards, there ſet up another ſtick; then come back to C and move in a line parallel to E B, till over the ſtick you can ſee the object A, and ſuppoſe C D the diſtance moved to be 300 yards, then it will be as D F 200 the difference between C D and B E : E F = B C 200 : : C D 300 to C A 300, or : : E B 100 : B A 100, the object's diſtance from C and B, for all the Δs are ſimilar.

PROBLEM CXLI.

To take a diſtance A B *as the breadth of a river &c. by a carpenter's or maſon's ſquare* E D F. Fig. 109.

1. At A the rivers ſide ſtick down a ſtreight ſtaff ⊥ to the horizon, upon its top D hang a ſquare F D E, ſo that along the ſide D E you can juſt ſee B the farther ſide of the river, and at the ſame time mind C, a point on the ground where D F the other ſide of the ſquare points, and let A C be two feet and A D 8 feet; then by problem 45, it will be as C A 2 : A D 8 : : A D 8 : A B 32 feet the anſwer.

Note. If you ſlit the head D of an upright ſtaff A D, and in that put a ſmall ſtick D E, bending it up or down till along the ſide D E you can juſt ſee B the farther ſide of the river, then turn the ſtaff D A about and obſerve where D E points on the land, which point meaſure from A, and you'll have the diſtance required.

The two following Problems include

The Art of LEVELLING.

PROBLEM CXLII.

To find the level between two places, suppose A *and* I. Fig. 110.

1. For this purpose you may make an instrument by fastening a square n c a upon the head of a streight staff A B, and at the ∠B fasten a thread and plumet c a e; now it is evident, when this staff A B is upright, the thread c e will (hanging at liberty) just touch the side c a of the square. 2. Having thus placed the staff at A in a ⊥ position, observe C along the side B n of the square, or, which is better, thro' two sights placed thereon; then measure the distance B C 10,6 feet for the level between A and C, also measure the height A B 5,5 feet for the height of C above the level of A. 3. Fix your instrument at C and do exactly as you did at A, and you'll see the point E, and thus do till you come to see I, the top of the hill, and let the dimensions be as here set down,

The several heights are	A B = 5,5 C D = 2 E F = 2,6 G H = 1,7	The several levels are	B C = 10,6 D E = 10,8 F G = 5 H I = 2
The sum is	I K = 11,8	and	A K = 28,4

Hence it appears, that if you dig a pit at I 11,8 feet deep, there will be level for the water, and that level will be 28,4 feet long. But that water in such cases may have a little current there is an allowance made of 4 or five inches for every mile or 1760 yards; i. e. if a level be 1760 yards long, one end of that level must be 4 or 5 inches lower than the other, that the water may run off; and so in proportion for any other length.

PROBLEM CXLIII.

It is required, to know whether water can be carried from I *to* A *(by the level, mentioned in definition* 33.) Fig. 110.

1. Take two streight poles, of any proper length each divided into inches and decimal parts of an inch. 2. Set one of these poles upright

at A, and the other any where elſe conveniently, as at E. 3. Fix your inſtrument at ſuch a point C as you may therefrom ſee both theſe poles then look thro' the ſights on the level for the pole ſet up at A, cauſing ſome aſſiſtant to move a piece of white paper up or down the ſaid pole L A, that you may juſt ſee the upper edge of it which in this figure is at B, ſee what inches the ſaid edge cuts, which ſuppoſe 90 inches from A to B; obſerve the ſame method in looking for a paper ſo moved upon the ſtaff at E, and you'll ſee the point E, ſo is A B the height, and E B the level of the point E. Then remove the ſtaff at A to another point as H, letting the ſtaff ſtand at E, and on ſome point between E and H do as before, and thus going on you may ſoon level any ſuch place, as is plain by the figure.

This is the apparent level and will do in ſhort diſtances without correction, but if the diſtance be long there muſt be made an allowance, on account of the earth's ſpherical figure. Thus, let A be the center of the earth (fig. 44) A E and A G two half diameters, E d a tangent to the ſurface at E, now if the level be placed at E it will give the apparent level E d, between the places E and G, whereas the true level is the arch E G, ſo that the apparent level exceeds the true one, E G by the line G d, for which reaſon this line G d muſt be found and taken from the apparent lev el to get the true level between the places E and G; thus, (by theorem) $2\,AG + Gd \times dG = \square Ed$, or, becauſe A G is great in reſpect to G d, at any diſtance under 30000 yards, we may ſafely take $2\,AG \times dG = \square Ed$, and then $Gd = \frac{\square Ed}{2\,AG}$ $= 0{,}222$ parts of a yard, when E d is taken $= 1760$ yards or one mile, (the radius A G of the earth being about 6966412 yards) now, $0{,}222 \times 36 = 8$ inches nearly. Hence the difference between the true and apparent level is 8 inches per mile, and ſo in proportion for any other diſtances.

Note. By this method you may eaſily calculate a table of corrections for diſtances.

PROBLEM CXLIV.

To survey mines, &c. Fig. 111.

1. Over the top of the pit lay a piece of wood, from the middle of which let down a line and plumet to the bottom, by this you'll know how to place your instrument so as its center may be in the same ⊥ line both at top and bottom, on this instrument see definition 32) the degrees begin at north and are numbered eastward to 90°, south 180°, west 270°, and north 360°, where they again begin; wherever you place this instrument for observation be careful to place it horizontally on a board, &c. so that it be perfectly level, and let the north end of the needle point north where the degrees begin; having thus placed it at the bottom of the pit, hold the end of a small line over its center, and let an assistant go with the other end as far as he can go in a direct line, observe what degrees are cut by this line, suppose 200 or 20° westward of south, measure the line which suppose 7 fathoms, set these down, and remove your instrument to where your assistant stood, and there work as before; and thus go on as far as is necessary, and let the dimensions be as follow;

Distances	7	degrees cut	200	or,	20° westward of south.
	5		180		due south.
	11.5		335		25° west. of north.

This done it is common to place the instrument with its center on the top of the aforesaid ⊥ line, and so by the degrees measure out the degrees exactly as they were taken at the bottom, and instead of measuring these distances in fathoms, &c. they have small pieces of wood numbered 1, 2, 3, 4, &c. put into the twine of the line at the ends of the 1st, 2d, 3d, and 4th, &c. distances, which marks shew how far you are to go above ground in every turn; but this method may sometimes create great if not impracticable labour, in going thro' woods, water, &c. to prevent which it will be best to lay down a figure carefully from the dimensions taken, and so lay off the thing at once, thus draw a line N. S. which make at large, N representing north and S south, on N make the ∠ S N A = 20°, and from a diagonal scale lay

7 on N A, from N to A, then thro' A and parallel to N S, draw n s, for another north and south line, and lay 5 from A to B (because the second distance was south) on B make an ∠ A B C = 25°, laying 11,5 from B to C, join C N, which is found by the same diagonal scale = 8 fathoms, also thro' C draw the north and south line d S parallel to n S, and measure the ∠ N C d, which is found = about 56° from the north eastward; therefore, if from the top of the pit you measure out 8 fathoms over 236° or south 56° westward, it will point out the place at top where you measured to below.

The 7 following problems will exercise the skill of the young student in

GEOMETRY.

PROBLEM CXLV.

Three ships A, B, *and* C, *sail from one port* P; A *sails due south* 24 *miles*, B *south westerly and* C *south easterly, each a distance unknown, but* B's *is greater*, B *and* C *are* 78 *miles asunder, and the sum of their courses is* 98°, *the three ships are all in the same east and west line. Quere, the courses and distances sailed of* B *and* C. Fig. 112.

1. Draw E W for the east and west line making it = 78 miles. 2. Make the ∠s C W E and C E W each = 8°, and with the radius C E = C W upon C describe the circle. 3. Parallel to E W and at 24 distant from it draw Q P, cutting the circle in P and Q either of which may be the port according as B's or C's distance is greater, join P E and P W, draw P A ⊥ to E W, so doth P W represent B's distance sailed, the ∠ A P W her course, P E C's distance sailed, the ∠ A P E her course, P A = 24, A's distance sailed; then by the same = parts that E W and A P were taken from, you'll find P W = 69,3

miles, P E = 27,2, and by the chords, ∠ W P A is found = 69,45′, and ∠ E P A = 28,15′ as required. If the ∠ W P E = 98°, then (by theorem 6) the arch W D E must be = twice 98 = 196°. Then the arch W P E must be = (360 – 196°) 164°, and because C W = C E, the ∠ C W E = ∠ W E C, each = 8° = ∠ W C E 164° taken from 180 and divided equally.

PROBLEM CXLVI.

A ship at S *observes three ports* A, B, *and* C, *whose distances are known to be* A B = 99, A C = 71, B C = 86, *and the angles of observation are* A S B = 127°, A S C = 128°, *so the* ∠ B S C *must be* = 105°, *(for they all must be* = 360°) *how far is she distant from each port.* Fig. 114.

1. With the three given distances 71, 86, and 99 make a △ A B C, now because ∠ A S B = 127° is opposite to A B, therefore, as in the last problem, make ∠ E A B = ∠ E B A each = 37°, and upon E where the sides meet with the radius E A = E B describe the arch A S B, also ∠ B S C being = 105° you must make ∠ D C B = ∠ D B C each = 15°, (see the last problem) and upon D with the radius D C or D B describe the arch D S B, crossing the last arch in S the place of the ship, whose distance from the three ports (by the diagonal scale) is found S A = 40,5, S B = 67, and S C = 42.

PROBLEM CXLVII.

A ship at S *observes three ports* A, B, C, *whose distances ar known to be* A B = 80, A C = 72, *and* B C = 120, *the angles under which she observes these ports are* C S A = 25°, B S A = 19°, *so* C S B *must be* 44°. *Quere, her distance from each port.* Fig. 113.

1. With the three sides make the △ A B C. 2. Upon B make the ∠ D B C = 25°, and upon C make the ∠ B C D = 19° drawing C D and B D till they meet in D. 3. About the △ B D C describe a circle, lay a ruler to D and A, and draw the line A D S cutting the circle in

S the place of the ship, join S B and S C, which by the diagonal scale are found S A = 164, S B = 95, and S C = 158, the distances sought, if the port A was farthest from the ship, but if it was nearest, then the distances are P a = 96, P B = 167, and P C = 149. For, by theorem 7, the ∠s D S B and D C F standing both on the same chord &c. are =, and for the same reason ∠ C S D = ∠ C B D, the same holds when a is one of the 3 ports. These 3 last problems shew how a figure may be constructed, when an angular point or points is required to be found, &c. The 3 following ones are on the same subject.

PROBLEM CXLVIII.

In a plane △ A B C (Fig. 115.) *is given the base* A C, *with its opposite* ∠ A B C, *and length of a line* B D *drawn from the said* ∠ *to bisect the said side* A C, *to construct the* △.

1. On the given side A C (by problem 61) describe the segment A B C of a circle so as to contain an ∠ A B C = the given one. 2. With the line B D in your compasses, and one foot in D the middle of A C, sweep an arch to cross the segment in B. 3. Join B A and B C, so is A B C the △ required. If the line B D were to bisect the given ∠ A B C instead of the base A C, then in practice, you may make the ∠ A B C = the given one, which bisect with B D, making it the given length, then prick A C on the edge of the ruler, which apply to the point D, so as these two pricks may fall on the lines B A and B C, and its evident the △ will be formed.

PROBLEM CXLIX.

In a △ *is given an* ∠, *the ratio of the containing sides, and the ratio of the segments of the base, to make the* △. Fig. 116.

1. Draw a line P C at pleasure, in which take it as A E : E B : : greater side : lesser side, and as A F : F B : : the greater segment of the base : the lesser segment. 2. Upon A B (by problem 61) make a segment of a circle to contain an ∠ A R B = given one P Q T. 3. Make it as E C : E A : : E B : E A — E B, and with the radius F C sweep an

arch E Q R, cutting the laſt arch in R, join R B and B A, ſo is A R B the △ required, for (by theorem 20) as A R : B R : : A E : B E, if the ⊥ were F Q given inſtead of the ∠, then upon E raiſe the ⊥ E Q, and thro' Q parallel to R A and R B draw Q P and Q T, ſo will P Q T be the required △. For by reaſon of theſe parallel lines it will be every way ſimilar to the △ A R B.

PROBLEM CL.

Given the 3 ∠s 45°, 56° 15', and 78° 45' with the 3 ſides in one ſum = 100, to find the ſides ſeverally. Fig. 117.

1. From a ſcale of equal parts make K P = 100 the given ſum. 2. Make the ∠s P K p = 45° and p P K = 56° 15'. 3. Biſect theſe two ∠s and at D where the biſecting lines meet, draw D B and D C parallel to K p and p P, ſo is B C D the △ required. And by the ſame ſcale of equal parts, you'll find D B = 33,01, D C = 28,06, and B C = 38,93. Arith. you may ſuppoſe any one of the ſides = what number you pleaſe, and then by it and the ∠s (by axiom 2.) find the other two ſides, and it will be (by ſimilar △s) : ſum of the 3 ſides thus had : any one of them : : 100 the given ſum : the true ſide, and ſo may you find them all three.

Demonſtration. The ∠ D B C = p K P and ∠ P K D = ∠ K D B = ½ ∠ p K P, whence, D B = K B ; in like manner it may be proved, that D C = P C, therefore D C + D B + B C = 100 = K P. Q. E. D.

PROBLEM CLI.

Let A B C D *repreſent a rectangular billiard table ; required in what direction a ball from a given point* P *muſt be ſtruck ſo that after three reflections it may fall into a purſe at the* ∠ B. Fig. 118.

1. Parallel to B D draw P K, making H K = twice B D, upon K raiſe the ⊥ K M making it = A H + C D, join M P ſo is d P M the direction required. For if the ball P be ſtruck in the oblique direction d s it will ſtrike A B in E, and from thence reflected back to F, from F to G, and from G to B, ſtill making the ∠s of reflection and incidence

$\doteq$, (see theorem 164) whence the ∠s H, A, C, and D being right ones, the △s P H E, F E A, F C G and B G D are all similar, and per fig. P H + A F + F C + B D is given = P H + 2 B D and E H + E A + G C + G D is also given = A H + C D, therefore, as P H + 2 B D : A H + C D : : P H : H E, which gives the above construction.

The 12 following problems are the most useful in

SOLID GEOMETRY,

For shewing the nature of solids, &c.

PROBLEM CLII.

From a given point in any plane to erect a ⊥ to that plane. 11. E. 12.

In common practice you may take a carpenters square, which applied to the given point will direct the ⊥.

PROBLEM CLIII.

To know if any wall, &c. be ⊥ to the horizon.

Stick one end of a staff into the ground with a line and plumet fastened to the other, then if the wall, &c. be ⊥ you will by standing at a small distance from it observe it to be parallel to the line and plumet.

PROBLEM CLIV.

To make a pyramid or solid angle. Fig. 119.

1. Suppose you would have a square pyramid, then make 4 = isosceles △s, join their vertexes together laying their sides along each other, and you'll have the form of a square pyramid A v B C D made of the 4 = △s A B v, A D v, C B v, and C D v; after the same manner may a pyramid of any number of sides be made, for every side of the pyramid has a △, be its sides more or less. But here it is to be observed, that the sum of the vertical ∠s of the plane ∠s which constitute a solid ∠

must be less than 360°, otherwise the angular points when connected will be all in one plane, as in a circle's center. Also, any two of the said ∠s however taken must be greater than any one of the other ∠s, or they cannot possibly form a solid ∠.

PROBLEM CLV.

How to make a cone. Fig. 120.

1, With any convenient radius describe a sector A v B upon pasteboard, stiff paper, or the like, then cut it out close by the edges A v and B v, as also by the arch A B, then this turned together so as v A lies upon v B will form a cone. By the same method you may make a cylinder with a rectangle,

PROBLEM CLVI.

Given d c *the radius of a sphere, to find the side of any of the five regular bodies inscribed therein.* Fig. 121.

1. Having described the semicircle divide the diameter into 3 = parts, viz. d a = b a = b r, erect the ⊥s a e and c f, and draw e r, e d, d f, cut e d in extreme and mean proportion in h, upon r raise the ⊥ r G making it = r d, join c G cutting the circle in n, join r n, now if we take r d the axis of the sphere = 2, then by the nature of a circle we have

1. r e = 1,62299	for a side of the greatest	Tetrahedron	inscribed in that sphere.
2. d f = 1,15470		Hexahedron	
3. d e = 1.41421		Octahedron	
4. d h = 0,71364		Dodecahedron	
5. r n = 1,05146		Icosehedron	

See the five regular bodies in the explanations, as well as the five following problems.

PROBLEM CLVII. CLVIII. CLIX. CLX. CLXI.

To represent all the five regular bodies.

1. A tetrahedron is a ſolid contained under four equilateral Δs and is no more but a triangular pyramid whoſe baſe and three ſides are all = equilateral Δs, and is formed by turning up the three Δs A B and E upon the ſides of the Δ D. Fig. 122.

2. A hexahedron, or cube, is form'd by turning up the four = □s mark'd 2, 4, 5, 6, upon the ſides of the = □ mark'd 3, over which as a cover bend the □ mark'd 1. Fig. 123.

3. An octahedron, is a ſolid contained under 8 = equilateral Δs, and is formed by folding the Δs in Fig. 124.

4. A dodecahedron, is a ſolid contained under 12 = equilateral and equiangular pentagons, and may be formed by folding up the 12 regular pentagons, in Fig. 125.

5. An icoſehedron, is a ſolid conſiſting of 20 triangular pyramids, whoſe vertexes meet in the center of a ſphere, which is imagined to circumſcribe it, and may be form'd by folding &c. the ∠s in Fig 126.

Note. Theſe figures ſhould be drawn on ſtiff paper or paſte-board, and then in the paſte-board cut the lines half way thro', and bend or fold in the ſeveral planes of which the body conſiſts, and they will cloſe together in ſuch ſort as to form the body deſigned.

PROBLEM CLXII.

To find the axis of any cone or pyramid, or of any fruſtum thereof.

Fig. 127.

1. Meaſure any length from B the baſe as B C, with a ſtreight ſtaff or line. 2. Meaſure from C to u, (in a direction parallel to the baſe A B) to the ſide u B of the ſolid, then it is evident, that B C is = L D = u o, the axis of the fruſtum A B C G, and C u (= B o) taken from B D leaves L u (= D o) which doubled gives G u the breadth &c. of the leſſer baſe of the fruſtum, make A a = G F and A e = G u, ſo will E a and u e be parallel to A v, whence the Δs A v B, a E B, and e u B will be alike, and it will be as e B (A B — G u) : o u, or : e a : : A B : D v or : A v, and : : a B (A B — E F) : n E, or : D E, &c. whence we have a rule to take the dimenſions of a growing tree, &c. thus, ſuppoſe a tree be 20 feet high, 40 inches girt at bottom, and $38\frac{1}{2}$ inches 3 feet

above the bottom, it will be as 3 feet is to $1\frac{1}{2}$ inches $(40 - 38\frac{1}{2})$ so is 20 feet to 10 inches, which taken from 40 leaves 30 inches the girth at top. The same holds true in any frustum of a pyramid, by using the peripheries at the base and what distance from it you please; or instead of the peripheries you may take two similar sides, and so find a like side at the top.

PROBLEM CLXIII.

To take the dimensions of any solid, suppose of a cask, H B H L d D d E. Fig. 128.

1. Close to each end of the cask set a streight ruler as A P and C Q ⊥ to E L its axis. 2. Upon the bung B lay another streight ruler A C, parallel to the said axis E L, and meeting the former two rulers in A and C, then it is evident that C A will be = H H = E L the cask's length, and that twice A H added to H d is = B D the bung diameter. Also, twice N m taken from B D leaves a diameter m D, as taken at m, between the bung and the head.

Note. The thickness of the cask and what the staves overshoot the heads must be considered. Also if the heads of the cask be =, then A C will be parallel to E L when A H is = C H, but if the ruler A C is not parallel to E L, it is plain by the figure, that the sum of the parts C H and A H added to H d will give B D.

If the cask stand upright, as fig. 129, you may do it easier thus, lay a streight ruler d Q over the top of the cask with a line and plumet Q P at the end Q; now it is plain that if the cask be ⊥ Q P will be parallel to E L its axis, and that twice H Q added to H d gives the diameter B D, the line touching the cask in the point B, also twice n m taken from B D leaves the diameter n D, &c. after the same manner you may take the dimensions of other solids as occasion may require.

The End of the First Part.

THE UNIVERSAL
MEASURER
AND
MECHANIC.

CONTAINING

The Principles of ALGEBRA, GEOMETRY, TRIGONOMETRY, MENSURATIONS, CONIC SECTIONS, SURVEYING, GAUGING, MECHANICS, PROJECTILES, HYDROSTATICS, HYDRAULICS, MECHANIC POWERS, PENDULUMS, CENTERS of GRAVITY, PERCUSSION, OSCILLATION, LAWS of MOTION, DESCENT of HEAVY BODIES, PNEUMATICS, PUMPS, BAROMETERS, STRESS and SRENGTH of WALLS, BEAMS, &c. WHEEL-CARRIAGES, RESISTANCE of FLUIDS, &c. &c.

IT is common for writers on algebra and geometry, to add a collection of questions by way of illustration; however I have ventured to deviate from the common method, by supplying this speculative part with useful articles and general theorems analytically demonstrated, which I hope will be thought as good exercise as the solutions of many questions, and in the mean time furnish a book with a large collection of principles applicable to the sollutions of questions in many branches of the mathematics and natural philosophy: such a collection and so digested, I presume cannot be found in any other book, which I hope will be thought worthy of my reader's notice.

DEFINITION. An algebraic procefs is no more than a comparative way of reafoning, and is founded on thefe fix fundamental axioms, viz.

1. Quantities that are equal to one and the fame thing, are equal themfelves.

2. If quantities laid on each other agree in each part, they muft be equal. By this axiom many of the propofitions in Euclid may be demonftrated.

3. If equal quantities are added to or taken from equal ones their fums or differences will be equal.

4. and 5. The whole is equal to all its parts taken together; and if quantities are equal, their like parts, as halves, doubles, trebles, &c. will be equal.

6. But if quantities are unequal thefe parts or fums, &c. will be unequal, and that which was greateft will remain fo, &c.

PROBLEM CLXIV.

Algebra is an art by which the moft difficult problems in arithmetic and geometry are folved, by affuming fymbols or letters for the things required, and thofe given alfo if you chufe, for this purpofe the initial letters in the alphabet are ufed, and the final ones for unknown quantities. Thefe letters or quantities muft be connected together by addition, fubtraction, multiplication, divifion, &c. as the nature of the problem requires; which done, obferve if the unknown quantities exceed the known ones in number, the problem is often impoffible, but if contrary it is capable of many anfwers; and when the number of known quantities is equal to that of the known ones, the problem can have but one anfwer, which anfwer or anfwers are had by working out all the unknown quantities except one, and this one being on one fide of the fign = or equation by itfelf, and the known ones on the other fide thereof, the value of this unknown one is thereby known, by taking the values of the known ones in numbers and working with thefe numbers as the connections of their refpective numbers direct. Any letters may be taken, or reprefent any numbers univerfally; fo that in folving any queftion algebraically you'll have a rule or theorem for all queftions of that nature. Obferve carefully that whatever you do on one fide of

the equation, you must with the same value do the like on the other side, otherwise, it is plain, the thing will lose its equality. In vanishing the unknown letters, you must use such means as you think will best and easiest do it, whether by addition, subtraction, &c. Also, for every different value in your problem or question, make choice of different letters to prevent mistakes.

Of SIGNS and CHARACTERS,

their names and significations in algebra and geometry.

$=$ Equal to. $10=10$, $a=e$, i. e. 10 is equal 10, or a equal e.

$+$ Plus or more. $a+e=z$, i. e. a more or added to e is equal z.

$-$ Minus or less. $a-e=z$, i. e. e taken from a leaves z.

$\times$ Multiplied. $a\times e$ or a e, either of which denotes the prod. of a and e.

$\div$ Divided by $a\div e$, or $\frac{a}{e}$ shews that a is divided by e.

$\sqrt{}$ or $\overline{}|^{\frac{1}{2}}$ Square root. $\sqrt{9}$, or $\overline{9}|^{\frac{1}{2}}=3$, the square root of 9 is equal 3.

$\square$ or $\overline{}|^{2}$ Square. $\square 9$ or $\overline{}|^{2}$ the square of 9 is to be taken.

:, : :, : Proportional. 2 : 4 : : 6 : 12, as 2 is to 4 so is 6 to 12.

$\angle$ Less than. $4 \angle 6$, 4 is less than 6.

$\lrcorner$ Greater than. $6 \lrcorner 4$, 6 is greater than 4.

$\because$ Ergo, or therefore. ——— $\angle$ Angle.

R $\angle$ or $\llcorner$ Right angle. ——— $\square$ Parallelogram.

$\triangle$ Triangle. ——— $\odot$ Circle, or the sun.

$\|$ Parallel. —— $\mp$ Minus and Plus. ——— $\pm$ Plus and Minus.

C $\square$ Complete square. —— ⅁ Involution. —— ω Evolution.

$a\times e\times u$, or a . e . u, or a e u, denotes the product of the quantities a e and u.

(47 E. 1) the 47 proposition of Euclid's first book.

$\angle$s Angles. ——— $\llcorner$s Right angles. ——— $\square$s Squares.

$\square$s Parallelograms. —— $\triangle$s Triangles. —— $\perp$s Perpendiculars

$\|$. Parallel to. —— $\propto$ Proportional to. —— $\sqrt[3]{}$ Cube root.

$\sqrt[n]{}$ The nth root. —— $\overline{}|^{3}$ Cubed power. —— $\overline{}|^{n}$ The n power.

$\sqrt[n]{a+e}$ or $\sqrt[n]{}:a+e$, or $\overline{a+e}|^{\frac{1}{n}}$ all shew or express the root, whose index is n is to be taken of $a+e$, also $\overline{a+e}|^{n}$ shews the sum of a and e is to be raised to the power denoted by n, and $\overline{a+e}\times u$, or : $a+e$: $\times u$ denotes the product a and e multiplied u. 2°, 5′, 6″ denote 2 degrees 5 minutes 6 seconds.

Numbers prefixed to quantities are called coefficients, and shew how often the quantity is to be taken, as 5 a denotes 5 times a, 6 a e, 6 times the product of a and e, $\frac{7a}{2e}$ or $7a \div 2e$ is 7 times a divided by twice e. When letters are joined together without the signs + or — among them they are called simple quantities, as a, 2 a e, $\frac{2e}{a}$ otherwise compound quantities such as $a + e$. $a - e + cd$, $-ae - gh e$. Quantities that have the sign + or no sign before them are positive, or affirmative quantities, and those that have — before them as $-a - cd$ are negative quantities. Every quantity has its sign on the left hand side implying its positive or negative relation. Multiplication joins the quantities together, and quantities divided as vulgar fractions, the dividend above and the divisor under a line drawn between them.

N. B. Quantities may be ranged in any order without altering their value, as $5ab - c$, or $-c + 5ab$, or $5ba - c$, &c. are all the same, for c is still negative and 5 a into b positive.

Figures following quantities are called indexes or indices, or exponents; thus, $a^{\frac{3}{2}}$ is $\sqrt{aaa}$, $:\overline{a+e}|:^{\frac{m}{n}}$ implies $a + e$ is to be involved to the m power, and evolved to the n power, such are called surd quantities. To illustrate these things more clearly;

Let $a = 6$, $b = 5$ and $c = 4$, then will

$$aa + 3ab - cc = a^2 + 3ba - c^2 = 36 + 90 - 16 = 110$$

$$2aaa - 3aab + ccc = 2a^3 - 3a^2b + c^3 = 432 - 540 + 64 = -44$$

$$a \times : a + b : - c = aa + ba - c = 36 + 30 - 4 = 62$$

$$\sqrt{2ac + cc} = \overline{2ac + c^2}|^{\frac{1}{2}} = \sqrt{: c^2 + 2ac :} = \sqrt{64} = 8$$

$$\frac{aaa}{a + c} = \frac{a^3}{a + c} = \frac{216}{10} = 21.6$$

$$\frac{a^3}{a + c} - cc = \frac{216}{10} - 16 = 21,6 - 16 = 5,6$$

$\frac{\sqrt{a}}{c} = \frac{\sqrt{6}}{4}$, but $\sqrt{\frac{a}{c}} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}} = \sqrt{1,5}$. And so on.

If no figure be before a quantity, you are to suppose an unit before it, so 1 a, 1 e, are the the same as a, e.

	1	$a + e = s$
	2	$a - e = d$
$1 + 2$	3	$2a = s + d$
$3 \times \bar{5}$	4	$10a = 5s + 5d$

On the left hand ſide of algebraic operations is ruled two lines with figures between them called ſteps, and thoſe on the left hand ſide of the ſteps are to ſhew how theſe operations are performed; ſo here againſt the third ſtep we have $1 + 2$, i. e. the quantities in the 1ſt and 2d ſtep are added together and their ſum placed in the 3d ſtep; alſo againſt the 4th ſtep $3 \times \bar{5}$, i. e. 5 times the quantities in the 3d ſtep are placed in the 4th ſtep. Obſerve, where a daſh is over a figure as $\bar{5}$, it ſhews that $\bar{5}$ denotes the number 5, without which it would denote the 5th ſtep. Some authors do not make uſe of ſteps, tho' I think them much eaſier, as they ſhew how every ſtep is wrought. Quantities as a and — a, or 2 b c and c b, or — 8 d a e and + 9 d a e, &c. conſiſting of the ſame letters are called like quantities, let the ſigns and coefficients be what they will, and vice verſa.

PROBLEM CXCV.

To add algebraic quantities.

Rule. When the quantities are unlike they can be no way added but by ſetting them down with their reſpective ſigns before them; but if they are alike and have the ſame ſigns, the coefficients are added together and to their ſum the quantities are annexed and ſigns prefixed, but if the ſigns are unlike, you muſt take the difference inſtead of the ſum of the coefficients, and ſo prefix to it the ſign of the greater coefficient, for a negative quantity is always leſs than nothing, ſuppoſe a man is only worth 1005l and owes 2000, the true ſum or ballance will be —995l his real worth, or 995l worſe than nothing.

EXAMPLES.

		1.	2.	3.	4.	5.	7.
	1	a	$-a$	$-a$	$17a-6b$	$5a+bd$	$bd-ce-\sqrt{a}+d$
	2	a	$-a$	a	$12a-b$	$2bd-3a$	$\sqrt{a}+db-10ce-\sqrt{e}$
$1+2$	3	$2a$	$-2a$	0	$29a-7b$	$3bd+2a$	$2db-11ec+d-\sqrt{e}$

		8.	9.
	1	$a+\sqrt{:a-e:}+5$	$a+\sqrt{:a+e:}+5$
	2	$2a+\sqrt{:a-e:}-10$	$2a+\sqrt{:a-e:}-10$
$1+2$	3	$3a+2\sqrt{:a-e:}-5$	$3a-5+\sqrt{:a+e:}+\sqrt{:a-e:}$

10.

	1	$-10\sqrt{:e+u:}+\sqrt{e-u:}$
	2	$-\sqrt{:e+u:}-\sqrt{:e-u:}$
1 + 2	3	$9\sqrt{:e+u:}$

11.

	1	$aaa+aa-a$
	2	$\sqrt{aaa}+{}^3\sqrt{aa}-\sqrt{a}$
	3	$\sqrt{:aaa+aa:}-{}^3\sqrt{a}$
1 + 2 + 3	4	$aaa+aa-a+\sqrt{a^3}+\overline{a^2}\vert^{\frac{1}{3}}-\sqrt{a}+\sqrt{:a^3+a^2:}-\overline{a}\vert^{\frac{1}{3}}$

If the indices of surds be unlike, as $\sqrt{a}$ and ${}^3\sqrt{a}$, or the quantities within the surd sign be unlike, as $\sqrt{:a+e:}$ and $\sqrt{:a+ee:}$ or have unlike signs as $\sqrt{:a-e:}$ and $\sqrt{:e-a:}$ it is evident they have different values, and so must be added as if they were different simple quantities, as in the foregoing examples.

PROBLEM CLXVI.

Subtraction of algebraic quantities.

Rule. Subtraction being the reverse of addition. If therefore you suppose all the signs in the subtrahend to be changed, viz. — into + and + into —, you may work as in addition.

		12.	13.	14.	15.	16.
	1	$2a$	$-2a$	0	$29a-7b$	$3bd+2a$
	2	a	$-a$	$+a$	$12a-b$	$2bd-3a-e$
1—2	3	a	$-a$	$-a$	$17a-6b$	$bd+5a+e$

		17.	18.	19.
	1	$2bd-11c-e$	$a+\sqrt{a}$	$aaa+\sqrt{a^3}$
	2	$e+11c-2bd$	$\sqrt{a}-a$	$a^3-\sqrt{aaa}$
1—2	3	$0 \quad 0 \quad 0$	$2a+2\sqrt{a}$	$2\sqrt{aaa}$

20.

	1	$\frac{\sqrt{:a-e+2:}}{a+c}$
	2	$-\sqrt{\frac{:a-e+2:}{a+c}}$
1—2	3	$\sqrt{\frac{:a-e+2:}{a+c}}+\sqrt{\frac{:a-e+2:}{a+c}}$

PROBLEM CLXVII.

Multiplication.

Rule. Multiply the coefficients of ſimple quantities together, and to the product annex the quantities, and prefix the ſign + if both ſigns were + or both —, but if one was + and the other — you muſt prefix —, i. e. + into + or — into — gives + in the product, but — into + gives —. See examples 86 and 87.

		21.	22.	23.	24.	25.	26.
	1	a	—a	—a	—16a	+180e	$2\surd$: a — ee:
	2	a	—a	+a	+2aa	—2ab	7aaec
1×2	3	aa	aa	—aa	—32aaa	—360abe	14aaec$\surd$: a — ee:

By ex. 21 and 22, it appears, that to multiply two different powers a^2 and a^5 &c. of the ſame quantity a, is but to take the ſum of their indexes 2 and 5, i. e. $a^2 \times a^5 = a^7$, and univerſally, (ex. 27) $a^n \times a^m = a^{n+m}$ Alſo, (ex. 28) $^{\frac{1}{n}} + a^{\frac{1}{m}} = a^{\frac{1}{n}+\frac{1}{m}}$, and, $a^{\frac{v}{n}} + a^{\frac{e}{m}} = a^{\frac{v}{n}+\frac{e}{m}} = a^{\frac{vm+en}{nm}}$ By reducing the indices to a common denominator and adding them together, (ex. 29) If any number be ſquared or cubed, &c. and the ſurd ſign of that power prefixed to it, ſuch a ſurd will be = to the firſt taken number, i. e. $3 \times 3 = 9$ and $\sqrt{9} = 3$, alſo $3 \times 3 \times 3 = 27$, and $\sqrt[3]{27} = 3$ &c. whence $\overline{a^n}|^m$ or $a^{n\,m} + b = \overline{a^n}|^m b = b\,\overline{^{\frac{1}{m}} a^n}|$ alſo, $a\sqrt{a - e} = \sqrt{a^3 - a^2 e}$ &c. If the multiplier be a compound quantity you muſt work with every term therein as a ſimple quantity, and the ſum of all theſe ſimple products will be the product ſought.

30.

	1	a + b
	2	a — b
1 × a	3	aa + ba
1 × b	4	—ab — bb
3 + 4	5	aa — bb

31.

	1	$a + e - ue + \frac{ae}{c}$
	2	a — e
1 × a	3	$aa + ae - uea + \frac{aae}{c}$
1 × —e	4	$-ae - ee + uee - \frac{aee}{c}$
3 + 4	5	$aa - ee - uea + uee + \frac{aae}{c} - \frac{aee}{c}$

		32.		33.
	1	$\sqrt{aa-ee}$	1	$\overline{a+e}\vert^{n}$
	2	$a-e$	2	$\overline{a-e}\vert^{n}$
1 × a	3	$\sqrt{a^4-a^2e^2}$	3	$\overline{aa+ae}\vert^{n}$
1 × −e	4	$\sqrt{a^2e^2-e^4}$	4	$\overline{-ae-ee}\vert^{n}$
3 + 4	5	$\sqrt{a^4-a^2e^2}+\sqrt{a^2e^2-e^4}$	5	$\overline{aa-ee}\vert^{n}$

If surds to be multiplied have the same index (as in ex. 33) you may multiply them as whole quantities, and to the product join the surd sign.

PROBLEM CLXVIII.

Division.

Rule. In simple quantities cast such quantities out of the dividend as you find in the divisor, and what remains will be the quotient; observing that + by + or — by — gives +, but + by — or — by + gives — in the quotient: that is, division is just the reverse of multiplication. See example 51.

		34.	35.	36.	37.	38.	39.
	1	aa	aa	$-aa$	$-32a^3$	$7ae$	$-8ae$
	2	a	$-a$	a	$2a$	eu	$2ue$
1 ÷ 2	3	a	$-a$	$-a$	$-16aa$	$\frac{7a}{u}$	$-\frac{4a}{u}$

		40.	41.	42.	43.
	1	ba^m	ba^n	$8ab\sqrt{ay}$	$15\times\overline{e^3+y^3}\vert^{\frac{1}{n}}\times:a+b:$
	2	ba^n	ea^m	$16abb\sqrt{ay}$	$7c\times\overline{e^3+y^3}\vert^{\frac{1}{n}}\times:a+b:$
1 ÷ 2	3	a^{m-n}	$\frac{b}{e}a^{n-m}$	$\frac{1}{2ab}$	$\frac{15}{7c}$

In any operation where the quantities are much compounded as in ex. 43, it will be best to put a single letter = to such compounded parts, (called substitution) and work with this letter as the nature of the operation directs, and then in the answer you may write such powers &c. of the compound quantity as those of the substituted one directs, (this is called restitution;) thus, ex. 41, put $A = \overline{eee+yyy}\vert^{\frac{1}{n}}$ and $B = a+b$, then ex. 43 will stand $+15BA^{\frac{1}{n}} \div 7cBA = \frac{15AB}{7cBA}$ where by the rule it plainly appears that $\frac{15}{7c}$ is the quotient required. If the di-

visor and dividend be both compound quantities, let each be ranged in one order that the greatest term may stand first &c. in order, or they may stand in any order to be most convenient for division, then take such a term for the first term in the quotient as that the first term in the divisor being multiplied thereby, the product may be = to the first term in the dividend, multiply all the divisors by this quotient term, and make subtraction as in common division, and look upon the remainder as a new dividend, then find a quotient term as before; and thus go on until division is finished, and if at last there be a remainder you may proceed on and so get a series, which is the method of putting a fraction into a series by division. It is to be observed, that after a few of the first terms are discovered by division, the law of continuation will appear, and may be carried as far as you please without dividing.

Ex. 45 and 46. Divide $aa-ee$ by $a+e$, and $a^3+5a^2e+5ae^2+e^3$ by $a+e$.

$$
\begin{array}{l}
a+e)\,aa-ee\,(a-e \\
\quad \underline{aa+ae} \\
\quad\quad -ae-ee \\
\quad\quad \underline{-ae-ee}
\end{array}
\qquad
\begin{array}{l}
a+e)\,a^3+5a^2e+5ae^2+e^3\,(a^2+4ae+e^2 \\
\quad \underline{a^3+a^2e} \\
\quad\quad +4a^2e+5ae^2 \\
\quad\quad \underline{+4a^2e+4ae^2} \\
\quad\quad\quad\quad +ae^2+e^3 \\
\quad\quad\quad\quad \underline{+ae^2+e^3}
\end{array}
$$

Ex. 47. Divide ee by $e-a$.

$$
\begin{array}{l}
e-a)\;ee\;\left(e+a+\frac{a^2}{e}+\frac{a^3}{e^2}+\&c.\right. \\
\quad \underline{ee-ae} \\
\quad\quad +ae \\
\quad\quad \underline{+ae-a^2} \\
\quad\quad\quad +a^2 \\
\quad\quad\quad \underline{+a^2-\frac{a^3}{e}} \\
\quad\quad\quad\quad \frac{a^3}{e} \\
\quad\quad\quad \underline{+\frac{a^3}{e}-\frac{a^4}{e^2}} \\
\text{remainder}\quad +\frac{a^4}{e^2}
\end{array}
$$

= the quotient required. In which you see the signs are all positive, and every following term = the next foregoing one multiplied by $\frac{a}{e}$ whence the terms may be continued at pleasure, which appears to be a decreasing geometrical series, whose first term is e, second a, and common ratio $\frac{a}{e}$ or e to a. And because every whole $(e-a)\,ee$ or $\frac{ee}{e-a}$ is = to the sum of all its parts, $(e+a+\frac{a^2}{e}+\frac{a^3}{e^2}$ &c.) we have this general rule for the sum of such a series; viz. divide the square of the first or greatest term by the difference of the first and second terms, and the quotient is the sum of all the terms in the series, the terms continued until the last term be infinitely small; as for instance, if a body move 12 the first hour, 10 the

ſecond $\frac{100}{12}$ the third, $\frac{1000}{144}$ the fourth &c, here e = 12 and a = 10, ſo if the body move for ever at that rate it cannot exceed $\frac{ee}{e-a} = 72$.

Ex. 48. Divide e y by e — 1, or which is the ſame, put $\frac{ey}{e-1}$ into an infinite ſeries

$$
\begin{array}{l}
e-1\,)\,ey\,\left(y+\frac{y}{e}+\frac{y}{ee}+\frac{y}{eee}\right\}\ \&c. \\
\quad \underline{ey-y} \\
\qquad +y \\
\qquad \underline{y-\frac{y}{e}} \\
\qquad\quad +\frac{y}{e} \\
\qquad\quad \underline{\frac{y}{e}-\frac{y}{ee}} \\
\qquad\qquad +\frac{y}{ee} \\
\qquad\qquad \underline{\frac{y}{ee}-\frac{y}{e^3}} \\
\text{remains} \qquad +\frac{y}{e^3}
\end{array}
$$

&c. where it appears that each following term is had by multiplying its foregoing term by $\frac{1}{e}$ which being the common factor or ratio, ſhews the ſeries to be a geometrical diverging one, 1ſt term y, common ratio or diviſor e; hence as before we have another general rule, divide the product of the greateſt term and common ratio by the difference between the ſaid ratio and unity, and the quotient will be the ſum of all the terms in the ſeries, the laſt or leaſt term being indefinitely ſmall or in effect = 0, ſo in the laſt queſtion, $\frac{a}{e} = \frac{10}{12} = \frac{1}{1.2}$ whence the common ratio is = 1,2 = e, therefore $\frac{ey}{e-1} = \frac{12 \times 1,2}{1,2-1} = \frac{14,4}{0,2} = 72$ as before, but if inſtead of the laſt or leaſt term being taken = 0, we take it = ſome aſſignable quantity as a, then it is evident that $\frac{ey}{e-1}$ will be the ſum too much by $\frac{a}{e-1}$ and therefore $\frac{ey}{e-1} - \frac{a}{e-1} = \frac{ey-a}{e-1}$ will be the ſum of thoſe terms between y and a; and if inſtead of ſuppoſing the ſeries to begin at y the greateſt term and decreaſe to a, we ſuppoſe them to begin at a and increaſe to y, i. e. if for $y + \frac{y}{e} + \frac{y}{ee} + \frac{y}{eee}\Big\}$ &c. we take $a + ae + aee + ae^3 +$ &c. we ſhall have $\frac{ey-a}{e-1}$ for the ſum of any geometrical increaſing ſeries, whoſe firſt term is a, common ratio or multiplier e, which is a new, ſhort and eaſy inveſtigation of geometrical progreſſions.

Ex. 49 and 50. If $\frac{aae}{a-e}$ and $\frac{z^6-z^3}{z^6+2z^3+1}$ be the sums of two series, what are the terms of these two series?

$$a-e)aae\ (ae+ee+\frac{eee}{a}+\frac{e^4}{aa}+\&c.$$ where the law of continuation appears.

$$\begin{array}{l} aae-aae \\ \hline +ae^2 \\ ae^2-e^3 \\ \hline +e^3 \\ e^3-\frac{e^4}{a} \\ \hline +\frac{e^4}{a} \\ \frac{e^4}{a}-\frac{e^5}{aa} \\ \hline \text{remains } +\frac{e^5}{aa} \end{array}$$

$$z^6+2z^3+1)z^6-z^3\ (1-\frac{3}{z^3}+\frac{5}{z^6}-\frac{7}{z^9}\&c.$$

$$\begin{array}{l} z^6+2z^3+1 \\ \hline -3z^3-1 \\ -3z^3-6-\frac{3}{z^3} \\ \hline +5+\frac{3}{z^3} \\ +5+\frac{10}{z^3}+\frac{5}{z^6} \\ \hline -\frac{7}{z^3}-\frac{5}{z^6}\&c. \end{array}$$

PROBLEM CLXIX.

Of algebraic fractions, or broken quantities.

The rules in vulgar fractions hold true in algebraic fractions. The most useful cases in reduction of broken quantities are these 4. 1st, To reduce fractions to their lowest term; 2, to a common denominator; 3, a mixed expression to a fraction; 4, and the contrary.

Lemma. If the numerator and denominator of any fraction be each multiplied or divided by the same quantity, the new fraction will be = in value to the old one; ex. 5, so $\frac{a}{e}=\frac{a}{e}\times\frac{u}{u}=\frac{au}{eu}=\frac{a}{e}\times\frac{zz}{zz}=\frac{auzz}{euzz}$ &c. which is plain from the nature of division in common arithmetic. Also ex. 52, $e\times\frac{aa}{e}=\frac{eaa}{e}=aa$, and $z^6\times\frac{5}{z^6}=\frac{5z^6}{z^6}=5$, &c. whence to reduce a fraction to its lowest terms, we have this general rule, cast like quantities out of both numerator and denominator.

Ex. 53. so $\frac{eaau}{auc}=\frac{ea}{c}$ and $\frac{ezzbd}{defc}=\frac{zzb}{fc}$ also, $\frac{aa-ee}{a+e}=a-e$, also, $\frac{zz-ee}{bz-be}=\frac{z-e}{b}$ &c. Hence also it appears, the method of reducing fractions to common denominators, for if you can multiply the denominator of a fraction by such a quantity so that it may be = the denominator of another fraction, the numerator of that fraction multiplied by the same quantity, the fraction will (per lemma) keep its value, and have a denominator = or common with the other fraction.

Ex. 54. so $\frac{a}{e} \times \frac{u}{u} = \frac{au}{eu}$ therefore, $\frac{a}{e}$ and $\frac{nm}{eu}$ when reduced to 1 denom. will be $\frac{au}{eu}$ and $\frac{nm}{eu}$ also $\frac{a}{b}$ and $\frac{c}{d}$ in a common denominator will be $\frac{ad}{bd}$ and $\frac{cb}{bd}$, which gives the common rule, viz. multiply the denominators continually for a common denominator, and every numerator into all the denominators except its own denominator for a new numerator. Fractions thus reduced to a common denominator may be added by taking the sum of the numerators, or subtracted by taking their difference and writing it above the common denominator; thus,

Ex. 55. $b + \frac{a}{e}$ added to $\frac{nm}{eu}$ gives $b + \frac{au + nm}{eu}$, also, the difference between $\frac{a}{b}$ and $\frac{c}{d}$ is $= \frac{ad - bc}{bd}$ or $= \frac{bc - ad}{bd}$

Ex. 56. Because (per lemma) $a = \frac{ae}{e}$ and $\frac{cde}{c} = de$, &c. therefore, $b + \frac{a}{e} = \frac{eb + a}{a}$ whence $b + \frac{a}{e} + \frac{d}{u} = \frac{eub + au + de}{eu}$ &c. Mixt quantities being thus ordered, their product is had by taking the product of the numerators and placing it over that of the denominators.

Ex. 57. so $b + \frac{a}{e} : \times \frac{n}{m} = \frac{:be + a:}{e} \times \frac{n}{m} = \frac{neb + na}{em}$

Ex. 58. $\frac{:a - e:}{e} \times \frac{:a + e:}{c} = \frac{aa - ee}{ec}$ &c. If fractions have a common denominator, and you divide the numerators of two such fractions the quotient will be the quotient of these two fractions.

Ex. 59. so $\frac{a}{b} \div \frac{c}{d} = \frac{da}{db} \div \frac{bc}{db} = \frac{da}{bc}$, also $\frac{a+e}{cd} \div \frac{aa - ee}{dc} = \frac{a+e}{aa - ee} = \frac{1}{a - e}$ in its lowest terms, otherwise the numerator of the dividend and denominator of the divisor is a new denominator.

Ex. 60. So $\frac{a}{b} \div \frac{c}{d}$ is $= \frac{da}{cb}$ and $\frac{ea}{b} \div \frac{rs}{c} = \frac{cea}{brs}$, &c. These few examples with the reasons and plainess of the rules, are sufficient for a perfect knowledge of algebraic fractions.

PROBLEM CLXX.

Of involution, evolution, and infinite series.

Rule. Involution is the raising of quantities to any proposed power, and is nothing but a continual multiplication of the given quantities or root into itself.

		61.	62.	63.	64.	65.	66.
	1	a	$-a$	$-ab$	$2e$	$\sqrt{a}$	$\overline{a+e}\vert^{n}$
1 G 2	2	aa	$+aa$	$+aabb$	$4ee$	$a=\sqrt{aa}$	$\overline{a+e}\vert^{2n}$
1 G 3	3	aaa	$-a^3$	$-a^3b^3$	$8e^3$	$a^{\frac{3}{2}}$	$\overline{a+e}\vert^{3n}$
2 G 3	4	a^6	$+a^6$	$+a^6b^6$	$64e^6$	$a^{\frac{6}{2}}=a^3$	$\overline{a+e}\vert^{6n}$
3 G 3	5	a^9	$+a^9$	$-a^9b^9$	$512e^9$	$a^{\frac{9}{2}}$	$\overline{a+e}\vert^{9n}$
5 G $\frac{1}{n}$	6	$a^{\frac{9}{n}}$	$-a^{\frac{9}{n}}$	$-\overline{ab}\vert^{\frac{9}{n}}$	$\overline{512e}\vert^{\frac{9}{n}}$	$a^{\frac{9}{2n}}$	$\overline{a+e}\vert^{9}$

Ex. 67, and 68. Let it be required to raiſe the compound quantities a + e called a binomial, and a — e called a reſidual to the fourth power.

	1	$a+e$ root or ſingle power.
	2	$a+e$
1 × a	3	$aa+ae$
1 × e	4	$+ae+ee$
3 + 4	5	$aa+2ae+ee$ ſquare, or ſecond power of $a+e$
	6	$a+e$
5 × a	7	$aaa+2aae+aee$
5 × e	8	$aae+2aee+eee$
7 + 8	9	$aaa+3aae+3aee+eee$ cube or 3d power of $a+e$
	10	$a+e$
9 × a	11	$a^4+3a^3e+3aaee+e^3a$
9 × e	12	$a^3e+3aaee+3ae^3+e^4$
12 + 11	13	$a^4+4a^3e+6a^2e^2+4ae^3+e^4$ biquadrat of $a+e$

Again	1	$a-e$ root
	2	$a-e$
1 × a	3	$aa-ae$
1 — e	4	$-ae+ee$
3 + 4	5	$aa-2ae+ee$
	6	$a-e$
5 × a	7	$aaa-2aae+eea$
5 × e	8	$-aae+2eea-e^3$
7 + 8	9	$a^3-3aae+3aee-e^3$ cube
	10	$a-e$
9 × a	11	$a^4-3a^3e+3aaec-ae^3$
9×—e	12	$-a^3e+3aaee-3ae^3+e^4$
11 + 12	13	$a^4-4a^3e+6a^2e^2-4ae^3+e^4$ biquadrat

From hence it appears, that if any binomial $a+e$, be raiſed to a power whoſe index is n, the terms without their coefficients will ſtand $a^n + ea^{n-1} + eea^{n-2} + eeea^{n-3} + e^4a^{n-4} +$ &c. until the index of e be $= n$, where it is evident, that as the index of a decreaſes that of e increaſes in arithmetical progreſſion, common difference = unity. If the index of the power be n, it alſo appears that the coefficients will be 1, n, $n\times\frac{n-1}{2}$, $n\times\frac{n-1}{2}\times\frac{n-2}{3}$, $n\times\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{n-3}{4}$ &c. until the number taken from n leave o; i. e. every coefficient is had by multiplying the foregoing one by n, leſſened and divided by an unit more than in that before it. Ex. 69. Theſe coefficients prefixt to their reſpective terms will ſtand thus $a^n + nea^{n-1} + n\frac{n-1}{2}eea^{n-2} + n\frac{n-1}{2}\frac{n-2}{3}eeea^{n-3} +$ &c. or taking $A =$ the 2d coefficient, $B =$ the 3d, $C =$ the 4th, $D =$ the 5th, &c. we ſhall have $A=n$, $B=A\times\frac{n-1}{2}$, $C=B\times\frac{n-2}{3}$, $D=C\times\frac{n-3}{4}$, &c. ſo $a^n + Aea^{n-1} + Beea^{n-2} + Ceeea^{n-3} + De^4a^{n-4} +$ &c. $=\overline{a+e}|^n$ by which any binomial may be raiſed to any power without the trouble of multiplication, ſo if $n=4$, then $A=4$, $B=4\times\frac{4-1}{2}=6$, $C=B\times\frac{n-2}{3}=6\times\frac{4-2}{3}=4$, $D=C\times\frac{n-3}{4}=4\times\frac{4-3}{4}=1$ which put in the laſt expreſſion for A, B, C, &c. and 4 for n, we ſhall have $\overline{a+e}|^4 + 4ea^3 + 6eeaa + 4e^3a + e^4$, the ſame with the ſtep 13 in ex. 67. And if for $a+e$ we take $a-e$, it will be the ſame, only every other ſign will be negative, as appears by ex. 68; as for Ex, 70. Let it be required to raiſe $a-e$ to the 6th power. Here $n=6=A$, then $B=A\times\frac{n-1}{2}=6\times\frac{6-1}{2}=15$, and $C=B\times\frac{n-2}{3}=15\times\frac{6-2}{3}=20$, and $D=C\times\frac{n-3}{y}=20\times\frac{6-3}{4}=15$, ſo the coefficients will be 1, 6, 15, 20, 15, 6, 1, for the firſt half of the coefficients being found, the ſecond is alſo found, the latter decreaſing as the other increaſes, (ſee ex. 67 and 68) now n being $=6$, the terms will be $a^6 - a^5e + a^4e^2 - a^3e^3 + a^2e^4 - ae^5 + e^6$, to which prefix their coefficients, then $a^6 - 6a^5e + 15a^4e^2 - 20a^3e^3 + 15a^2e^4 - 6ae^5 + e^6 = \overline{a-e}|^6$ Ex. 71. Raiſe $a+u+z$ to the nth power. This is the ſame as ex. 69, by writing $u+z$ inſtead of e, ſo we ſhall have $a^n + A\overline{u+z}a^{n-1} + B\overline{u+z}|^2a^{n-2} + C\overline{u+z}|^3a^{n-3} +$ &c. $= \overline{a+u+z}|^n$ In this manner may any expreſſion be raiſed to any power, or put into an infinite ſeries by involution.

Evolution is the reverſe of involution; i. e. as the one raiſes roots to powers, ſo the other finds roots to powers. But to put any ſurd into an infinite ſeries by evolution is the ſame as the laſt example or ex. 69, if for the index you take $\frac{1}{n}$ inſtead of n. Exam. 72. Find the nth root of $a+e$, or put $\overline{a+e}|^{\frac{1}{n}}$ into an infinite ſeries. Here, by ex. 69, we ſhall have $a^{\frac{1}{n}} + Aea^{\frac{1}{n}-1} + Beea^{\frac{1}{n}-2} + Ceeea^{\frac{1}{n}-3} +$ &c. $= \overline{a+e}|^{\frac{1}{n}}$ for it is evident the method of involution makes no difference whether the index be a whole or a fractional quantity, as n or $\frac{1}{n}$ Ex. 73 What's the □ root of $a+e$, or put $\overline{a+e}|^{\frac{1}{2}}$ into an infinite ſeries. Here $A=\frac{1}{2}$, $B=A, \times \frac{\frac{1}{n}-1}{2}\ \frac{1}{2} \times \frac{\frac{1}{2}-1}{2} = -\frac{1}{8}$ $C = B \times \frac{\frac{1}{n}-2}{3} = -\frac{1}{8} \times \frac{\frac{1}{2}-2}{3} = \frac{1}{16}$, $D = C \times \frac{\frac{1}{n}-3}{2}$ $= \frac{1}{16} \times \frac{\frac{1}{2}-3}{4} = -\frac{5}{128}$ &c. which ſubſtituted in the laſt ſeries, for A, B, C, &c. and $\frac{1}{2}$ for $\frac{1}{n}$ gives $a^{\frac{1}{2}} + \frac{1}{2}ea^{-\frac{1}{2}} - \frac{1}{8}eea^{-\frac{3}{2}} + \frac{1}{16}eeea^{-\frac{5}{2}}$ &c $= \overline{a+e}|^{\frac{1}{2}}$ But becauſe by the nature of diviſion, a^{-m} is the ſame as $\frac{1}{a^m}$ (ſee ex. 40, 41.) viz. $a^{1-m} = \frac{a}{a^m} = \frac{1}{a^{m-1}} = a^{-m+1}$ alſo, $\frac{a^n}{a^n}$ $= a^{n-n} = a^0$, but $\frac{a^n}{a^n}$ is known to be $= 1$, conſequently $a^0 = 1$ or unity, let the quantity a be what it will. Likewiſe $a^n \div a^{2n} = \frac{a^n}{a^{2n}} = a^{n-2n} =$ $a^{-n} = \frac{1}{a^n}$, which ſhews that quantities with negative indexes become diviſors, and therefore $\overline{a+e}|^{\frac{1}{2}} = a^{\frac{1}{2}} + \frac{e}{2\sqrt{a}} - \frac{ee}{8a^{\frac{3}{2}}} + \frac{eee}{16a^{\frac{5}{2}}} -$ &c. But the above coefficient may be taken otherwiſe; thus, $A=\frac{1}{2}$, $B=A \times \frac{\frac{1}{2}-1}{2} = \frac{-1}{2,4}$, $C = \frac{+1}{16} = \frac{+13}{2,46}$, $D = \frac{-5}{128} = \frac{-1,3,5}{2,4,6}$, which ſhews the numerators to be a ſeries of odd numbers, and the denominators a ſeries of even ones, obſerved with the indices of a, &c. in a few of the firſt terms, the law of continuation (as in ex. 47) is evident, and may be continued to what number of terms you pleaſe.

Ex. 74. What is the ſquare root of $a-e$? By the above work it will be found $\sqrt{a} - \frac{e}{2\sqrt{a}} - \frac{1,ee}{2,4\sqrt{a^3}} - \frac{1,3,eee}{2,4,6\sqrt{a^5}} - \frac{1,3,5eeee}{2,4,6,8\sqrt{a^7}}$ $- \frac{1,3,5,7e,5}{2,4,6,8,10\sqrt{a^9}} -$ &c. here all the ſigns are negative; by negative ſigns of the terms, (ſee ex. 70) and poſitive ſigns of the coefficients (ſee ex. 73.)

Ex. 75. Put $\overline{aaa-eee}|^{\frac{1}{3}}$ into an infinite series. Here $\frac{1}{n}=\frac{1}{3}=A$, and $B=A\times\frac{\frac{1}{3}-1}{2}=\frac{1}{3}-\frac{2}{6}$, $C=\frac{1}{3}-\frac{2}{6}\frac{\frac{1}{3}-2}{3}=-\frac{1,2}{3,6}-\frac{5}{9}=\frac{1,2,5}{3,6,9}$, $D=\frac{1,2,5}{3,6,9}\times\frac{\frac{1}{3}-3}{4}=-\frac{1,2,5,8}{3,6,9,12}$ &c. now by writing aaa—eee for a + e, and $\frac{1}{3}$ for $\frac{1}{n}$ in the general series in ex. 72, with these values of A, B, C, &c. we shall have $a-\frac{1,eee}{3,aa}-\frac{1,2,e^6}{3,6,a^5}-\frac{1,2,5e^9}{3,6,9,a^8}-\frac{1,2,5,8,e^{12}}{3,6,9,12a^{11}}$ — &c. cube root of a^3-e^3. By this method we may also put any fraction into an infinite series. As for

Ex. 76. $\frac{ee}{e-a}$ (by ex. 73) $=ee\times\overline{e-a}|^{-1}$ Here $A=-1=\frac{1}{n}$, $B=-1,\frac{-1-1}{2}=1$, $C=1,\frac{-1-2}{3}=-1$, $D=-1:\frac{-1-3}{4}=$&c. so by putting these values of A, B, C, &c. with -1 for $\frac{1}{n}$, and e — a for a + e in the said series in ex. 72, we shall have $e^{-1}+ae^{-2}+aae^{-3}+aaae^{-4}+$ &c, $=\overline{e-a}|^{-1}$ which multiplied by e e gives $e+a+\frac{aa}{e}+\frac{a^3}{ee}+$ &c. $=\frac{ee}{e-a}$, as by ex. 47.

Ex. 77. Put $\frac{\sqrt{a+e}}{\sqrt{:a-e:}}$ viz. $\sqrt{:a+e:}\times\overline{a-e}|^{-\frac{1}{2}}$ into a series.

First, $\overline{a+e}|^{\frac{1}{2}}=\sqrt{a}+\frac{e}{2\sqrt{a}}-\frac{ee}{8\sqrt{a^3}}+$ &c. by ex. 73.

2dly, $\overline{a-e}|^{-\frac{1}{2}}=\frac{1}{\sqrt{a}}+\frac{e}{2\sqrt{a^7}}+\frac{3ee}{8\sqrt{a^5}}+$ &c. by ex. 74, $-\frac{1}{2}$ for $+\frac{1}{2}$

Now by multiplying these terms together and taking the sum

$$1+\frac{e}{2a}-\frac{ee}{8aa}+\text{\&c.}$$

$$\frac{e}{2a}+\frac{2ee}{8aa}-\text{\&c.}$$

$$\frac{3ee}{8aa}+\text{\&c.}$$

we get $1+\frac{e}{a}+\frac{ee}{2aa}+$ &c. $=\overline{a+e}|^{\frac{1}{2}}\div\overline{a-e}|^{\frac{1}{2}}$

PROBLEM CLXXI.

Of equations, converging ſeries, &c.

An equation is when two different expreſſions equal in value are ſet down with the ſign = between them, as $10 - 5 + 2 = 20 - 10 - 3$. In ſtating algebraic queſtions, you muſt be careful to connect the ſuppoſed letters as the nature of the queſtion requires, viz. ſo as the verbal and ſymbolical expreſſions be exactly the ſame in effect, which done, the next and moſt difficult part is to clear the equations of all the unknown quantities except one (ſee prob. 164) which muſt be done by a perfect knowledge of the foregoing work.

Ex. 78. If the product of two numbers be 35, and their quotient 1,4 what are theſe numbers? Firſt put $a =$ the greater number ſought, $e =$ the leſſer, $p =$ their product: and $q =$ their quotient.

Note. It is beſt if there be fractions in the terms, to clear them out by multiplying each term in the equation by the denominator of the fraction. Alſo, if there be ſurd terms, clear them out by involution, evolution, &c.

Then	1	$ae = p = 35$	by the queſtion.
	2	$\frac{a}{e} = q = 1{,}4$	
$2 \times e$	3	$a = qe$	
$1 \div e$	4	$a = \frac{p}{e}$	
$3 = 4$	5	$qe = \frac{p}{e}$	
$5 \times e$	6	$qee = p$	
$6 \div q$	7	$ee = \frac{p}{q} = 25$	
7 w 2	8	$e = \sqrt{\frac{p}{q}} = 5$	for the leſſer number
3	9	$a = qe = 1{,}4 \times 5 = 7$	for the greater number.

When there are more unknown quantities in the queſtion than one, ſome of them may be left out in ſtating, by ſubſtituting their values, and the work much ſhortened; ſo here, becauſe $\frac{a}{e} = q$, or $a = qe$, by writing qe for a, we ſhall have $qe \times e = qee = p$, and dividing each ſide by q we get $ee = p \div q$, the ſame with ſtep 7.

Ex. 79. If $a + e = s = 12$ be the ſum of two numbers a and e, and $aa + ee = z = 74$ be the ſum of their ſquares, what are theſe numbers? Here, becauſe $a + e = s$, or $s - e = a$, by taking e from each ſide

we shall have	1	$\overline{s-e}\vert^{2} + ee = z = ss - 2se + ee + ee$ per [question.
$1 - ss$	2	$z - ss = 2ee - 2se$
$\overline{2 \div 2}$	3	$\frac{z-ss}{2} = ee - se$
3 c □	4	$\frac{z-ss}{2} + \frac{1}{4}ss = ee - se + \frac{1}{4}ss$ (see ex. 80)
4 ɯ 2	5	$\sqrt{:\frac{z-\frac{1}{2}ss}{2}:} = e - \frac{1}{2}s$
$5 + \frac{1}{2}s$	6	$\frac{1}{2}s + \sqrt{:\frac{z-\frac{1}{2}ss}{2}} = e = 6 + \sqrt{:\frac{74-72}{2}:} = 6 + \sqrt{1}$

$= 7$, one of the required numbers, and $s - e = a$, viz. $12 - 7 = 5$ the other of them.

Otherwise. If we take $u = 6$, half the sum, and $d =$ half the difference; then will $a = u + d$, and $e = u - d$, (a denoting the greater and e the lesser number,

Then, per question	1	$aa + ee = \overline{u+d}\vert^{2} + \overline{u-d}\vert^{2} = uu + 2ud + dd + uu - 2ud + dd = z$
1 +	2	$2uu + 2dd = z$. By addition.
$2 \div 2$	3	$uu + dd = \frac{1}{2}z$
$3 - uu$	4	$dd = \frac{1}{2}z - uu = 37 - 36 = 1$
4 ɯ 2	5	$d = \sqrt{:\frac{1}{2}z - uu:} = \sqrt{1} = 1$
whence	6	$a = u + d = 6 + 1 = 7$ and $e = u - d = 6 - 1 = 5$ as before

By the foregoing examples it is observable, that algebraic questions may be performed several ways, but that which is shortest and easiest is to be preferred.

Ex. 80. In step 3d foregoing we have $ee - se$, where if for s we write $2a$, then it is plain, (by step 5th in ex, 68) there wants but aa to be added $ee - 2ae$ to make it a complete square, viz. $aa - 2ae + ee$, whose square root is $a - e$, (as in step 5th foregoing) which gives this general rule for compleating the square, and so solving this sort of equations, viz. square half the coefficient of the unknown quantity, and add it to each side of the equation, &c. (as per steps 45 and 46 above) which sort of equations admits of three cases, all solvable this way, viz.

	to which			
1. $aa + 2ae = s$	$+ee$	$aa + 2ae + ee = s + ee$	whence	$a + e = \sqrt{\ s + ee:}$
2. $aa - 2ae = s$	we	$aa - 2ae + ee = s + ee$		$a - e = \sqrt{:s + ee:}$
$2ae - aa = s$	have	$aa - 2ae + ee = ee - s$		$a - e = \sqrt{:ee - s:}$

Ex. 81. But to express these equations universally, let us suppose $a^{2} = a$, then

$$\left.\begin{array}{l} 1.\ a^{2n} + 2a^n e + ee = s \\ 2.\ a^{2n} - 2ea^n + ee = s \\ 3.\ a^{2n} - 2ea^n + ee = -s \end{array}\right\} \begin{array}{c}\text{which by} \\ \text{c}\,\square\ \text{and} \\ \text{transposing} \\ \text{e gives}\end{array} \left\{\begin{array}{l} a = \sqrt{}: -e + \overline{s+ee}|^{\frac{1}{n}}: \\ a = \sqrt{}: +e \pm \overline{s+ee}|^{\frac{1}{n}}: \\ a = \sqrt{}: +e \pm \overline{ee-s}|^{\frac{1}{n}} \end{array}\right.$$

In the 2d and 3d cases you may take either the sign — or before $\overline{s+ee}|^{\frac{1}{n}}$ and so get two different roots: either of which must be taken as the nature of the problem requires. The reason of two roots is, because a — e or e — a squared gives the same thing, viz. $aa - 2ae + ee$. But in case 2d, one of these roots will be negative or less than nothing, so it can have but one positive root; but in case 3d, both the roots are positive; these are called adfected quadratic equations; because there are but two dimensions of the unknown quantity, and the index 2 n of the one double to n that of the other; and if a^n be taken out, it is called a simple quadratic equation. Also, such as $a + dda - ca = s$, wherein the unknown quantity a is only of the first power, or index unity, are called simple equations; all other forms of equations are called adfected equations, and cannot be solved by compleating the square; but all kinds of equations may be solved by the following general method, called converging series.

Ex. 82. Required the value of e, in $ae^n + be^{n-1} + ce^{n-2} + de^{n-3} + \&c. = Q$; wherein n, a, b, c, d, &c. represent any quantities positive, or negative; to solve which put $r + z = e$,

$$\left.\begin{array}{l}\text{Then} \\ \text{per exam.} \\ \quad 69.\end{array}\right\} \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \left\{\begin{array}{l} e^n = r^n + nr^{n-1}z + \&c. \\ e^{n-1} = r^{n-1} + :n-1:r^{n-2}z + \&c. \\ e^{n-2} = r^{n-2} + :n-2:r^{n-3}z + \&c. \end{array}\right.$$

continuing the terms no further than the first power of z.

Now these values of e^n, e^{n-1}, &c substituted in the given equation, we have $ar^n + anr^{n-1}z + br^{n-1} + :n-1:br^{n-2}z + cr^{n-2} + :n-2:cr^{n-3} + \&c. = Q$; now, by transposing all the terms wherein z is not concerned, and dividing by the coefficient of z, we have

$$z = \frac{Q - ar^n - br^{n-1} - cr^{n-1} - dr^{n-3} - \&c.}{nar^{n-1} + :n-1:br^{n-2} + :n-2:cr^{n-3} + \&c.}$$

Ex. 83. As an use of this general expression, let it be required to find the value of e in this equation $-eee + 300e = 1000$.

First, by supposition and trial get a number to come as near the value of e as you can, and call that number r, and what r differs from e call it z; then if r is greater than e it will be $r - z = e$, otherwise $r + z = e$.

ſo here by trial, e is found to be ſomewhat more than 3, ſo take $r = 3$ and $r + z = e$, then in this ex. n is $= 3$, $a = -1$, $b = 0$, $c = 300$, and $Q = 1000$, d, &c. being not in this ex. becauſe there are no e terms for them, they are taken $= 0$, and ſo the general equation becomes $\dfrac{Q + a r^{n} - c r^{n-2}}{-n a r^{n-1} + : n - 2 : c r^{n-3}} = \dfrac{1000 + r^{3} - 300 r}{-3 r^{2} + 300} =$ $\dfrac{1000 + 27 - 900}{-27 + 300} = \dfrac{127}{273} = 0{,}5 = z$, whence, $r + z = 3{,}5 = e$.

But this method cannot at one operation give the juſt value of e, becauſe in the general equation (ex. 82) all the powers of z above z^{1} are left out, yet by making this value of e, $3{,}5 = r$, and repeating the operation z will be had to two or three more places of figures; and by making a third operation, you'll find z or e to 4 or 5 places more, and thus proceeding e may be found to any exactneſs required, every operation doubling at leaſt the places of figures; ſo by writing 3,5 for r in the laſt equation, we have $z = \dfrac{1000 + 42{,}875 - 1050}{-36{,}75 + 300} = \dfrac{-7{,}125}{263{,}25} = -$ $0{,}027$, ſo $(r + z)$ $3{,}5 - 0{,}027 = 3{,}473 = e$ nearly; and if this 3,473 be taken for a new r and proceed as before, you'll find $e = 3{,}47296351$ and ſo on if neceſſary.

Ex. 84. If the root of a ſimple power be required, then a, b, c, d, &c, are each $= 0$, for $e^{n} = Q$ in this caſe, ſo we ſhall have $z = \dfrac{Q - r^{n}}{n r^{n-1}}$

A general method for extracting roots by converging ſeries.

Ex 85. Every adfected equation hath as many roots poſitive and negative as it conſiſts of dimenſions; thus, if $e = b$, and $e = c$, then $b - e = 0$ $c - e = 0$, and $: c - e \times : b - e : = ee - ce - eb + cb = 0$, or $ee - 2ze + bc = 0$ (by putting $-2z = -c - b$) the ſame with caſe 2d in ex. 81.

Again, $e - b \times c - e = eb - ee - cb + ce = 0$, or $-ee + 2ze = cb$, by putting $2z = b + c$; this is caſe 3d in ex. 81. Alſo, $: e - a : \times : e - b : \times : e - c : = eee - ae^{3} - be^{2} - ce^{2} + abe + ace + bce - abc = 0$, an adfected cubic equation. Such equations are ſet down by ſome writers thus,

$$eee + \left.\begin{matrix} -a \\ -b \\ -c \end{matrix}\right\} ee + \left.\begin{matrix} ab \\ ac \\ bc \end{matrix}\right\} e - abc = 0$$

Here e has three values or roots, viz. a, b, &c. and ſo for other higher equations, it appears that $-a-b-c$, the coefficient of the ſecond term is $=$ ſum of all the roots with a contrary ſign; that of the third term is $ab+ac+bc$, the ſum of all their products, that can be made by two at a time, the laſt term of all is $=$ the product of all the roots, viz. $-a, b, c, +$. From a due conſideration of which, rules might be invented for finding each of theſe roots; but as one poſitive root commonly anſwers the end, and that may be had univerſally by the rules in ex. 83, it is thought ſufficient. However, it is eaſy to ſee that if any ſuch equation be divided into ſo many roots or parts, $a-e=0$, $e-b=0$, &c. as being multiplied together will produce the given equation, that you will have (a, b, c, &c.) all the required roots.

Ex. 86. That $-$ multiplied by $+$, or $+$ by $-$ produces $-$, may be thus proved, viz. if $+b \times -c$, I ſay $p=-bc$,

put	1	$b+c=a$
$1-c$	2	$b=a-c$
$2\times b$	3	$bb=ba+b\times -c$
that is	4	$bb=ba+p$
$1\times b$	5	$bb+bc=ba$
$5-bc$	6	$bb=ba-be$
$4=6$	7	$ba+p=ba-bc$
$7-ba$	8	$+p=-bc$, Q. E. D.

Ex. 87. That $-$ multiplied by $-$ produces $+$ may be thus proved, viz. If $=-b\times -c$, the product p is $=+bc$;

put	1	$a=b+c$
$1-b$	2	$a-b=c$
$2\times -c$	3	$-ca=-ca+-b\times -c$. By ex. 86.
that is	4	$-cc=-ca+p$
$1\times -c$	5	$-ca=-cb-cc$
$5+cb$	6	$bc-ca=-cc$
$4=6$	7	$bc-ca=-ca+p$
$7+ca$	8	$+bc=+p$. Q. E. D.

PROBLEM CLXXII.

Ex. 88. To find s, the ſum of any ſeries $a + b + c + d + e + \&c.$ by means of their differences.

Here we have	1	$a + b + c + d + e \; \&c. = s$	By taking the firſt term from that which follows it, firſt in one row and then in another.
	2	$-a + b, \; -b + c, \; -c + d, \; -d + e,$	
	3	$a - 2b + c, \; b - 2c + d, \; c - 2d + e$	
	4	$-a + 3b - 3c + d, \; -b + 3c - 3d + e$	
	5	$a - 4b + 6c - 4d + e, \; \&c.$	
put	6	$A = -a + b$	the firſt ariſing difference or remainder in the 2 ſtep, or row.
	7	$B = a - 2b + c$	the firſt ariſing difference or remainder in the 3 ſtep, or row.
	8	$C = -a + 3b - 3c + d$	the firſt ariſing difference or remainder in the 4 ſtep, or row.
	9	$D = a - 4b + 6c - 4d + e$	the firſt ariſing difference or remainder in the 5 ſtep, or row.
then	10	$a = a$, the firſt term in the ſeries.	From the 14 ſtep the law of continuation is evident, and ſhews the coeff, of A, B, C, &c. to be thoſe of a binomial raiſed to the ſecond, third, &c. powers.
per 6,	11	$b = a + A$	
per 7, 10,	12	$c = a + 2A + B$	
8, and 11,	13	$d = a + 3A + 3B + C$	
9, and 12,	14	$e = a + 4A + 6B + 4C + D$	
ſum of the laſt five ſteps	15	$a + b + c + d + e = 5a + 10A + 10B + 5C + D, \; \&c. = s.$	

Hence, If n = the number of terms in the proposed series $a + b + c + d +$ &c. we shall have $s = na + : n \times \frac{n-1}{2} A : + : n \times \frac{n-1}{2} \times \frac{n-2}{3} B : + : n \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} C : +$ &c.

Ex. 89. Required the sum of the series of square $9 + 16 + 25 + 36 +$ &c. to n terms in number.

Here $a = 9$ the first or least term, $A = 7$ the first term of the first differences, $B = 2$ that of the second differences, $C = 0$; so, $na + n \times \frac{n-1}{2} A$, &c. becomes $= 9n + n \times \frac{n-1}{2} 7 + n \times \frac{n-1}{2} \times \frac{n-2}{3} 2 = S = 9n + \frac{7n^2 - 7n}{2} + \frac{2nnn - 4nn + 2n}{6} = \frac{2nnn + 17nn + 35n}{6} = \frac{nnn}{3} + \frac{17nn}{6} + \frac{35n}{6}$.

7	9	11
	2	2
		0

Ex. 90. Required the sum of the series $2 + 6 + 12 + 20 + 30 +$ &c, to n terms.

Here $a = 2$, $A = 4$, $B = 2$, and $C = 0$, so, $na + n \times \frac{n-1}{2} A + n \times \frac{n-1}{2} \times \frac{n-2}{3} B$, &c. $= 2n + \frac{4nn - 4n}{2} + \frac{2n \times \overline{n-1} \times \overline{n-2}}{6} = \frac{nnn + 3nn + 2n}{3} = s$

4,	6,	8,	10,
	2,	2,	2,
		0,	0,

Ex. 91. Required the sum of the rectangles $m \times n$, $+ \overline{m-1} \times \overline{n-1}$, $+ \overline{m-2} \times \overline{n-2}$, $+ \overline{m-3} \times \overline{n-3} +$ &c. These terms actually multiplied stand

nm, $nm - m - n + 1$, $mn - 2m - 2n + 4$, $mn - 3m - 3n + 9$ &c.
$-m - n + 1$, $-m - n + 3$, $-m - n + 5$, &c.
,2 ,2
,0

Here $a = nm$, $A = -m - + 1$, $B = 2$, C, D, &c. each $= 0$, whence we shall have $n^2m \times : -m - n + 1 : \times n \times \frac{n-1}{2} + 2n \times \frac{n-1}{2} \times \frac{n-2}{3} = \frac{3mn^2 + 3mn - n^3 + n}{6}$ for the required sum, continued to n terms. As an instance of which, let it be required to find the number of shot in an oblong pile, whose length at the base is 46, and breadth 15 shot.

Here m = 46 and n = 15, so $\frac{3mnn+3mn-n^3+n}{6} = 4960$, the number of shot in such a pile. And if m = n = 15, the pile will be a square pyramid, and we shall have $\frac{2n^3+3nn+n}{6} = 1240$ for for the number of shot.

Ex. 92. By taking a = 1, and working as before, we shall have

$$1+2+3+4+5+\&c. \quad +n=\frac{nn+n}{2}=s$$

$$1^2+2^2+3^2+4^2+5^2+\&c. \quad +n^2=\frac{nnn}{3}+\frac{nn}{2}+\frac{n}{6}=s$$

$$1^3+2^3+3^3+4^3+5^3+\&c. \quad +n^3=\frac{n^4}{4}+\frac{n^3}{2}+\frac{n^2}{4}=s$$

$$1^4+2^4+3^4+4^4+5^4+\&c. \quad +n^4=\frac{n^5}{5}+\frac{n^4}{2}+\frac{n^3}{3}-\frac{n}{30}=s$$

$$1^5+2^5+3^5+4^5+5^5+\&c. \quad +n^5=\frac{n^6}{6}+\frac{n^5}{2}+\frac{5n^4}{12}-\frac{n^2}{12}=s$$

$$1^6+2^6+3^6+4^6+5^6+\&c. \quad +n^5=\frac{n^7}{7}+\frac{n^6}{2}+\frac{n^5}{2}-\frac{n^3}{6}+\frac{n}{42}=s$$

93. Hence the first term of the sum of $1^d+2^d+3^d+4^d+\&c.$ $+\ n^d$ is $=\frac{n^{d+1}}{d+1}$ and if n be indefinitely great, viz. divided into an indefinite number of equal parts, then this $\frac{n^{d+1}}{n+1}$ will be the true sum of $1^d+2^d+\&c.\ +n^d$, for an indefinitely great number or quantity cannot be increased or diminished by addition or subtraction, and therefore all the following terms in the said sum may be rejected. See theorem 78.

Ex. 94. Req. the sum of the arith. series $\overline{a}\ \overline{\pm e} + \overline{a \pm 2e} + \&c,$
Here a = a, A = $\pm$ e, B, C, &c. each = o $\quad + e \quad \pm e$, &c.
o, &c,

whence $na+n\times\frac{n-1}{2}$ A &c. $= na + \frac{nn-n}{2} : \pm e :$

$$= \frac{2na \pm n^2e \mp ne}{2} = s.$$ See ex. 103.

PROBLEM CLXXIII.

Of proportional and progressional quantities, analogies, &c.

Ex. 96. If the quantities a, b, c, d, are in direct proportion, viz. $a : b :: c : d$, the product of the two means b and c is $=$ the product of the two extreams a and d; i. e. $bc = ad$, for $a : b :: c : \frac{bc}{a} = d$, i. e. the product of the second and third terms divided by the first term is $=$ the fourth or required term, by the nature of the golden rule; whence, if we multiply each side of the equation $\frac{bc}{a} = d$, by a, we'll have $\frac{abc}{a} = ad$, viz. $bc = ad$, (by ex. 53.)

Ex. 97. Because these two terms of the proportion become a fraction $\frac{b}{a}$ it is easy (by ex. 53) to reduce compounded ratios into more simple ones. As for

Ex. 98. If it be as $abc : adc :: A : A^2$; it will be the same thing if we say as $ab : ad$, or as $b : d :: A : A^2$. Also $aa - ee :: a - e$, or $a + e : 1$ the same ratio; because each term is divisible by $a - e$, without any remainder, &c.

Ex, 99. Also we see how to turn equations into analogies or proportions; for it is but to divide each side of the equation into two factors, one side for the two means, and the other for the two extreams; so, if $aa = db$, then $a : b :: d : a$, or $d : a :: a : b$, for $a \times a = b \times d$.

Ex. 100. If $a = bd + cd$, then, $d : \sqrt{a} :: \sqrt{a} : b + c$, or, $d : 1 :: a : b + c$, &c.

Ex. 101. If	1	$ad = bc$, then $a : b :: c : d$, in direct proportion.
Then	2	$a : c :: b : d$, called alternate proportion
and	3	$b : a :: d : c$, called inverse proportion.
$1 + db$	4	$da + db = bc + db$, then $a + b : b :: c + d : d$, comp. pr.
$1 + cd$	5	$a + c : c :: b + d : d$, alternately compounded.
$1 - bd$	6	$ad - bd = bc - bd$, then $a - b : b :: c - d : d$, divided
$1 - cd$	7	$ad - cd = bc - cd$, then $a - c : c :: b - d : d$, alter. div.
$1 \pm ac$	8	$ad \pm ac = bc \pm ac$, then $a : b \pm a :: c : d \pm c$ convert.
Lastly	9	$a + b : a - b :: c + d : c - d$, mixtly.
9 ∴	10	$ac + ad - cb - bd = ac + bc - ad - bd$, viz. $da = bc$.

Thus you see how many ways it is possible to vary four proportional quantities; which is the substance of Euclid's fifth book.

102. If a quantity a, be continually increased or diminished by a quantity e, the results $a, a+e, a+2e, a+3e$, &c. are called arithmetical series or progressionals increasing, and $a, a-e, a-2e, a-3e$, &c. are called arithmetical series or progressionals decreasing.

Let $a =$ the first term, $e =$ the common excess or difference, $y =$ the last term, $n =$ the number of terms, and $s =$ the sum of all the terms in the series; then y will be $= a + : \overline{n-1}\, e$, or, $a - \overline{n-1}\, e$, as the series increases or decreases, as is plain from the series themselves; whence we get, first $a = y - ne + e$; 2. $e = \frac{y-a}{n-1}$, and 3. $n = \frac{y-a+e}{e}$ the series being increasing. Then

Example 102.

per Ex. 94.	4	$\frac{2na+:nn-n:e}{2} = s$, or $2na+:nn-n:e = 2s$, when n, a and e are given.
and also	5	$nne+2na+a-y = 2s+e$, for $y = a+ne-e$, as above.
per 5 and 2.	6	$\frac{:a+y:\times n}{2} = s$, when a, y, and n are given, (to find s.
per 2 and 4.	7	$2na+:nn-n:\times:\frac{y-a}{n-1}: = 2s$
7 reduced	8	$\frac{na+ny}{2} = s$, as per step 6.
per 3 and 8.	9	$:\frac{a+y}{2}:\times:\frac{y-a+e}{e}: = s = \frac{yy-aa+ae+ye}{2e}$ (when a, e, and y are given.
per 1 and 8	10	$\frac{n:y-ne+e:+ny}{2} = \frac{2ny-nne+ne}{2}$ (= s, when n, e, and y are given.

The 4th, 6th, (or 8th) 9th or 10th steps, contain the principal theorems in arithmetical progression; by which any of the other letters, as well as s, may be found.

103. Geometrical progression is had by multiplying and dividing, as that of arithmetical is by adding and subtracting. Thus,

$$\left.\begin{array}{l} a,\ ae,\ aee,\ aeee,\ \&c.\ \text{increasing,} \\ \frac{a}{1},\ \frac{a}{e},\ \frac{a}{ee},\ \frac{a}{eee},\ \text{decreasing,} \end{array}\right\}\ \text{geometrical series}$$

Whence it appears, that if a = the first term, e = common ratio, n = number of terms, s = the sum of all the terms in the series, the last term y will be $= a e^{n-1}$, whence, and by ex. 48, we have

	1	$s = \frac{ep - a}{e - 1} = \frac{e a e^{n-1} - a}{e - 1} = \frac{a e^{n} - a}{e - 1}$, when (a, e, and n are given.
	2	$a = \frac{y}{e^{n-1}}$, for $y = a e^{n-1}$, by the above.
1 and 2.	3	$s = ey - \frac{y}{e^{n} - 1} = \frac{y e^{n} - y}{e^{n} - e^{n-1}}$, when e, n, and (y are given, as also e, y, and a,
	4	$e^{n-1} = \frac{y}{a}$, for $y = a e^{n-1}$, as before.
$4\,\omega : n-1$:	5	$e = \overline{\frac{y}{a}}\Big\vert^{\frac{1}{n-1}}$, or log. e = log. $\frac{y \div a}{n - 1}$:
1 and 6	6	$s = y + \frac{y - a}{\overline{\frac{y}{a}}\Big\vert^{\frac{1}{n-1}} - 1}$, when n, y, and a, are (given.

These are the principal theorems in geometrical progression; from which any of the other letters may be found if necessary.

104. Musical, or harmonical proportion, is that between those numbers, which assign the lengths of musical intervals; or the lengths of strings sounding musical notes. Thus, if a string be divided in proportion as 3, 4, 6, these lengths will sound an octavo 3 to 6, a fifth 4 to 6, a fourth 3 to 4: for these numbers, 3, 4, 6, are as in theorem 178. Also, 5, 6, 8, 10, are 4 numbers in harmonical proportion; for strings of such lengths will sound an octavo 5 to 10, a 6th greater 6 to 10, a 3d greater 8 to 10, a 3d lesser 5 to 6, a 6th lesser 5 to 8, a 4th 6 to 8; whence, if for 3, 4, 6, we take a, b, c, we shall have, as a : c : : a — b : b — c, ergo, a b — a c = c a — c b. Any two of these three letters being given, the third may be had thus, $a = \frac{bc}{2c - b}$, when b and c are given, and $c = \frac{ab}{2a - b}$ (= 6) when a and b are given. Also $b = \frac{2ac}{a + c}$ = 4, a musical mean proportional between the whole string c = 6 and a = 3 the octavo; which 4 is the length answering to the fifth, viz. 4 to 6. Again, if for 5, 6, 8, 10, we take a, b, c, d, by the nature of these numbers we shall have, as a : d : : a — b : c — d, ergo, a c — a d = d a — d b: any three of these 4, a, b, c, d, being given, the fourth is also had as before.

105. From a view of these numbers, it appears, that a series of numbers in harmonical proportion, are inversely as another series in arithmetical proportion: thus,

Harmonical	10,	12,	15,	20,	30,	60.
Arithmetical	6,	5,	4,	3,	2,	1.

That is, as 10 : 12 :: 5 : 6, and as 12 : 15 :: 4 : 5, &c.

106. Hence, If a, b, c, d, e, f, &c. denote an harmonic series, and $m = b - a$, we shall have $a = \frac{ab}{b}$, $b = \frac{ab}{b-m}$, $c = \frac{ab}{b-2m}$, $d = \frac{ab}{b-3m}$, $e = \frac{ab}{b-4m}$, &c. and the last term $= \frac{ab}{b - :n-1:m}$ n being = the number of terms in the series. These terms divided by a b gives the series $\frac{1}{b}$, $\frac{1}{b-m}$, $\frac{1}{b-2m}$, $\frac{1}{b-3m}$, &c. = to $\frac{1}{b - :n-1:m}$, in harmonic proportion.

PROBLEM CLVI.

To find a vulgar fraction in few figures, equal to a given decimal fraction q; *or, (which is the same thing) to find the ratio of the decimal's denominator to its numerator in fewer figures.*

Take the following solution, as a general one.

Ex. 107. Required the ratio of 1 to 3,14159265, (in fewer figures) being the ratio of a circle's diameter to its periphery.

Here the nearest ratios in the least whole numbers are 1 to 4, but nearer as 1 to 3, which in fractions are $\frac{4}{1}$ and $\frac{3}{1}$. Let $\frac{a}{b} = \frac{3}{1}$, the nearer ratio, and $\frac{c}{d} = \frac{4}{1}$, the other ratio, viz. $a = 3$, $b = 1$, $c = 4$, $d = 1$, and suppose $\frac{ae + c}{eb + d} = q = 3{,}14159265$, whence $e = \frac{qd - c}{qb - a}$ or $= \frac{c - qd}{qb - a}$, having thus found e, substitute it in the supposed fraction $\frac{ae + c}{ab + d}$, so you'll have the first approximated value of q, which call a second $\frac{a}{b}$, and for the first taken $\frac{a}{b}$ take $\frac{c}{d}$, and proceed on as before, for a second approximated value of q; and thus repeat the ope-

tation with the two last nearest values of q, you may come to an approximation as near q's value as you please; so here, $e = \frac{c - q\,d}{q\,b - a} = \frac{4 - 3{,}1415}{3{,}1415 - 3} = \frac{,8575}{,1415} = 6$ nearly, whence, $\frac{a\,e + c}{e\,b + d} = \frac{3 \times 6 + 4}{1 \times 6 + 1} = \frac{22}{7} = q$ nearly. Again, Let $\frac{a}{b} = \frac{22}{7}$, and $\frac{c}{d} = \frac{3}{1}$. Then e will be $= \left(\frac{c - q\,d}{q\,b - a}\right) \frac{3 - 3{,}14159 \times 1}{3{,}14159 \times 7 - 22} = \frac{,1415}{,00887} = 16$ nearly whence, $q = \frac{a\,e + c}{e\,b + d} = \frac{22 \times 16, + 3}{7 \times 16, + 1} = \frac{355}{113}$, the second approximate which call $\frac{a}{b}$ and $\frac{22}{7} = \frac{c}{d}$, and we shall have $e = \frac{c - q\,d}{q\,b - a} = \frac{,00885145}{-,00003055} = -290$ nearly, whence $\frac{a\,e + c}{e\,b + d} = \frac{-102950 + 22}{-32770 + 7} = \frac{102928}{32763} = q = 3{,}14159265$, &c.

Note: The value of e is still to be had only in the nearest whole numbers, the second and third approximations are the ratios in common practice, viz. $\frac{22}{7}$, or as 7 to 22, called Archimede's ratio, or nearer, as 113 to 355, called Metius's ratio; this ratio is truer than the common factor, 1 to 3,1416, for $355 \div 113 = 3{,}141592$. Therefore in using this ratio, the error cannot exceed ,0000003, but $\frac{22}{7} = 3{,}142$ &c. so this ratio may err, nearly ,002, tho' it is the most common one in practice and indeed may serve for ordinary uses well enough.

Whence, as 1 to 4, or nearer, as 1 to 3, or nearer, as 7 to 22, or nearer as 113 to 355, or nearer as 32763 to 102928, so is any circles diameter to its periphery, &c. for any other.

PROBLEM CLXXIV.

How to compute logarithms.

108. What natural numbers do, multiplication, division, involution, and evolution, logarithms, or artificial numbers, performs by addition, subtraction, multiplication, and division, i. e.

109. The sum of the log. of any two numbers, is = to the log. of the product of those two numbers.

110. The difference between the logarithms of any two numbers, is = to the log. of the quotient of the one number divided by the other.

111. If you mult. the log. of any number by the index of any power, the product will be the log. of that number involved to that power.

Ex. 112. If you divide the log. of any number, by the index of any root, the quotient will be the log. of that root. See ex. 72.

113. Lemma. If from each of the indefinite roots of two numbers, an unit be taken, the sum of the two remainders will be = to the product of the said two indefinite roots minus unity; or which is the same, if from the indefinite root of any number you subtract 1, the double of the remainder will be = to the square of the said indefinite root lessened by unity, i. e. if $1 + e$, be any finite number, and n an indefinite number, then we are to prove that $\overline{1+e}|^{\frac{1}{n}} - 1 \times 2$ is $= \overline{1+e}|^{\frac{1}{n}} \times \overline{1+e}|^{\frac{1}{n}} - 1$, viz. that $2:\overline{1+e}|^{\frac{1}{n}} - 2 = \overline{1+e}|^{\frac{2}{n}} - 1$. First by ex. 72 and 71, &c.

we shall have	1	$2:\overline{1+e}\vert^{\frac{1}{n}} = + \frac{2}{n}e + \frac{2}{n} \times : \frac{1}{2}\frac{2}{n} - 1 : ee + \frac{2}{n} \times : \frac{1}{2}\frac{2}{n} - 1 \times \frac{1}{3}\frac{2}{n} - 2 : eee;$
	2	$\overline{1+e}\vert^{\frac{2}{n}} = 2 + \frac{2}{n}e + \frac{2}{n} \times : \frac{1}{2}\frac{2}{n} - 1 : ee + \frac{2}{n} \times : \frac{1}{2}\frac{2}{n} - 1 \times \frac{1}{3}\frac{2}{n} - 2 : eee$, &c.
1 − 2	3	$2:\overline{1+e}\vert^{\frac{1}{n}} - 2 = \frac{2}{n}e + \frac{2}{n} \times : \frac{1}{2}\frac{2}{n} - 1 : ee +,$
2 − 1	4	$\overline{1+e}\vert^{\frac{2}{n}} - 1 = \frac{2}{n}e + \frac{2}{n} \times : \frac{1}{2}\frac{2}{n} - 1 : ee +,$
3 or 4	5	$\frac{2}{n}e + \frac{2}{n} \times - \frac{1}{2}ee + \frac{2}{n} \times - \frac{1}{2} \times - \frac{2}{3}eee +$ &c.
5 reduced	6	$\frac{2}{n}e - \frac{2}{2n}ee + \frac{2}{3n}eee, -$ &c.
6 reduced	7	$\frac{2}{n} \times : e - \frac{1}{2}ee + \frac{1}{3} -$ &c.

Note. Log. or l. stands for logarithm, and logs. or l's. for logarithms.

Here the series in the third and fourth steps, are exactly the same. Q. E. D. But because n, by supposition is indefinitely great, it is evident that $\frac{2}{n}$ must be indefinitely small, therefore, $\frac{2}{n}$ added to, or taken from any number, can make no sensible alteration in the said

number, ſo in the co-efficients $\frac{2}{n} - 1\ \frac{2}{n} - 2$, &c. this $\frac{2}{n}$ may be omitted, and then the ſeries will be as in the 5th ſtep, which contracted, gives that in the 7th ſtep.

114. From hence it appears that if one be taken from $\overline{1+e}|^{\frac{1}{n}}$ the indefinite root of $1+e$, of any number, the remainder $\frac{1}{n} \times : e - \frac{1}{2}e^2 + \frac{1}{3}e^3 - \frac{1}{4}e^4 +$ &c. would be the log. of $1+2$, were it not indefinitely ſmall, viz. multiplied by $\frac{1}{n}$, but if we put $Q = \frac{10000\ \&c.}{n}$ (where 10000 &c. is ſuppoſed to conſiſt of as many places of figures as n doth) the ſeries will then become finite, and we ſhall have $Q \times : e - \frac{1}{2}e^2 + \frac{1}{3}e^3 - \frac{1}{4}e^4 + \frac{1}{5}e^5 -$ &c. for the log. of $1+e$.

115. It is further evident, that there may be as many forms of logs. as n can be ſuppoſed different indefinite numbers. Thus, if $Q = 1$, then $n = 10000$ &c. and the above ſeries will become $e - \frac{1}{2}e + \frac{1}{3}e^3 -$ &c. which is called the hyperbolical log. of $1+e$. Again, if $n = 2{,}30251$ &c. then Q will be $= 0.434291$ &c. and $Q \times : e - \frac{1}{2}e^2 + \frac{1}{3}e^3 -$ &c. is the tabular or Brigg's log. of $1+e$, but this ſeries tho' it be the moſt natural one, yet it converges ſo ſlowly that it is of little uſe in the conſtruction of logs. ſo to find a ſeries that will converge ſwifter, let us ſuppoſe $\frac{1}{1-e}$ to denote the number whoſe log. is req. then by ordering $\overline{\frac{1}{1-e}}\Big|^{\frac{1}{n}} - 1 = \frac{1}{\overline{1-e}|^{\frac{1}{n}}} - 1 = \overline{1-e}|^{-\frac{1}{n}} - 1$ as you did $\overline{1+e}|^{\frac{1}{n}} - 1$ viz. by taking the nth root of $\overline{1-e}|^{-\frac{1}{n}}$, and ſubtracting 1 from that root, you'll have $Q \times : e + \frac{1}{2}e^2 + \frac{1}{3}e^3 + \frac{1}{4}e^4 +$ &c. for the log. of $\frac{1}{1-e}$, to which add $Q \times : e - \frac{1}{2}e^2 + \frac{1}{3}e^3 - \frac{1}{4}e^4 +$ &c. the log. of $1+e$, and you'll have $2Q \times : e + \frac{1}{3}e^3 + \frac{1}{5}e^5 + \frac{1}{7}e^7 +$ &c. for the log. of $\frac{1}{1-e} \times 1+e$, or $\frac{1+e}{1-e}$, and thus you have 3 different ſeries viz. one for the log. of $1+e$, one for the log. of $\frac{1}{1-e}$, converging faſter, and 1 for the log. $\frac{1+e}{1-e}$ converging yet faſter.

O

116. Ex. required the log. of any number N; firſt, let $N = \frac{1+e}{1-e}$ and then we ſhall have $e = \frac{N-1}{N+1}$, which being ſubſtituted in the laſt ſeries for e, will give the logarithm of the number N, as required.

117. Required the log. of a fraction as $\frac{N}{D}$. Here, as before, we muſt put $\frac{N}{D} = \frac{1+e}{1-e}$, and then we'll have $e = \frac{N-D}{N+D}$, which put in the laſt ſeries for e, gives the anſwer.

118, What is the hyperbolical, or Neper's log. of 2? Here $N=2$, ſo $e = \frac{N-1}{N+1}\ \frac{2-1}{2+1} = \frac{1}{3}$, therefore, $e + \frac{1}{3}e^3 + \frac{1}{5}e^5 + \frac{1}{7}e^7 + \frac{1}{9}e^9 + \frac{1}{11}e^{11} + \frac{1}{13}e^{13} + \frac{1}{15}e^{15} + \frac{1}{17}e^{17} + \frac{1}{19}e^{19} + \frac{1}{21}e^{21} =$ 0,346573590280, which multiplied by 2 Q 2 gives 0,693147180560 for Neper's log. of 2, true to the laſt figure, by theſe 11 terms of the ſeries only.

119. Required the tabular or Briggs' log. of 2. Here, if inſtead of multiplying 0,346573590280 by 2 = 2 Q you multiply it by 0,868588963926 = 2 Q (by 115.) you'll have the tabular log of 2; or, which is eaſier, Let 2 Q = R = 0,8685889, &c. then the foregoing ſeries becomes $R \times : e + \frac{e\ e\ e}{3} + \frac{e^5}{5} + \frac{e^7}{7}$; now e being before found $= \frac{1}{3}$, we ſhall have,

$$Re = \frac{0,8685889}{3} = .2895296 +$$

$$\frac{Re^3}{3} = \frac{0,8685889}{81} = ,0107233 +$$

$$\frac{Re^5}{5} = \frac{0.8685889}{1215} = ,0007149 -$$

$$\frac{Re^7}{7} = \frac{0,8685889}{15309} = ,0000568 -$$

$$\frac{Re^9}{9} = \frac{0,8685889}{177147} = ,0000049 +$$

$$\frac{Re^{11}}{11} = \frac{0,8685889}{1948617} = ,0000005 -$$

Sum ,3010300 for the tabular log. of 2, as required. Having thus found the log. of 2, you may from it

find the logs. of 4, 8, 16, 32, 64, &c. for twice the log. of 2 gives the log. of 4, (because $2 \times 2 = 4$, see ex. 109.) to which and the log. of 2 and you'll have the log. of 8, &c. and by the above series you may find the log. of the prime numbers, 3, 7, 11, 17, &c. But as we go to higher numbers, the series converges slower, therefore take the following method for such cases.

120. In this manner the logs. of small numbers may be readily found. But it will be more expeditious to find the logs. of large numbers, from those of small ones already found. Thus, suppose

	1	$a = b - 1$ } a, b, &c. being three numbers in arithmetical progression whose common difference is unity.
	2	$c = b + 1$
1×2	3	$ac = bb - 1$
$3 + 1$	4	$ac + 1 = bb$
$4 \div ac$	5	$\frac{ac+1}{ac} = \frac{bb}{ac}$
put	6	$ac + 1 = N$, and $ac = D$, then $\frac{ac+1}{ac} = \frac{N}{D}$
by ex. 117	7	$\frac{N-D}{N+D} = e = \frac{1}{2ac+1}$, per last step
hence	8	log. $\frac{ac+1}{ac} = Re + \frac{Re^3}{3} + \frac{Re^5}{5} +$ &c. which series call S.
then per 5	9	2 log. b $-$ log. a $-$ log. c $= S$, viz. $=$ log. $\frac{ac+1}{ac}$,
transpo. 9th step we get,	10	$\frac{1}{2}$ log. a $+ \frac{1}{2}$ log. c $+ \frac{1}{2} S =$ log. b,
	11	2 log b $-$ log. c $- S =$ log. a. Hence, if any tw of these 3 logs. be given, the third may be found.

Ex. 121. Required the tabular logarithm of 7, having found the log. of 2 and 3, as directed in ex. 119, you may from them have the logs. of 8 and 9, viz. 3 times log. of 2 = log. of 8, and twice log. of 3 = log. of 9, and per 11th step, log. of 7 = 2 log. 8 — log. 9 — S, (a being = 7, b = 8 and c = 9) now to find S, we have per step 7, $e = \frac{1}{2ac+1} = \frac{1}{127}$, so $S = Re + \frac{Re^3}{3} + \frac{Re^5}{5} +$, &c. $=$ log. $\frac{ac+1}{ac} =$ log. $\frac{64}{63}$, whence, by resolving the series, we shall have

$$Re = \frac{,868588963}{127} = ,006839283$$

$$\frac{Re^3}{3} = \frac{,868588963}{5145149} = ,000000141$$

$$\frac{Re^5}{5} = \frac{,868588963}{165191847035} = ,000000000$$

here S, or sum $= ,006839424 =$ tabular log.

of $\frac{64}{63}$. Whence 2 log. 8 — log. 9 — S = 0,845098040, equal tabular log. of 7, true to the last figure, and only 3 terms of the series used, and also turns out the tabular log. of the fraction $\frac{64}{63}$ or $\frac{1}{63}$, and if we have greater numbers fewer terms will do; thus, if the logs. of 2, 3 and 5 be known, then those of 48 and 50 may be known, from whence the log. of 7 ($=\frac{1}{2}$ log. 49) may be also found, for let a = 48, b = 49 &c. = 50, then $e = \frac{1}{2ac+1} = \frac{1}{4801}$, and $R\,e = \frac{,868588\ \&c.}{4801} =$,0001809 &c. = S, then per step 10, we have $\frac{\text{log. }48 + \text{log. }50 + S}{2}$ = log. 49, or $\frac{\text{log. }48 + \text{log. }50 + S}{4}$ = ,845098040 = log. 7 as before, and takes in only the first term of the series, which would hold true to a place or two more if continued, and greater the numbers are, the log. by only the first term of the series will still be truer to more places of figures, by which a table of logarithms may readily be made, or examined.

PROBLEM CLXXV.

Given any hyperbolical logarithm L, *to find its natural number* $1 + e$.

122. If you revert the series found in ex 114, you'll have the required value of $1 + e$. Otherwise, since we have $\overline{1+e}|^{\frac{1}{n}} - 1 = L$, and as n may be = any indefinite number, we may take $r : \overline{1+e}|^{\frac{1}{r}} - r$ for $\overline{1+e}|^{\frac{1}{n}} = 1$

That is		
	1	$r : \overline{1+e}\vert^{\frac{1}{r}} - r = L$
$1 + r \div r$	2	$\overline{1+e}\vert^{\frac{1}{r}} = 1 + \frac{L}{r}$
2 ☉ r	3	$1+e = 1 + \frac{rL}{r} + \frac{r \times : r - 1 : LL}{2 \quad rr} + \frac{r \times : r - 1 : \times : r - 2\ LLL}{2 \quad 3 \quad rrr} + \&c.$
3 reduced	4	$1 + e = 1 + L + \frac{LL}{2} + \frac{LLL}{2,3} + \frac{L4}{2,3,4} + \frac{L5}{2,3,4,5} + \&c.$

Here (as in ex. 113) r being supposed indefinitely great, the numbers — 1, — 2, — 3, &c. in step 3, are left out in the 4th step, which gives the answer.

PROBLEM CLXXVI.

Reversion of infinite series.

122. Before we do this it will be needful to know how to put, or raise an universal infinite series, $hz^{m} + bhz^{m+1} + chz^{m+2n} + dhz^{m+3n} + \&c. = hz^{m} \times : 1 + bz^{n} + cz^{2n} + dz^{3n}$ + &c. to an infinite power whose index is v, or to find the value of $\overline{hz^{m}}|^{v} \times : 1 + bz^{n} + cz^{2n} + dz^{3n} + \&c.$ in simple terms. To make this plain let us put $p = 1 + bz^{n} + cz^{2n} + dz^{3n} + ez^{4n} + fz^{5n} + \&c.$ then neglecting $h^{v} z^{vm}$ till afterwards, and working as per ex. 69, we'll have $\overline{1 + p}|^{v} = 1 + Ap + Bpp + Cppp + Dp4$ + &c. whence

$$1 + Ap = 1 + Abz^n + Acz^{2n} + Adz^{3n} + Aez^{4n} + Afz^{5n} + \&c.$$
$$Bpp = \quad + Bb^2z^{2n} + 2Bbcz^{3n} + Bccz^{4n} + 2Bbdz^{4n} + 2Bebz^{5n} + 2Bcdz^{5n}$$
$$Cppp = \quad + Cb^3z^{3n} + 3Cb^2cz^{4n} + 3Cbccz^{5n} + 3Cbbdz^{5n} + \&c.$$
$$Dp^4 = \quad + Db^4z^{4n} + 4Db^3cz^{5n} + \&c.$$
$$Ep^5 = \quad + Eb^5z^{5n} + \&c.$$

I here break off the feries at z^{5n}, i. e. I take in all the terms in p, p^2, p^3, p^4 and p^5, which come below z^{6n}, you may break off fuch a feries where you pleafe, but the more terms you take, the nearer the truth your anfwer will be, and nearer the law of continuation you'll approach, if not quite.

124. Now the like terms of z collected together and multiplied by $h^v z^{vm}$ gives $h^v z^{vm} + Abh^v z^{vm+n} + \overline{A + Bb^2} \times h^v z^{vm+2n} + \overline{Ad + 2Bbc + Cb^3} \times h^v z^{vm+3n} + \overline{Ae + 2Bbd + Bcc + Db^4} + h^v z^{vm+4n} + \overline{Af + 2Beb + 2Bcd + 3Cbcc + 3Cb^2 + 4Db^3c + Eb5} \times h^v z^{vm+5n} + \&c. = hz^m \times \overline{1 + bz^n + cz^{2n} + dz^{3n} + \&c.}|^v$

To illuftrate this theorem by one particular cafe.

125. Let it be required to raife $2z^2 - 6z^5 + 8z^8 - 12z^{11} + 16z^{14}$ &c. $= 2z^2 \times : 1 - 3z^4 + 4z^6 - 6z^9 - 8z^{12}$ &c. to the 5th power. Here we have $h = 2$, $m = 2$, $b = 3$, $c = 4$, $d = 6$, $e = 8$, &c. $n = 3$, $v = 5 = A$, $B = v \times \frac{v-1}{2} = 10$, &c. and fubftituting thefe values in the above general theorem, it will become, $32z^{10} - :32:\times:15z^{13} : + :32\times20 + 90z^{16}: + 32\times - 30 - 240 - 270z^{19}$ &c. $= 32z^{10} - 32:15z^{13} + 32:110z^{16}: - 540:32z^{19}$ &c. the required feries ; or, eafier thus, $32\times:z^{10} - 15z^{13} + 110z^{16} - 540^{19}$ &c.

126. Lemma. If the fum of a feries of terms confifting of the feveral powers of a variable quantity be = o, the fum of the terms, where any one and the fame power of that variable quantity is involved, will alfo be = o,

That is, If	1	$az^n + bz^n + cz^m + dz^m + ez^r + fz^r + gz^r + = 0,$
then $1 \div z^n$	2	$a + b + \&c. = \frac{0}{zn} = 0$, Ergo, $a + b = 0$,
$1 \div z^m$	3	$0 + 0 + c + d + \&c. = \frac{0}{zm} = 0$, whence $c + d = 0$
$1 \div z^r$	4	$0, + e + f + g = \frac{0}{zr} = 0$, so, $e + f + g = 0$,
consequen.	5	$a + b = 0$, $c + d = 0$, $e + f + g = 0$, &c.

127. Therefore, if a sum of a series of terms consisting of the several powers of a variable quantity be $= 0$, there must be at least two terms wherein each of the said powers are involved. Otherwise, the values of the co-efficients in such an assum'd series cannot be determin'd, now to do this universally.

128. Let it be required to revert the universal infinite series, $e^m + be^{m+n} + ce^{m+2n} + de^{m+3n} + Ee^{m+4n} + fe^{m+5n} + \&c. = z$, or which is the same thing, to put z into such a series, as may express the value of e, to effect which, assume the infinite series $z^{\frac{1}{m}} + Bz^{\frac{1+n}{m}} + Cz^{\frac{1+2n}{m}} + Dz^{\frac{1+3n}{m}} + \&c. = e$, then by what is said in ex. 122, we'll have,

$$1.\ e^m = z + mBz^{\frac{1+n}{m}} + mCz^{\frac{1+2n}{m}} + mDz^{\frac{1+3n}{m}} + m\frac{m-1}{2}B^2z^{\frac{1+2n}{m}} + m\frac{m-1}{2}BCz^{\frac{1+3n}{m}} + m\frac{m-1}{2},\frac{m-2}{3}B^3z^{\frac{1+2n}{m}} + \&c.$$

$$2.\ be^{m+n} = bz^{\frac{1+n}{m}} + \overline{m+n}Bbz^{\frac{1+2n}{m}} + \overline{m+n}Cbz^{\frac{1+3n}{m}} + \overline{m+n}\,\frac{m+n-1}{2}B^2bz^{\frac{1+3n}{m}} + \&c.$$

$$3.\ ce^{m+2n} = cz^{\frac{1+2n}{m}} + \overline{m+2n}Bcz^{\frac{1+3n}{m}} + \&c.$$

$4.\ de^{m+3n} = dz^{\frac{1+3n}{m}}$ + &c. breaking the series off at $z^{\frac{1+3n}{m}}$

&c. &c.

Here its plain, if these values of e^m, be $c e^{m+n}$, e^{m+2n} &c. be substituted in the assum'd series, it will express the value of e, but then the co-efficients B, C, D, &c. are not known, so to find them we have, first, those of $z^{\frac{1+n}{m}}$, are, b and m B, so (per lemma) $b+mB=0$, whence, $B=-\frac{b}{m}$. Secondly, those of $z^{\frac{1+2n}{m}}$, are, $c+mC+m\frac{m-1}{2}, B^2+m+nBb$, which (per said lemma) are $=0$, whence $C=\frac{m+1+2nbb,-2mc}{2mm}$, (for $B=-\frac{b}{m}$ and $B^2=\frac{bb}{mm}$ as just now found). Thirdly, the sum of the co-efficients of $z^{\frac{1+3n}{m}}$, are $d+mD+m\frac{m-1}{2}BC+m\frac{m-1}{2},\frac{m-2}{3}, B^3+,m+nCb+m+n\frac{m+n-1}{2}B^2b+m+2nBc$, which also, are $=0$ (per lemma) now by reduction and substituting the values of B and C, as above found, we have $D=\frac{:2mm+9mn+9n^2+3m+6n+1:-b^3}{6m^3}+\frac{:1+3n+m:bc}{m^2}-\frac{d}{m}$, now substituting these values of B, C, D, in the assum'd series, it will become $e=z^{\frac{1}{m}}-\frac{bz}{m}^{\frac{1+n}{m}}+\frac{:m+1+2n:b^2:-2mc}{2mm}z^{\frac{1+2n}{m}}+\frac{:2m^2+9mn+9n^2+3m+6n+1:-b^3}{6m^3}+\frac{:1+3n+m:bc}{mm}-\frac{d}{m}z^{\frac{1+3n}{m}}$. A general theorem whereby a series representing the value of e in any particular case may be readily found. Thus, Ex. 129. required the value of e in $e+be^2+ce^3+de^4+$ &c. $=z$. Here $e^m=e$, $e^{m+n}=e^2$, so $m=1$, and $n=1$; therefore, $e=z^{\frac{1}{m}}-\frac{bz}{m}^{\frac{1+n}{m}}$ &c. $=z-bz^2+:2b^2-c:z^3+:5bc-5b^3-d:z^4$ &c.

130. Required the value of e, in $e-be^3-ce^5-de^7-$ &c. $=z$, here $e^m=e$, $e^{m+n}=e^3$, so $m=1$, and $n=2$, b, c, d, &c. each negative. Therefore, $e=z^{\frac{1}{m}}-\frac{bz}{m}^{\frac{1+n}{m}}$ &c. $=z+bz^3+:3b^2+c:z^5+:8bc+12b^3+d:z^7$ &c.

130. Required the value of e, in $e - be^3 - ce^5 - de^7 -$ &c. $= z$. Here, $e^m = e$, $e^{m+n} = e^3$ so $m = 1$ and $n = 2$, b, c, d, &c. each negative; therefore, $e = z^{\frac{1}{m}} - \frac{b}{m} z^{\frac{1+n}{m}}$ &c. $= z + bz^3 + : 3b^2 + c : z^5 + : 8bc + 12b^3 + d : z^7$ &c.

131. Required e, in $e^2 + be^4 + ce^6 + de^8 +$ &c. $= z$.

Here, $e^m = e^2$ $e^{m+n} = e^4$, so $m = 2$ and $n = 2$, whence $e = z^{\frac{1}{m}} - \frac{b}{m} z^{\frac{1+n}{m}}$ &c. $= z^{\frac{1}{2}} - \frac{1}{2} b z^{\frac{3}{2}} + : \frac{7b^2 - 4c}{8} : z^{\frac{5}{2}} + : \frac{9bc - 2d}{4} - \frac{33b^3}{16} : z^{\frac{7}{2}}$ &c.

132. Required e, in $\sqrt{e} + be^4 + ce^{7\frac{1}{2}} + de^{11} +$ &c. $= z$. Here, $e^m = {}^{\frac{1}{2}}$ $e^{m+n} = e^4$ whence $m = \frac{1}{2}$ and $n = \frac{7}{2}$, so $e = z^2 - 2bz^9 + : 17b^2 - 2c : z^{16} + : 48bc - 200b^3 - 2d : z^{23}$ &c.

133. Let it be required the square (e) root out of this equation, $3e - 6e^2 + 8e^3 - 13e^4 +$ &c. $= 15y$.

Note. When the first or least term has a coefficient, it is best to divide the whole equation by it.

So this equation divided in this manner is $e - 2e^2 + \frac{8}{3}e^3 - \frac{13}{3}e^4 +$ &c. $= 5y = z$ whence, $b = -2$, $c = \frac{8}{3}$, $d = -\frac{13}{3}$, now by substituting these values of b, c, and d, in ex. 129, (because the powers of e there and here are alike) it will become $e = z - 2z^2 + \frac{16}{3}z^3 + 5\frac{1}{3}z^4$ &c.

134. If (in Ex. 128.) z were = another universal infinite series, $Qv^p + Rv^{p+t} + PE^{p+2t} + Tv^{p+3t} +$ &c. Q, R, S, T, &c. being known coefficients, and it were required to find e in terms of v, it is evident, the last general theorem will answer this end, by substituting this series in it, for z and the powers of z, i. e. raising the said series, to the powers of $\frac{1}{m}$ $\frac{1+n}{m}$, &c.

135. If you would have more terms in value of e, than what is in these examples, it will be easy in most numerical cases, from these four terms to give the law of continuation, but if in any case they do not, you may either continue the general theorem further by taking in terms beyond $z^{\frac{1+3n}{m}}$, before you break off, or you may from what is here done, easily revert any particular case by itself. See the following ex.

136. Required e in $e - be^3 + ce^5 - de^7 + oe^9 - \&c. = z$.

Here, as in ex. 130, $m = 1$, and $n = 2$, but every second coefficient negative, whence, (per 128, general theorem,) $e = z + bz^3 + : 3b^2 z^5 - cz^5 : + : 12b^3 + d - 8bc : z^7$ &c. here, if we take $-b = \frac{-1}{2,3} = \frac{-1}{6}$ $c = \frac{1}{2,3,4,5} = \frac{1}{120}$, $-d = \frac{-1}{2,3,4,5,6,7} = -\frac{1}{5040}$, we'll have $e = z + \frac{z^3}{2,3} + \frac{3z^5}{36} - \frac{z^5}{120} + \frac{12z^7}{216} + \frac{z^7}{5040} - \frac{8z^7}{720}$ &c. which terms duly reduced, gives $e = z + \frac{z^3}{6} + \frac{3z^5}{40} + \frac{5z^7}{112} + \&c.$ where it plainly appears that if we take $A = z$, $B = \frac{zz}{2} A$, $C = \frac{3zz}{4} B$, $D = \frac{5zz}{6} C$, &c. we'll have $e = A + \frac{1}{3} B + \frac{1}{5} C + \frac{1}{7} D + \frac{1}{9} E + \frac{1}{11} F + \&c.$ every of these denominators increasing 2, which evidently shews the law of continuation.

137. Now if we take $z = \frac{1}{2} = 0{,}5 = A$, then will $B = \frac{1}{8} A$, $C = \frac{3}{16} B$, $D = \frac{5}{24} C$, $E = \frac{7}{32} D$, $F = \frac{9}{40} E$, $G = \frac{11}{48} F$ &c. that is

A = ,5	,50000000 = A
B = ,0625	2083334 = $\frac{1}{3}$ B
C = ,01171875	234375 = $\frac{1}{5}$ C
D = ,00244141	34877 = $\frac{1}{7}$ D
E = ,00053406	5934 = $\frac{1}{9}$ E
F = ,00012016	1093 = $\frac{1}{11}$ F
G = ,00002754	212 = $\frac{1}{13}$ G
H = ,00000639	43 = $\frac{1}{15}$ H
I = ,00000149	9 = $\frac{1}{17}$ I

which collected gives, sum = ,52359877 = e, and by taking in more terms you may get e to what exactness, or places of figures you please.

138. If the radius of a circle be 1, the sine of 30° will be $\frac{1}{2}$ or 0,5, and then e = ,52359877, will be the length of the arch 30°, which multiplied by 6 (because 30° is $\frac{1}{6}$ of the $\frac{1}{2}$ periphery) gives 3,14159262, true to the last figure, which should be 5, instead of 2, for the periphery of a circle whose diameter or 1, or of that half circle whose radius is 1. See art. 173.

PROBLEM CLXXVII.

The uſe of infinite ſeries in the reduction of equations.

139. When an equation is given in two variable or unknown quantities, e, and y, to find (y) one of them in terms of (e) the other, and can be done no other way, we muſt have recourſe to infinite ſeries, now to find the indices of ſuch an aſſum'd ſeries as will beſt anſwer this end, viz. to make the found ſeries converge the faſteſt &c. take theſe rules.

140. Rule 1. Make the given equation $= 0$, then for y in the ſaid equation ſubſtitute $A\,e^n$, if fluctional letters be the ſame in every term, they may be rejected as conſtant ones.

141. Rule 2. In this new equation make two of the leaſt indices of e, which have contrary ſigns = each other, when e is ſmall in reſpect of a, otherwiſe the two greateſt powers of e with contrary ſigns muſt be made = each other, by which we find n, the index of the firſt term of the required ſeries.

142. By making the co-efficients $= 0$, of thoſe terms whoſe indexes were made = each other, we'll have the value of A, the co-efficient of the firſt term of the required ſeries.

143. Write the value of n found as above, in each of the indices of e, and ſet down the difference, that there is between one of the equal ones above-named, and every one of the different indexes of e, which call a ſeries of differences.

143. To this ſeries of differences ſet down all the leaſt different numbers, that can be made, by doubling, trebling, adding together &c. the terms of the ſaid ſeries, with thoſe that ſhall be thus produced, till you have thereby got as many different terms, as you deſign the required ſeries to conſiſt of.

145. Rule 6. Add each of theſe terms to the value of n, if the leaſt indices were made equal, otherwiſe they muſt be taken from the ſame, and the ſum or difference, are the indexes of the terms of the required ſeries.

146. If A, have two or more equal values (per 142) the ſeries of differences found per 143, muſt be divided by the number of thoſe equal values before they are added to, or taken from the value of n. But if A have another poſſible value beſides theſe equal ones this other value is uſed, and theſe equal ones not regarded.

147. If in the given equation after the ſubſtitution of $A\,e^n$ for y, there be any term that hath no power of e in it, then the index of e in ſuch a term is always $= 0$.

148. If there be any fraction, or surd in the given equation, it must be cleared thereof by division, or involution &c. if it can, if not, it must be cleared, by putting that part where the surd &c. is into a series.

149. The value, or values of A, found by (142), Rule 3. or by the comparing of like terms as by Art. 126, are the same, so having found A, as per art. 142, you may either use this found value, or A it's self in the assum'd series, for if $A e^n =$ the first term of any series, and S $=$ the rest, i.e. if $A e^n + s = y$ in any equation, then because the first term of any series is either the greatest, or the least of all the terms, consequently the terms in that equation, which have the least, or greatest indices of e, may be made $=$ per 126, for if the sum of a series be supposed $= 0$, the co-efficients are to be such as will actually make that series $= 0$.

150. Example. Given $a^3 e + a e^3 = a^3 y + y^4$, viz. $a^3 e + a e^5 - a^3 y - y^4 - 0$, to find y in terms of e, in a series converging faster as e is greater, or which is the same, as is supposed to be small in respect of e.

By putting $A e^n = y$, we'll have $- A e^n a^3 - A^4 e^{4n} + a^3 e + a e^3 = 0$, per rule 1, whence the indices of e are n, 4^n, 1 and 3, the two greatest of which with contrary signs (per rule 2,) are 3 and 4^n, so $n = \frac{3}{4}$, and their co-efficients are A^4 and a, so (per 142) $A = a^{\frac{1}{4}}$ and these indices of e will be $\frac{3}{4}$, 3, 3 and 1, (per 143) each of which differs from (3 or 4^n) one of the above equal ones by 0,2, $2\frac{1}{4}$, the series of difference (per 143) now, let the required assum'd series consist of 5026 terms, then the least different numbers that can be produced by doubling &c. and adding together of 0,2, $2\frac{1}{4}$ will be 0,2, $2\frac{1}{4}$ $4\frac{1}{4}$, $4\frac{1}{2}$ &c. Then (per rules 2 and 6,) each of these taken from ($n =$) $\frac{3}{4}$ leaves $\frac{3}{4}$, $-\frac{5}{4}$, $-\frac{6}{4}$, $-\frac{13}{4}$, $-\frac{14}{4}$ &c. the indices sought; whence, the assum'd series is $A e^{\frac{3}{4}} + B e^{-\frac{5}{4}} + C e^{-\frac{6}{4}} + D e^{-\frac{13}{4}} + E e^{-\frac{14}{4}}$ &c. $= y$, which being put in the given equation for y it will become

$$\left.\begin{array}{llll} A^4e^3+4A^3Be+4A^3Ce^{\frac{3}{4}}+4A^3De^{-1} & +4A^3Ee^{-\frac{5}{4}} & +4A^3Fe^{-\frac{6}{4}} \\ -ae^3-a^3e \qquad\qquad +6A^2Be^{-1} & +12A^2BCe^{-\frac{5}{4}} & +6A^2C^2e^{-\frac{6}{4}} \\ \qquad +a^3Ae^{\frac{3}{4}} & +a^3Be^{-\frac{5}{4}} & +a^3Ce^{-\frac{6}{4}} \end{array}\right\} \text{\&c.} = 0.$$

Now (per art. 126 and 127) by making these terms equal 0, that have the same power of e in them, as they here stand one below another, we'll have $A = a^{\frac{1}{4}}$ (as found before) $B = \frac{1}{4} a^{\frac{3}{4}}$, $C = -\frac{1}{4} a^{\frac{10}{4}}$,

$D = -\frac{3}{8}a^{\frac{17}{4}}$, $E = \frac{1}{8}a^{\frac{18}{4}}$, $F = -\frac{1}{32}a^{\frac{19}{4}}$, &c. whence, $y = a^{\frac{1}{4}}e^{\frac{1}{4}}$
$$+\frac{a^{\frac{3}{4}}}{4e^{\frac{5}{4}}} - \frac{a^{\frac{10}{4}}}{4e^{\frac{6}{4}}} - \frac{3a^{\frac{17}{4}}}{8e^{\frac{13}{4}}} + \frac{a^{\frac{18}{4}}}{8e^{\frac{14}{4}}} - \frac{a^{\frac{19}{4}}}{32e^{\frac{15}{4}}} \text{ \&c.}$$

151. Required the value of y, in $aay + y^3 - e^3 = 0$, e being small in respect of a, here, by putting Ae^n for y, we have $aaAe^n + A^3e^{3n} - e^3 = 0$, and (per rule 2) the indices of e are n, 3^n and 3, the two least of which with contrary signs are n and 3, whence, $n = 3$, and $A = \frac{1}{ay}$ (per art. 142) and the indices of e, in numbers are 3, 9 and 3, each differing from $n = 3$ (143) by 0,6, for the series of differences, then by doubling, trebling &c. and adding together of 0,6, we get 0,6, 12, 18, &c. (per rule 6) each of which added to $n = 3$, gives 3, 9, 15, 21, &c. for the indices of the required assum'd (universal infinite) series ($Ae^n + Be^{n+2m} + Ce^{n+3m} + De^{n+4m}$ + &c.) whence

$y = Ae^3 + Be^9 + Ce^{15} + De^{21} + Ce^{27}$ + &c. Then will

$y^3 = A^3e^9 + 3A^2Be^{15} + 3A^2Ce^{21} + 3A^2De^{27} + 3AB^2e^{21}$
$(+ 6ABCe^{27} + B^3e^{27}$ &c.

$aay = aaAe^5 + aaBe^9 + aaCe^{15} + aaDe^{21} + aaEe^{27}$ &c.

$y^3 - e^3 = -e^3 + A^3e^9 + 3A^2Be^{15} + 3A^2Ce^{21} + 3AB^2e^{21}$
$(+ 3A^2De^{27} + 6ABCe^{27} + B^3e^{27}$ &c.

Now by equating the like terms, &c, (as per 126) we shall have

$$\left.\begin{matrix} aaA \\ -1 \end{matrix}\right\} = 0, \quad \left.\begin{matrix} aaB \\ A^3 \end{matrix}\right\} = 0, \quad \left.\begin{matrix} aaD \\ 3A^2C \\ 3AB^2 \end{matrix}\right\} = 0, \quad \left.\begin{matrix} aaE \\ 3A^2D \\ 6ABC \\ BBB \end{matrix}\right\} 0, \text{ \&c.}$$

$$\text{whence, } \left\{\begin{matrix} A = \frac{1}{aa}, & B = \frac{-1}{a^8} \\ C = \frac{3}{a^{14}}, & D = \frac{-12}{20} \\ E = \frac{55}{a^{26}}, \text{ \&c.} & \end{matrix}\right\} \text{Therefore, } y = \frac{e^3}{aa} - \frac{e^9}{a^8}$$
$$+\frac{e^{15}}{a^{14}} - \frac{12e^{21}}{a^{20}} + \frac{55e^{27}}{a^{26}} \text{ \&c.}$$

152. Given $a^3y^3 - 4a^2e^2y^2 + 5ae^4y - 2e^6 - e^3y^3 - y^6 = 0$, to find y, when e is small in respect of a. Here by putting $Ae^n = y$ we have $a^3A^3e^{3n} - 4a^2A^2e^{2n+2} + 5aAe^{n+4} - 2e^6 - A^3e^{3n+3} - A^6e^{6n} = 0$: now by making any two of the least indices with contrary signs ($3n$, $-2n + 2$, $n + 4$, -6) equal each other, (because no such two of these can be less than the

reſt and yet equal each other) we have $n = 2$, and the coefficients belonging them are $A^3 a^3 - 4A^2 a^2 + 5 A a - 2 = 0$ (per 146,) which being diviſible by $A a - 1$, and the quotient again by $A a - 1$, the laſt quotient is $A a - 2$; whence, A has two equal values, each $= \frac{1}{a}$, and one poſſible odd value $\frac{2}{a}$, ſo the equal ones are not regarded, whence our indices will be 2, 5, 8, 11; ſo $y = \frac{2 e^2}{a}$ (or A) $+ B e^5 + C e^8 + D e^{11} + E e^{14}$ &c. which being involved, &c. as in the two laſt examples, we ſhall have $y = \frac{2 e^2}{a} + \frac{8 e^5}{a^4}$ $\frac{32 e^8}{a^7} + \frac{34 \times 32 e^{11}}{a^{10}} + \frac{24 \times 64 e^{14}}{a^{13}} +$ &c.

153. If in any ſuch equations e ſhould differ but little from a, (or the equation be of the logarithmetical kind, where it is ſometimes impoſſible without a different ſubſtitution) it will be beſt to work as follows.

154. Given $y^3 + a a y - e^3 = 0$, to find y; e being ſuppoſed nearly $= \frac{2}{3} a$, or $\frac{2}{3} a - z = e$. Then $e^3 = \frac{8 a^3}{27} + \frac{4 a a z}{3} - 2 a z z$ $- z z z$, whence, $y^3 + a a y - e^3 = y^3 + a a y - \frac{8 a^3}{27} + \frac{4 a a z}{3}$ $- 2 a z z + z^3$, and putting $A z^n = y$, the indices will be, 3 n, n, 0, 1, 2, 3, the two leaſt of which with contrary ſigns (becauſe e is leſs than a) are n and 0, viz. $n = 0$, and becauſe $n = 0$, 3 n is alſo $= 0$, and the co-efficients belonging them are $A^3 + A a a \frac{8 a^3}{27} = 0$, from which the value of A, may be had by ſolving an adfected equation (ſee ex. 83) &c. as before, you'll have $B = \frac{-4 a a}{3 a^2 + 9 A^2}$, $C = \frac{2 a - 3 A B^2}{a a + 3 A^2}$, $D = \frac{- B 3 - 1}{a^2 + 3 A^2}$ &c. the aſſumed ſeries being $A + B z + C z^2 + D z^3$ &c. $= y$.

155. If e be greater than a, or $\frac{3}{2} a - z = e$, then by writing $A z^n$ for y, our given equation $y^3 + a^2 y - e^3 = 0$ will become $A^3 z^{3n}$ $+ a a A z^n - \frac{27 a^3}{8} + \frac{27 a^2 z}{4} - \frac{9 a z z}{2} + z^3 = 0$, the two greateſt of theſe indices with contrary ſigns (becauſe e is greater than a),

are 3 n and 2, ſo $n = \frac{2}{3}$, their co-efficients are $A^3 - \frac{9a}{2}$ ſo $A = \overline{\frac{9a}{2}}\Big|^{\frac{1}{3}}$, and theſe indices in numbers will be 2, $\frac{2}{3}$, 0, 1, 2, 3, each of which differs from n, or $\frac{2}{3}$ by 0, $\frac{1}{3}$, $1\frac{1}{3}$, $2\frac{1}{3}$, or 0, $\frac{1}{3}$, $\frac{4}{3}$, $\frac{7}{3}$, the ſeries of differences, which by doubling &c. gives, 0, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, each of theſe taken from n, $\frac{2}{3}$, leaves, $\frac{2}{3}$, $\frac{1}{3}$, 0, $-\frac{1}{3}$, $-\frac{2}{3}$, $-\frac{3}{3}$, $-\frac{4}{3}$, ſo $y = Az^{\frac{2}{3}} + Bz^{\frac{1}{3}} + Cz^{0}$, viz. $+ C + Dz^{-\frac{1}{3}} + Ez^{-\frac{2}{3}} +$ &c. for the aſſum'd ſeries.

PROBLEM CLXXVIII.

CONTAINING, *The principals of Geometry, or chief theorems in Euclid compendiouſly demonſtrated.*

Before you enter upon the following theorems, make your ſelf maſter of what is contained in prob. 120.

156. If two right lines C G and D E (Fig. 130,) any way croſs each other, they will make four angles, the oppoſite of which are equal, viz. $\angle$ E B C $= \angle$ G B D, and $\angle$ E B G $= \angle$ C B D, this is ſo plain it needs no demonſtration. Theo. 1.

157. If a line C G, be cut by two or more parallel lines D E, and A C, it will make equal angles with all theſe parallel lines, viz. $\angle$ A $= \angle$ a, and $\angle$ C $= \angle$ c, &c. this is plain per figure. Theo. 2.

158. If a line B G meet another line D E in any point B, it will make two angles therewith whoſe ſum is 180°, for (fig. 130) upon B ſweep the ſemicircle, ſo are the arches G D and E F G, the meaſures of the angles G B D and G B E, which two arches are per fig. = 180° or a half circle. Theo. 3.

159. the ſum of three angles of any plane triangle are equal to 180° (fig. 130) for, thro' any angular point B, draw a line D E parallel to the oppoſite ſide A C, and produce the other two ſides A B and C B, then per theorem 1. angle b $= \angle$ B, and by theo. 2. $\angle$ a $= \angle$ A, and $\angle$ c $= \angle$ C, which three angles a, b, and c, are manifeſtly = a ſemicircle, or 180°, i. e. a + b + c = 180° = A + B + C. Theo. 4.

160. If any ſide C A of a triangle A B C, be produced, the outward angle B A I is equal to the ſum of the two inward oppoſite $\angle$s B and C (fig. 130) for by theorem 3. 180° − B A C $= \angle$ B $+ \angle$ C, conſequently, $\angle$ B A I $= \angle$ B $+ \angle$ C. Theo. 5.

161. In a circle L O B F (fig. 48) any angle O I F at the center I, is double of the angle O B F at the circumference, when the ſame arch O L F, or ſame chord F O is the baſe of the angles. For by theo. 5, $\angle$ L I F = $\angle$ I B F + $\angle$ F F B, but I B = I F (being ſemi-diameters) ſo their oppoſite angles I B F and I F B are equal, whence, $\angle$ L I F = 2 $\angle$ I B F = 2 $\angle$ I F B in like manner it will be prov'd that $\angle$ L I O = 2 $\angle$ I B O, conſequently, $\angle$ O I F = 2 $\angle$ O B F. Theo. 6.

This way of reaſoning holds true, let the point B be where it will in the arch O B F, which ſhews that all angles are equal which are in the ſame ſegment of a circle i. e. $\angle$ B O F = $\angle$ B L F &c. Theo. 7.

Becauſe the diameter of a circle ſubtends 180°, viz. half the periphery, its evident by theorem 6, that an angle C F D (fig. 41) in a ſemi-circle is a right angle, Theorem 8.

From whence it follows, that an angle in a ſegment O B Q F (fig. 48) greater than a half circle is acute, and an angle B Q F in a ſegment leſs than a ſemi-circle is obtuſe, conſequently, if any trapezia B Q F O be inſcribed in a circle, the ſum of any two oppoſite angles B Q F + B O F, or O F Q + O B Q, is = to 180°, and therefore theſe ſorts of trapezia's can only be inſcrib'd in circles.

162. If any two figures are alike, their like ſides are proportional, that is, thoſe ſides which ſubtend the equal angles, as alſo thoſe ſides which are about the equal angles, are proportional, and conſequently if any two or more figures have all their ſides proportional, their angles are equal. Theorem 9.

For ſuppoſe the ſides A B and A C (fig. 36) of a triangle A E B, to be produc'd by the motion of the ſide or line E B, parallel to itſelf, and in it's motion to increaſe in length, ſo that the points E and B may always be found in the produced ſides A E and A B, then it's evident that when E B becomes C D, that A E will become A D, and A B will be A C, i. e. as E B : D C : : A E : A D, and : : A B : A C, if we make C F = E B and join E F, then by the above ſaid motion E B, has increaſed or gained in length, D F, and A E has gained E D, alſo A B, has gained B C, i. e. as A E : A B : : E D : B C = E F and ſo on, for other like figures, which are but ſome number of triangles put together, for all plane figures may be ſuppoſed to be made up of triangles and therefore what holds in ſimilar triangles, muſt alſo hold in all ſimilar planes; that the triangles A E B and A C D are ſimilar, it's plain, for E F being parallel to A C, the $\angle$ F E D = $\angle$ E A B, (per theo. 2) as alſo $\angle$ E F D = $\angle$ A C D, becauſe C F is parallel to E B, and the $\angle$ D, is common to both triangles, in like manner it may be proved that all the 3 triangles, A E B, A D C, and E D F, are alike; there are other ways of demonſtrating this famous theorem, whoſe uſes are

so many, that few problems can be solved, or theorems demonstrated without it, as appears by what follows.

163. If two right lines F E and G I, cross each other within a circle they will form two similar triangles y E I and y G F. Fig. 131.

For by theorem 7. $\angle$ y I E = $\angle$ y F G, because they both stand on the same chord E G, and for the same reason, $\angle$ y E I = $\angle$ y G F, so by the last theorem, it will be as y I : y E : : y F : y G. Ergo, y F $\times$ y E = y I $\times$ y G. Theo. 10.

164. If two right lines A G and A F, meet each other without a circle, (the other ends terminating in the periphery) the angles A I G and A E F, will be similar, for theorem 7. $\angle$ A F E = $\angle$ A G I, and $\angle$ A common, so it will be as A G : A F : : A I : A E. Ergo, A G $\times$ A E = A F $\times$ A I. Theo. 11.

165. If a triangle G I F, inscribed in a circle have one of its angles G I F bisected, with the line I D, which produced meets the circle in e, there will be two similar triangles I G e and I F D, for by hypothesis, $\angle$ D I F = $\angle$ D I G, and by theo. 7. $\angle$ I F D = $\angle$ I e G, both standing on the same chord I G (fig. 131) and therefore as I D + D e (I e) : I G : : I F : I D, ergo, $\square$ I D + D e $\times$ I D = I G + I F, but by theo. 10, F D $\times$ D G = D e + I D, whence $\square$ I D + F D $\times$ D G = I G $\times$ I F, or by tranposition, $\square$ I D = I G $\times$ I F — F D $\times$ D G. Theorem 12.

If we suppose the triangle G I F to be isosceles, viz. I F = I G, then its manifest the bisecting line I D, will also bisect the base G F and be a perpendicular thereunto, i. e. D I F and D I G, will be two equal right angled triangles, and then the last theorem will become $\square$ I D = $\square$ F I — $\square$ F D, or = G I — $\square$ G D. Theo. 13, which is the famous theorem in 47, E. 1, of so much service in all parts of mathematics and in words is, thus; In any right angled plane triangle, the square of the hypothenuse is equal to the sum of the squares of the two legs. Now if (in fig. 40) we put u = D E, c = D C, p = D F, a = E F and e = C F, b = the hypothenuse C E = a + e, we may prove this theorem another way; Thus, the 3 Δs C D E, C D F, and E D F, are all similar, for (by problem 129) $\angle$ C F D 90° — $\angle$ F C D = $\angle$ C D F, and $\angle$ C D E 90° — $\angle$ F C D = $\angle$ C E D, whence $\angle$ C D F = $\angle$ C E D, in like manner it is proved that $\angle$ F D E = $\angle$ F C D, the 3 angles C D E, C F D, and E F D, being each = 90°,

Then by theo. 9	1	$a:p::p:e$, ſo, $ae=pp$, whence $\sqrt{ae}=p$, Theo. 14
	2	$b:c::c:e$. Ergo, $cc=eb=e\times\overline{a+e}=ae+ee$
	3	$b:u::u:a$. Ergo, $uu=ab=a\times\overline{a+e}=aa+ae$
2+3	4	$cc+uu=aa+2ae+ee=\overline{a+e}\vert^2=bb$ Q.E.D.
per 2 and 3	5	$cc:uu::ae+ee:aa+ae::e:a$, Theo. 15
	6	$a+e(b):aa+ae\,(uu)::a:\dfrac{aaa+aae}{a+e}=aa$, Theo. 16

166. (Fig. 131) If e I be the diameter of a circle, and n I a perpendicular in the triangle I G F, then the △s I e F, and I G n are alike, for (by theorem 7) ∠ I e F = ∠ I G F, and by ſuppoſition ∠ I n G = 90° and (by theo. 8) ∠ I F e = 90°, ſo it will be (by theo. 9) as n I : I G : : I F : I e. Theo. 17.

167. (Fig. 131) If any trapezia, I G e F, be inſcribed in a circle, and the diagonals I e and F G be drawn, and ∠ G I n be made = ∠ e I F, the triangles A e F and I G n will be ſimilar, for ∠ n I F = ∠ G I e by the figure, and ∠ I e F = ∠ I G n, by theo. 7, as alſo, ∠ I F n = ∠ I e G, whence the triangles I F n and I e G are alſo alike. So per theorem 9,

	1	I e : e F : : I G : G n, and as I F : F n : : e I : e G
1 ∵	2	I e × G n = e F × I G, and I e × F n = e G × I F
1 + 2	3	I e × : G n + F n : = e G × I F + e F × I G
viz.	4	I e × G F = e G × I F + e F × I G, for F n + G n = G F, per fig.

Theorem 18. This is called Ptolemy's theorem, ſaid to be found by him. If any two ſides of the aboveſaid trapezia be parallel, ſuppoſe e F ‖, to I G, then the other two ſides e G and F I will be equal as will alſo the diagonals, viz. F G = e I, and then the laſt theorem will become □ I e = □ e G + e F × I G, or □ I e − □ G e = e F × I G. Theo. 19.

168. (Fig. 67) If any point C be taken in a right line A B, and it be made as A C − B C : B C : : A C : C O, and with the radius C O a circle be deſcribed upon O, and lines drawn from A and B to any point P in the periphery, it will be as B C : A C : : B P : A P. For

Since	1	A C − B C : B C : : A C : C O, ergo, B C × A C = C O × : A C − B C.
1 ÷	2	: A C + C O : × B C = C O × A C = A O × B C, per fig.
2 in : :	3	C O : A O : : B C : A C, but per fig. C O − B O = B C, and A O − C O = A C.
ſo per 3	4	C O : A O : : C O − B O : A O − C O.
4 ÷	5	C O : A O : : B O : C O, or P O : B O : : A O : P O, : : A P : B P.
3 and 5	6	B C : A C : : B P : A P, Q. E. D. Theo. 20.

169. In these theorems, when I have found any two angles of one triangle respectively equal to two angles in any other △, I say these two triangles are similar or equiangular which is evident from theorem 4, and because the sides opposite to such equal angles are proportional (by theo. 9) it is manifest that in all △s, equal sides subtends equal ∠s, the greatest side subtends the greatest ∠, the least side, the least ∠, and contrary. Theo. 21.

170. In any rectangle (A B C D), fig. 132, it's area is had by taking the product of the length and breadth, viz. A B × B C = area ▭ A B C D, this will appear if you number the little parallelograms in the figure for you'll find their number equal 66 = A B 11 × B C 6. Theo. 22.

If D E = C F, then because A D and B C, as also A B and D C, are parallel, the sides and angles (by theo. 21 and 22) in the △s A D E and B C F, are respectively equal, consequently their areas will be equal. Hence, if from the end B C of the ▭ A B C D, we take △ B C F (= △ A E D) and add it to the end A D thereof, it's plain the ▭s A B C D and A E F B are equal, whence A B × A D = area of the rhomboides A E F B. Theo. 23. By the same way of reasoning it will appear that $\frac{A\,B \times e\,F}{2} = \left(\frac{A\,B \times B\,C}{2}\right)$ area triangle AFB. Theorem 24.

Whence if d = the diagonal of any trapezia, e and n two ⊥s, falling thereon, then half d e = area one of the △s, and half d n = area of the other △, so half d e + half d n = $\frac{d\,e + d\,n}{2} = \frac{:e + n: \times d}{2}$ = area trapezia. Theo. 25.

But if a trapezia E A e F, have two parallel sides e A and E F, then half A D × D E (E F — D F) = △ A D E, and A D × D F = ▭ A D F e, and the sum of these two is, half A D × : E F — D F : + A D × D F = $\frac{A\,D \times :E\,F + D\,F\,(A\,e):}{2}$ = area of the trapezia E A e F. Theo. 26.

If S = the side of any regular polygon, R = the radius of its inscribed circle, and n = the number of its sides, then half R S = the area of one of the triangles of which the polygon is composed, which triangles being all equal, we have $\frac{R\,S\,n}{2}$ = it's area. Theo. 27.

If the sides be infinitely small so as to lie in the periphery of a circle, then R will be the radius of that circle, and S being infinitely small may be rejected, then will n = the periphery of the said circle, whence the last theo. will become half R n = a circles area. Theo. 28.

If inſtead of taking n equal the whole periphery, we take it = any part thereof, we'll have half R n = area of a ſector, whoſe arch is n, and radius R. Theo. 29.

171. From theo. 21, it is evident that any parallelogram G H m Q is divided into two equal parts or triangles H G m and Q G m, by a diagonal m G, as alſo are the ▭s S G T R and L R y m, by the diagonals G R and R m, whence, it is manifeſt, that from the ▭ G H m Q, you take △s L m R = L m y, there will leave ▭ S G Q y = ▭ G T L H, from each of which, take away the = △s G T R and G S R, and there leaves ▭ T R y Q = ▭ S R L H, &c. Theo. 30. Fig. 133.

If I B be parallel to A D, (fig. 133) and I C = B D = p, be ⊥ to them, and a = A C, e = C D, then half p a = △ A I C, and half p : × : a + e : = △ A B D, the former taken from the latter leaves $\frac{1}{2}$ p e = △ A B D – △ A B C, which ſhews that triangle A B C is = half a p = △ A I C, conſequently, all triangles (and alſo ▭s, they being the halves of △s) which ſtand on the ſame baſe or ſide and have the ſame height, are equal. Theo. 31.

From this laſt method of reaſoning it appears, that there is no difference whether a and e, be taken as lines or as areas, whence it follows that all priſms and cylinders, or all pyramids and cones, which ſtand upon the ſame or equal baſe A C, and between the ſame parallel planes I B and A D, are equal, viz. ſolid A I C = ſolid A B C (fig. 133) Theo. 32.

172. If B and b, be the baſes, P and p the perpendiculars, A and a the areas of two triangles, then by theo. 24, half P B = A and half p b = a, ſo it will be as P B : p b : : A : a, (for the $\frac{1}{2}$ being conſtant may be rejected) See prob. 173. If A = a, then P B = p b, whence as B : b : : p : P, if P = p, then $\frac{A}{B} = \frac{a}{b}$, or A b = a B, whence as A : B : : a : b. If B = b, then A p = a P, whence as A : a : : P : p, i. e. all △s, or all ▭s, are in proportion as the products of their baſes and heights, and if the areas are equal, the baſes and heights, are inverſely proportional, but if the baſes or heights are equal, then the areas are as the heights or baſes. Theo. 33.

The very ſame things hold in priſms or cylinders, cones or pyramids, by looking upon B and b, as the areas of their baſes. Theo. 34.

If theſe figures are ſimilar, viz. their ſides &c. proportional, thus, if it be as B : P : : b : $\frac{Pb}{B}$ = p, then a = p b = $\frac{Pbb}{B}$, ſo as B P : A : : $\frac{Pbb}{B}$: a. Ergo, a B P = $\frac{APbb}{B}$, that's a B B P = A P b b, or a B B

$= A\,b\,b$, whence, as $A : a :: BB : b\,b$, and becauſe all planes are made up of triangles it follows from this proceſs, that all plane figures are as the ſquares of their like ſides. Theo. 36.

If D, E, F, and d, e, f, be the dimenſions of two ſolids, and if it be as $a : b :: d : \frac{bd}{a} = D$, and $:: e : \frac{be}{a} = E$, and $:: f : \frac{bf}{a} = F$, or putting $r = \frac{b}{a}$, we'll have $rd = D$, $re = E$, and $rf = F$, then theſe two ſolids are ſimilar becauſe their dimenſions are proportional, now if $pdef =$ the one ſolid, $pdefrrr = pDEF$ muſt by the ſame rule, be equal the other ſolid and therefore as $pdef : pdefrrr :: ddd : \frac{pdefrrrddd}{pdef} = rrrddd = DDD$, &c. for E and F, whence all like ſolids, are in the triplicate ratio of their homologous, or like ſides, i. e. as any ſolid is to the cube of any part of it ſo is any other like ſolid, to the cube of it's like part, or ſide. Theo. 37.

PROBLEM CLXXIX.

Of ſines tangents &c. with the axioms &c. in plane trigonometry.

173 If there be three arches A F, A N, A P, (fig. 134) in arithmetical progreſſion, viz. if $AF = A$, then $AN = 2A$, and $AP = 3A$ draw the lines as you ſee in the figure, then the $\triangle$s z C N and C O B, being ſimilar it will be, as $CN : CO :: Nz : \frac{CO \times Nz}{CN} = OB$ and $:: Cz : \frac{Cz \times CO}{CN} = CB$, but ſince $FN = NP$, F O will be $=$ O P, whence it's plain O B will be an arithmetical mean between P G and F H, and is therefore $= \frac{PG + FH}{2}$ (half their ſum) and for the ſame reaſon $CB = \frac{CG + CH}{2}$, conſequently, $\frac{PG + FH}{2} = OB = \frac{CO \times Nz}{CN}$ and $\frac{CG + CH}{2} = \frac{Cz \times CO}{CN}$, whence, $PG = \frac{Nz \times 2\,OC}{CN} - FH$ and $CG = \frac{Cz \times 2\,OC}{CN} - CH$, and if radius C N be taken equal unity, then $PG = Nz \times 2\,OC - FH$, and $CG = Cz \times 2\,OC - CH$, i. e. If z N the ſine of the mean arch A N, be multiplied by 2 O C, twice the co-ſine of the (arch $NF = NP$) common difference, and from that product, the ſine F H of either ex-

treme (arch as A H) be taken, there will leave P G, the ſine of the other extreme arch A P. Theo. 38.

Alſo, If C z the co-ſine of the mean A N, be multiplied by twice the co-ſine of the common difference, and C H the co-ſine of either extreme A F, be taken from that product, there will leave C G, the co-ſine of A P the other extreme. Theo. 39,

Now if e denote the ſine of any arch A, and hrlf y it's co-ſine, we may find the ſine and co-ſine of n A, or n times that arch, thus let A be conſidered as an arithmetical mean between o and 2 A. Then by

$$\text{Theo. 38 we have}\begin{cases}\text{ſine } 2A = ey = \text{ſine } A \times y - \text{ſine } o. \text{ Theo. 40. radius} = 1.\\ \text{ſine } 3A = eyy - e = \text{ſine } 2A \times y - S\,A\\ \text{ſine } 4A = eyyy - 2ey = \text{ſine } 3A \times y - S:2A\\ \text{ſine } 5A = eyyyy - 3eyy + e = \text{ſine } 4A \times y - S:3A,\end{cases}$$

From whence it appears that the ſine of n A will be $= e \times : y^{n-1} - \frac{n-2}{1} y^{n-3} + \frac{n-3}{1} \times \frac{n-4}{2} y^{n-5} -$ &c.

In like manner by theo. 39, we have co-ſine $2A = \frac{yy}{2} - 1 = \frac{yy+2}{2} =$ co-ſine $A \times y -$ co-ſine o, viz. $A \times y - 1$ (for $1 =$ radius $=$ co-ſine of o,) co-ſine $3A = \frac{yyy - 3y}{2} =$ co-ſine $2A \times y -$ co-ſine A, and ſo on as before, we'll get co-ſine $nA = \frac{y^n}{2} - \frac{ny^{n-2}}{2} + \frac{n}{2} \times \frac{n-3}{2} y^{n-4} - \frac{n}{2} \times \frac{n-4}{2} \times \frac{n-5}{3} y^{n-6} +$ &c. Theorem 41. which ſeries muſt be continued till the indices of y become $= o$, if n A be given ſuppoſe $= S$, then by ſolving the laſt equation, we'll have half $y =$ the co-ſine of $\frac{1}{n}$ and by the former equation you may alſo find the ſine of $\frac{1}{n}$, this is evident, for half y being the co-ſine of A, the ſeries expreſſes the co-ſine of n A, &c. Alſo, becauſe the ſine of any arch is half the chord of double that arch, it follows that if 2 e be written for e, we may find the ſide of any regular polygon inſcribed in a circle, or if we double the ſide when found by the ſaid ſeries, of the multiple or ſub-multiple arches n A or $\frac{1}{n}$, it will be the ſide of a regular polygon, or twice ſine of the arch, &c. be-

cause by the nature of the circle, $ee + \frac{1}{4}yy = 1$ (viz. square radius) therefore, if n be equal any odd whole positive number, we may have the sine of n A in terms of e only, (because the sine of 3 A, 5 A, &c. turns out in even powers of y) without surds, thus let $4 - 4ee$, be put in these terms instead of yy it's equal, so we'll have sine 3 A $=$ $3e - 4eee$, sine 5 A $= 5e - 20e^3 + 16e^5$ sine 7 A $= 7e - 56e^3 + 112e^5 + 64e^7$ &c. whence sine n A $= ne - \frac{n}{1} \times \frac{n^2 - 1}{2,3} e^3 + \frac{n}{1} \times \frac{n^2 - 1}{2,3} \times \frac{n^2 - 9}{4,5} e^5 - \frac{n}{1} \times \frac{n^2 - 1}{2,3} \times \frac{n^2 - 9}{4,5} \times \frac{n^2 - 25}{6,7} e^7$ + &c. now if the arch A be taken very small it will be equal e, it's sine very nearly, then by writing A instead of e, we'll have sine n A $= nA - \frac{n}{1} \times \frac{n^2 - 1}{2,3} A^3 + \frac{n}{1} \times \frac{n^2 - 1}{2,3} \times \frac{n^2 - 9}{4,5} A^5 - \frac{n}{1} \times \frac{n^2 - 1}{2,3} \times \frac{n^2 - 9}{4,5} \times \frac{n^2 - 25}{6,7} A^7$ + &c. now if n be supposed indefinitely great, then n A will be equal some asignable quantity suppose equal z, and the numbers 1, 9, 25, &c. taken from n may be rejected as small in respect of n, and then the last series will become $z - \frac{z^3}{2,3} + \frac{z^5}{2,3,4,5} - \frac{z^7}{2,3,4,5,6,7}$ + &c. (the same series with that given in art. 136) which series gives the sine of any arch z. Theo. 42.

In like manner you'll get $1 - \frac{z^2}{2} + \frac{z^4}{2,3,4} - \frac{z^6}{2,3,4,5,6} + \frac{z^8}{2,3,4,5,6,7,8}$ — &c. for the co-sine of any arch z. Theo. 43.

174. (In fig. 134) The triangles C N z, C O B and P N v are alike, so, as C N : C O : : N z : $\frac{CO \times Nz}{CN} =$ O B, and as C N : P O : : C z : $\frac{Cz \times PO}{CN} =$ P v, then P G $=$ (O B + P v) $\frac{CO \times Nz + Cz \times PO}{CN}$ and F H $=$ (O B — v I or — it's equal P v) $\frac{CO \times Nz - Cz \times PO}{CN}$ that is, the sine (P G) of the sum (P A) of any two arches (A N and N P) is equal sum of the products of the sine of the one into the co-sine of the other (radius $= 1$). Theo. 44.

And the sine (F H) of their difference (F A) is $=$ the difference of the same products (radius C N being $= 1$). Theo. 45.

175. To find the sine of any arch, suppose of 5°. First as 10800 the minutes in 180° is to 3,14159265358 &c. (see article 136) so is 300 the minutes in 5° to ,08726646, for z, or the length of the given arch, radius being equal 1, then for the sine, (by theo. 42) for the co-sine (by theo. 43.)

$$z = +\,,08726646 \qquad +1 = +1,$$

$$-\frac{z^3}{6} = -\,,00011076 \qquad -\frac{z^3}{2} = -\,,00380771$$

$$+\frac{z^5}{120} = +\,,00000004 \qquad +\frac{z^4}{24} = +\,,00000241$$

so sine 5° = ,08715574, and it's co-sine = ,9961947

Thus may the sine and co-sine of any arch be found, but greater the arch is, the slower the series will converge, and therefore a greater number of terms must be taken, but having thus found a few sines, you may by the foregoing theorems, soon find as many as you please, and so construct new tables of sines, or prove old ones, and from a table of logs. you may have the log. sines of these natural sines so found. Thus, to find the log. sine and log. co-sine of 5°, first, because the natural numbers in the common log. tables are but to 4 places of figures, take the log. of 8715 = 940296, and of 9962 (near = 9961947 the co-sine of 5°) = 998344, then because the co-sine has one place of figures more in it than the sine has, (in this case) and the log. radius of these tables is set at 10,000000 = log. sine of 90° = log. tangent of 45°, the indices of these logs. must be 8 and 9, so 8,940296 is the log. sine of 5° and 9,998344 = it's log. co-sine.

176. Let r = radius, s = sine, c = co-sine, t = tangent, z = co-tangent, n = secant, e = co-secant, v = versed sine, then by theo. 9, and 13, any two of these letters being given, any of the next may be found, as appears by the right angled similar triangle in figure 44. prob. 54.

$$1.\ S = \sqrt{rr - cc} = \frac{A}{r} = \frac{tr}{n} = \frac{tr}{\sqrt{rr+tt}} = \frac{rc}{z} = \frac{rr}{\sqrt{rr+zz}} = \frac{rr}{e}$$

$$= \frac{cn}{e} = \frac{tz}{e} = \frac{r}{n}\sqrt{nn - rr} = \overline{2rv - vv}\big|^{\frac{1}{2}} = \sqrt{v}\,\big|^{\frac{1}{2}},$$ V being = versed-sine supplement.

$$2.\ c = r - v = V - r = \sqrt{rr - ss} = \frac{rs}{t} = \frac{rr}{\sqrt{rr+tt}} = \frac{rr}{n} = \frac{tz}{n}$$

$$= \frac{es}{n} = \frac{sz}{r} = \frac{rz}{e} = \frac{rz}{\sqrt{rr+zz}} = \frac{r}{e}\sqrt{ee - rr},$$ and so on for any of

the reſt. Whence

177. It follows, that if log. radius $= r = 10$, $s =$ log. ſine, $c =$ log. co-ſine, $t =$ log. tangent, $z =$ log. co-tangent, $n =$ log ſecant, and $e =$ log. co-ſecant, then by the nature of logs. we'll have

$s = c + t - r = t + r - n, = c + r - z = 2r - e$, and $t = s + r - c$ $(= 2r - z$,

$c = s + r - t = 2r - n = s + z - r = z + r - e$, alſo, $z = c +$ $(r - s = 2r - t$,

$n = t + r - s = 3r - s - z = e + r - z = 2r - c$, and $e = z + r$ $(- c = 3r - c - t = n + r - t = 2r + s$,

ſo that if all the log. ſines or log. co-ſines &c. be found firſt, all the reſt are eaſily found by them.

178. By the ſimilar Δs (fig. 134) as 2 O B, or P G + F H : 2 O D, or P I : : O B : O D : : O L : O F : N M : N K, that is, as the ſum of the extream arches is to their difference (ſine greater — ſine leſſer) ſo is tangent mean arch to tangent common difference of the arches, and becauſe the mean arch $= \frac{1}{2}$ ſum of the extreams, therefore, in any two arches (A F and A P) as ſum of their ſines is to their difference (s greater — s leſſer) : : tangent $\frac{1}{2}$ their ſum to tangent half their difference. Theo. 46.

179. In the right angled Δ A B F (fig. 135) H I is the tangent a e the ſine and e B = I B the radius of the arch or $\angle$ A B F (by prob. 54) then per ſimilar Δs, B I H and B F A, as B I (rad.) : I H (tangent $\angle$ A B F) : : B F : A F. Theo. 47.

Alſo, as e B (rad.) : B A : : e a (ſine $\angle$ A B F) : F B, whence in the Δ B A G, (divided into the two right angled ones A B F and A G F by the perpendicular A F) it will be as R : A G : : ſine $\angle$ A G F : $\dfrac{AG \times S\angle AGF}{R = \text{radius}} = AF$, and by the laſt proportion, as R : : A B : : S$\angle$ A B F : $\dfrac{AB \times S\angle ABF}{R} = AF$, as before, whence, $AB \times S\angle ABF = AG \times S\angle AGF$, in like manner, by drawing G K $\perp$ B A produced, it will be as R : A G : : S$\angle$ G A K : $\dfrac{AG \times S\angle GAK}{R} = GK$, and alſo, as R : G B : : S$\angle$ A B G : $\dfrac{GB \times S\angle ABG}{R} =$ the ſame G K, whence $AG \times S\angle GAK$ (= comp. $\angle$ G A B to 180°) $= GB \times S\angle ABG$. Theo. 48.

Let a = A B, c = A G, u = B G and e = G F, then by theo 13, c c — e e = □ A F a a — u u + 2 u e — e e = (□ A B — □ B F) □ A F, therefore c c — e e = a a — u u + 2 e u — e e, that is, c c = 2 e u — u u + a a, or a a — c c = u u — 2 e u, which turned to an analogy gives as u : a + c : : a — c : u — 2 e. Theo. 49.

In any triangle A B C (fig. 135) make B D = B A, and join A D, which bisect in F with the line B G, draw F E, parallel to A C, then will A F = F D, D E = E C, the ∠ B A F = $\frac{BAD + BDA}{2}$ = $\frac{BAC + ACB}{2}$ and ∠ G A F = B A C — B A F = $\frac{BAC - ACB}{2}$) also A B + A C = 2 E B and B C — A B = 2 D E, then the △s B E F and B G C being similar it will be, as B E : E C : : B F : F G, but (by theo. 47) as A F : R : : B F ; tangent B A F : : F G : tangent F A G, or as B F : F G : : tangent B A F : tangent F A G, consequently, as B E : E C (D E) : : tangent B A F : tangent F A G. Otherwise, As B C : B A : : sine ∠ A : sine ∠ C (by theo. 48) and as B C + B A : B C — B A : : S ∠ A + S ∠ C : S ∠ A — S ∠ C : : (by theo. 46, tangent $\frac{A + C}{2}$: tangent $\frac{A - C}{2}$. Theo. 50.

These 4 last theorems demonstrate the 4 axioms in plane trigonometry.

180. Given, I a = e, e B = I B = r, the radius, and e the versed-sine of an arch e I (fig. 135) = z, to find the length thereof. Suppose the given versed sine I a, to be divided into an infinite number of = parts and let a q be one of these parts, which put = 1, draw v q parallel to e a, which because q a, is very small, the small arch v e may be looked upon as a streight line, and then the △s B a e and v e n are similar, (v n being = and || q a) so as e n$(\overline{2re - ee}|^{\frac{1}{2}})$: e B (r) : : v n (1) : $\frac{r}{\sqrt{:2re - ee:}}$ = v e, now because there are as many such parts v e in the arch e I, as there are units in I a, = e, its plain that ve × e = z = $\frac{re}{\sqrt{:2re - ee:}}$, then by comparing, this equation with ex. 77, that is putting it into a series, we'll have z = $\frac{2re}{2\sqrt{2re}} + \frac{2re,ee}{2,2\sqrt{re,2re}} + \frac{2,3re,ee}{2,82re|^{\frac{5}{2}}}$ &c. = $\sqrt{2r} \times : \frac{e^{\frac{1}{2}}}{2} + \frac{2e^{\frac{3}{2}}}{2,4r} + \frac{2,3e^{\frac{5}{2}}}{2,8,4rr}$ + &c. which by dividing each term in the series by the index of e in that term (see

rule to prob. 185,) gives the true value of $z = \sqrt{2re} \times : 1 + \frac{2}{2^2, 3r} + \frac{3ee}{2^3, 4, 5rr} + \frac{3, 5eee}{2^4, 4, 6, 7r^3}$ + &c. or, $z = \sqrt{de} \times : 1 + \frac{e}{2, 3d} + \frac{3ee}{2^2, 4, 5dd}$ + &c. by putting $d = 2r$. Theo. 51.

Because e the versed sine in arches under 40° or 35° is small in respect of the arch, or of its radius &c. its plain this series in such a case will converge very fast, and that the two first terms thereof will give the length of the arch near enough for most uses, that is $z = \sqrt{2re} + \frac{e\sqrt{2re}}{12r} = \frac{\sqrt{2re} : \times 12r + e :}{12r}$, nearly, or $z = \frac{\sqrt{2re}}{12r} \times : 12r + e :$

PROBLEM CLXXX.

Given any conic section &c. to find its property. Fig. 136.

Definition. If a cone U E F (formed as in prob. 155) be cut by a plane passing thro' the vertex u and center of the base dividing the cone into two equal parts of sections, the plane of each section will be an isosceles triangle. (182) If a cone be any where cut by a plane C P parallel to E F its base, the plane of that section will be a circle.

183. If a cone (fig. 136) be cut by a plane T S passing from T the extremity of the base to u F the opposite side, the plane of this section will be an ellipsis, whose transverse axis or diameter is T S, being the longest diameter, and the shortest diameter B A B cutting T S at right angles in A its middle, is called the conjugate diameter, any part S a or T a of the axis of any conick section, is called an abscissa, and the line b a b, which cuts off that abscissa is called an ordinate, which being at right angles to each other, the ordinate is said to be rightly applied. (184) If the cutting plane S A (fig. 137) pass any how so as A S produced will meet the other side E u of the cone produced as in T, the plane of this section is an hyperbola, whose transverse axis is T S, B A B and b a b, ordinates to the abscissa's S A and S a. (185) If the cutting plane S A (fig. 138) be parallel to u E, a side of the cone, then the plane of this section is a parabola whose axis is S A, b a b an ordinate to the abscissa S a, (186) of these five sections, viz. triangle, circle, ellipsis, parabola, and hyperbola, the 3 last are only called conick sections because the properties of the triangle and circle, may be known without help of the cone. Thus, in the triangle C S M (fig. 134) R L being parallel to S M, we have per similar Δs, as CN : SM

$:: CO : \frac{SM}{CN} \times CO = RL$, whence, if we put $p = \frac{SM}{CN}$, and $e =$ any abſciſſa C O, then, $e\,p = RL$, which equation is called, the nature, property, or equation of the figure, (in this caſe of a plane triangle.) Theo. 52.

Again, let $a = CH$ (fig. 44) the diameter of any circle, $z = AB$, $e = BC$ any abſciſſa, to the $\frac{1}{2}$ chord, or ſemi-ordinate $BG = y$, then by theorem 10, we will have, $\overline{a-e} \times e = y \times y$, viz. $ae - ee = yy$, which is an equation of the circle. Alſo, becauſe by the figure, $\frac{1}{2}a - e = z$, therefore, $\frac{1}{2}a - z = e$, and ſo, $\frac{1}{4}aa - zz = yy$, (that is, $\square AG - \square AB = \square BG$), a ſecond equation. Theo. 53.

187. It is eaſy to underſtand, that the ordinates B A B and b a b, are chords of circles of the cone whoſe diameters are E F and C D, therefore, by

Theo. 53	1	$\square BA = AF \times AE$, and $\square ba = aD \times aC$.
By theo. 9	2	$Sa : aD :: SA : AF$ } for the $\triangle$s S a D and S A F
	3	$Ta : aC :: TA : AE$ } are ſimilar as alſo are the $\triangle$s T A C and T A E.
2×3	4	$Sa \times Ta : aD \times aC :: SA \times TA : AF \times AE$.
1 and 4	5	$Sa \times Ta : \square ba :: SA \times TA : \square BA$, in fig. 136, 137.
but	6	$Sa : \square ba :: SA : \square BA$, in (fig. 138) where S A

is parallel to u E, for, then $Ca = EA$, and ſo in ſtep 3, T a becomes $= TA$. Whence, if we put $2a =$ the tranſverſe, and $2c =$ the conjugate diameters of the ellipſis, and hyperbola, but in the parabola, (fig. 138) $a = SA$, the axis, and $2c = BAB$ the greateſt ordinate, then, in any of theſe three ſections, if $E = Sa$ any abſciſſa, and $2y = bab$, an ordinate to that abſciſſa, we by ſtep 5th have, $aa : cc :: \overline{2a-e} \times e : \frac{cc}{aa} \times : 2ae - ee : = yy$. Theo. 54, for the ellipſis,

alſo, $aa : cc :: \overline{2a+e} \times e : \frac{cc}{aa} \times : 2ae + ee = yy$, for the hyperbola. Theo. 55, and by ſtep 6, $a : cc :: e : \frac{ecc}{a} = yy$ for the parabola. Theo. 56.

Theſe are the chief properties of the three conick ſections, from which other equations relating to them are deduced, if we take $z = Aa$ (fig. 136) $= a - e$, then we'll have $\frac{cc}{aa} \times : aa - zz : = yy$ for another equation of the ellipſis, and when the ellipſis becomes a circle then $a = c$, and then theſe equations are the ſame with thoſe of the

circle, which ſhews that the difference between the equations of the circle, and ellipſis is only the factor $\frac{cc}{aa}$, if T S, be taken for the conjugate diameter of the ellipſis, and B B, for the tranſverſe, the method of demonſtration will be the very ſame as before, which ſhews that the property of the ellipſis is the ſame in reſpect of each diameter and its ordinate. Theo. 57.

PROBLEM CLXXXI.

Given the property of a conick ſection, to find its latus, rectum, or parameter, and its focus.

188. The letters a, e, y, z, repreſenting the ſame things as in the laſt prob. if we take $\frac{cc}{a} = p$ in the parabola, or $\frac{4cc}{2a} = p$ in the ellipſis and hyperbola, then this p will be a conſtant factor, to any variable abſciſſa e, by which its ordinate is found, ſo this p being made as an ordinate is called the latus rectum, or right-parameter, and that point in the axis, or diameter of the ſection thro' which it paſſes, is called the focus, node, or burning point of the ſection, of which there are one in the parabola, one in the hyperbola, being the point K, in fig. 28, where H is the focus of the oppoſite hyperbola, which being taken in with K, in the conſtruction, the hyperbola is ſaid to have two focuſes, in the ellipſis are two focuſes, which are the points B and C in fig. 26. Now by theo. 54, 55, and 56, we have

$$2a : p :: \overline{2a-e} \times e : \frac{p\times : 2ae - ee}{2a} = \frac{p}{2a} \times : 2ae - ee : = yy, \text{ for the ellipſis. Theo. 58.}$$

$$2a : p :: \overline{2a+e} \times e : \frac{p}{2a} \times \overline{2ae + ee} = yy, \text{ for the hyperbola.}$$

Theo. 59.

$p : y :: y : e$, ergo $pe = yy$, or univerſally $pe^{n} = y$, for the parabola. Theo. 60.

189. Let the ordinate b a b (fig. 138) be the parameter of the parabola, given to find S a (e) the diſtance of a, the focus from S, the vertex of the parabola, here by theo. 60, $pe = yy = \frac{1}{4}pp$, (becauſe per laſt art. $\frac{cc}{a} = p$) hence, $e = \frac{1}{4}p$, again, for the ellipſis and hyperbola, it appears by the equations of theſe two curves, that the difference is only in the ſigns —, and +, viz. —, in the ellipſis and +,

in the hyperbola, whence $\mp$, being in the equation will serve both curves, so putting $z = 2a \mp e =$ the distance between any $\frac{1}{2}$ ordinate y, and the center of the figure we shall (by theo. 54, and 55) $\frac{p}{aa} \times : aa \mp zz : = yy$, for another equation of these two curves, now if 2y be the parameter, or $y = \frac{1}{2}p$, then, $\frac{p}{aa} \times : aa \mp zz : = yy = \frac{1}{4}pp$, hence $aa \mp zz = \frac{1}{2}pa$, and so, $z = \sqrt{:aa \mp \frac{1}{2}pa:}$ and writing $\frac{4cc}{2a}$ instead of its $= p$, we'll have $z = \sqrt{aa - cc}$ in the ellipsis. Theo. 61, and $z = \sqrt{:aa + cc:}$ in the hyperbola. Theo. 62, which shews that $PC + PB = TS$ (in fig. 26) &c. for the parabola and hyperbola. Fig. 27, and 28.

PROBLEM CLXXXII.

Shewing some other useful properties of conick sections.

190. By the similar $\triangle$s TAE and TSm, as also, SAF and STn (fig. 136) it is as $TS : Sm :: \frac{1}{2} TS\ (TA) : \frac{Sm \times TS}{2\,TS} = \frac{Sm}{2} = AE$, and, as $ST : Tn :: \frac{1}{2} ST\ (SA) : \frac{Tn \times ST}{2\,ST} = \frac{Tn}{2} = AF$, but by theo. 53, $AE \times AF = \square BA = \frac{Sm}{2} \times \frac{Tn}{2}$, or $4 \square BA = Sm \times Tn$, that is the conjugate diameter of any ellipsis, is a mean proportional between the diameters of the cone at each end of the transverse. Theo. 63.

191. Because casks are supposed to resemble these conick sections, now to find the true form of a cask &c. let $m = mD$ (fig. 128) a diameter taken in the middle between the head Hd and bung BD, $k = Hd$, the head, $b = BD$ the bung diameters, $a = \frac{1}{2} EL$, half the casks length, $c = vE$.

Then by the property of the parabola	1	$ce = hh$, $c \times : e + \frac{1}{2}a : = mm$, $c \times : e + a : bb$, c being = parameter.
	2	$aa : b - h :: \frac{1}{2} aa : \frac{b-h}{4} = \frac{1}{2} mD - \frac{1}{2} Hd.$
by proper ellip.	3	$\frac{aabb}{bb-hh} = tt$, and $\frac{\frac{1}{4}aabb}{bb-mm} = tt$, t being = half transverſe.
from 1ſt ſtep	4	$mm = \frac{bb+hh}{2}$. Theo. 64.
per 2	5	$m = \frac{3b+h}{4}$. Theo. 65.
per 3	6	$mm = \frac{3bb+hh}{4}$. Theo. 66.
alſo by writing z in the 3 firſt ſteps for $\frac{1}{2}$ a, they become.	7	$mm = : hh + \frac{bb-hh}{a} : \times z$. Theo. 67.
	8	$m = : b - \frac{b-h}{aa} : \times zz$. Theo. 68.
	9	$mm = : bb - \frac{bb-hh}{aa} : \times zz$. Theo. 69.

By the three firſt of theſe theorems, you may find a diameter in the middle between the bung by calculation, which compared, with one taken by your gauge rod &c. will ſhew what form the caſk is neareſt, or becauſe diameters are as their peripheries, you may uſe the girths inſtead of the diameters, the 3 laſt of theſe theorems are for ullaging ſtanding caſks.

Note. The firſt of the above ſteps, is for the fruſtum of a parabolic conoid. the ſecond ſtep for that of a parabolic ſpindle, and the third for the zone of a ſpheroid.

192. If a ſolid U E F (fig. 139) formed by the revolution of the ſemi curve U E P, about the axis u P, be cut by a plane S A parallel to the ſaid axis U P, its manifeſt, that B F B and B E B, the baſes of the two parts ſo cut, are ſegments of the circle of the baſe of the whole ſolid U E F, whoſe diameter E F, call 2 a, chord B A B = 2 b, verſedſine E A = e, then by theo. 53, we'll have $2ae - ee = bb$, then by the nature of the generating curve U E P, we may find the equation of the ſection B S B, made by the ſaid cutting plane S A. Thus, let U E P be a half parabola, whoſe parameter, let be = q, it's axis u P = c, the radius of its baſe E P = F P = a, the height S A of the cutting plane = h, and S I = A P, its diſtance from the axis (u P) = z.

Then by theo. 60, q c = a a, and q × : c — h : = z z, whose difference is q h = a a — z z = □ B A by theo. 53, hence the section made by this cutting plane, is a parabola, having the same parameter with the generating parabola. Theo. 70.

Again, if E u F, be a semi-spheroid, whose axis of revolution is u P = c and E F = 2 a the other diameter of the generating ellipsis, E A = e, then by the property of the ellipsis as $\frac{cc}{aa}$ × : 2a e — e e: ($\frac{cc}{aa}$ × □ B A) : 2 ae — e e, or lower, as $\frac{cc}{aa}$: 1, or as c c : a a : : □ S A : □ B A, whence, as c : a : : S A : B A, hence, S B B the plane of the section cut off, is an ellipsis similar to the generating one U E F. Theo. 71.

193. If from the vertex U of a cone (fig. 141) a perpendicular U Q let fall upon S A (produc'd) the axis of the conick section, this perpendicular, as also U p, the axis of the cone may be found by having given the dimensions of the frustum, M E F S, thus, make S I = S m, thro' I draw I G and I H parallel to E F and E U, then draw I T, H P and S R, ⊥s to F U, I G and E F, and the right angled △s H P G and I T G will be similar, having the common ∠ at G, as also will I T S and U S Q, because of the equal ∠s at S, likewise, the △s H I G and U m S, are similar because H I is by construction parallel to U m.

by theo. 9.	1	G H : G I : : H P : I T,
and	2	G H : G I : : S U : S m = S I, by constr.
1 = 2	3	H P : I T : : S U : S I,
by theo. 9.	4	I T : I S : : U Q : U S,
by 3 and 4.	5	U Q : U S : : H P : U S, so, U Q = H P.
by theo. 9.	6	F R : S R : G P : $\frac{GP \times SR}{FR}$ = H P = U Q } Theo. 72.
as also	7	F R : S R : : F p : $\frac{Fp \times SR}{FR}$ = U P }

The 7 following problems contain the theory of mensurations, or the investigation of theorems, for measuring all kinds of planes and solids.

PROBLEM CLXXXIII.

Having given the 3 sides of any plane △, to find its area.

Let the side be denoted by the same letters, a, c, and u, as in theo. 49. where, we'll have $e = \frac{cc + aa - uu}{2a}$, which squared, gives

$ee = \frac{c^4 + a^4 + u^4 + 2a^2c^2 - 2c^2u^2 - 2a^2u^2}{4aa}$, which taken (by theorem 13) from cc, will leave the ſquare of the perpendicular of the $\triangle$, whoſe ſquare root (ſee theo. 24) multiplied by $\frac{1}{2}a$, half the baſe, gives the area of the $\triangle = \frac{1}{4}\surd : -c^4 - a^4 - u^4 + 2a^2c^2 + 2c^2u^2 + 2u^2a^2 : = \surd : \frac{-c+a+u}{2} \times \frac{-a+c+u}{2} \times \frac{a-u+c}{2} \times \frac{a+c+u}{2}$. Theorem 73.

194. It appears from fig. 58, that if lines be drawn from each angular point, L, m, n, to E the center of the inſcrib'd circle F G H, that the $\triangle$ L m n, will be divided into 3 $\triangle$s, E L m, E m n, and E n L, whoſe $\perp$s are each = the radius (r) of the ſaid inſcribed circle, and therefore by theo. 24, $\frac{1}{2}ra + \frac{1}{2}ru + \frac{1}{2}rc =$ the area of the $\triangle$ L m n, whence, if the radius of the inſcrib'd, and 3 ſides be given the area is eaſily found, or if the three ſides be given the ſaid radius by the laſt theorem is eaſily had, for $\frac{1}{2}r \times : a + u + c : = \surd : \frac{-c+a+u}{2} \times \frac{c-u+a}{2} \times \frac{c+u-a}{2} \times \frac{a+c+u}{2}$, which by diviſion gives $\frac{1}{2}r = \surd : \frac{1}{16} \times \frac{-c+a+u \times c-a+u \times c+a-u}{a+c+u}$, Theorem 74.

PROBLEM CLXXXIV.

Given the 3 *angles* A, B, *and* C, *of any ſpherical triangle* A B C, *to find its area.* Fig. 142.

195. If the circle A R D, be moved equally round the globe, upon the two poles A and D, it will deſcribe equal ſpaces in equal times, and therefore, thoſe ſpaces muſt be as the angles at the ſaid poles, that is, putting D = the axis of the ſphere, and $p = 3{,}1416$, we'll have (by theo. —) p D D = the ſurface of the whole ſphere, then, as 360° : p D D : : $\angle$ A, or $\angle$ D : ſpace A Q D, &c. put G = area $\triangle$ A B C. Alſo, = $\triangle$ D E F = H (becauſe the ſides and $\angle$s in the one $\triangle$ are reſpectively = thoſe in the other $\triangle$) R = ſpace C B Q D, T = C E D, and S = A C E P.

Then by what is said before	1	$360 : pDD :: A : G + R$
	2	$360 : pDD :: B : G + S$
	3	$36 : pDD :: C : G + T = H + T$
$1 + 2 + 3$	4	$3 \times 360 : 3 pDD :: A + B + C : 3G + R + S + T$
$4 \because$	5	$3 \times 360 \times : 3G + R + S + T : = 3pDD \times : A + B + C;$
$5 \div 3$	6	$360 \times : 3G + R + S + T = pDD \times : A + B + C:$
per figure	7	$R + S + T = \frac{1}{2} pDD - G$
6 and 7	8	$360 \times : \frac{1}{2} pDD + 2G : = pDD \times : A + B + C$
$8 - 180 pDD$	9	$720 G = pDD \times : A + B + C - 180 :$
$9 \div 720$	0	$G = : \frac{A + B + C - 180 : \times pDD}{720}$. Theo. 75.

196. Becauſe all polygons are made up of triangles, let $S =$ ſum and $n =$ the number of angles of any ſpherical polygon, of what ordinate, or inordinate figure ſoever it be, we'll have $\frac{pDD}{720} \times : 360 + S - 180n$, for its area. Theorem 76.

PROBLEM CLXXXV.

Given $ae + \frac{1}{2} bee + \frac{1}{3} ceee + \frac{1}{4} de^4 + \&c. = A$, *the area of any curve-lined ſpace* TGC (fig. 31) e *being = the abſciſſa* TC, a, b, c, d, &c. *being known, or unknown coefficients, to find* y *the ordinate* GC *to the ſaid abſciſſa, and from the equation of the curve thus found, to determine its area.*

197. Let e z, be drawn very near, and parallel to GC, ſo may the ſmall part e G of the curve be taken as a ſtreight line, or the ſpace GCze, as a Trapezia, with two parallel ſides e z and GC, whoſe half ſum will be $= qd$, an ordinate in the middle between GC and e z, let z, = the abſciſſa Tz, then by the above given area, $az + \frac{1}{2} bzz + \frac{1}{3} cz^3 + \frac{1}{4} dz^4 + \&c.$ will be the area of the ſpace, Tez, then, its plain that if from the ſpace TGC, be taken the ſpace Tez, there leaves the trapezia GCze, which divided by Cz (e—z) gives (by theo. 26) the ordinate d q (q), thus

From $ae + \frac{1}{2}be^2 + \frac{1}{3}ce^3 + \frac{1}{4}de^4 + \&c. =$ ſpace TGC,
take $az + \frac{1}{2}bz^2 + \frac{1}{3}cz^3 + \frac{1}{4}dz^4 + \&c. =$ ſpace Tez,

$e - z)\ ae - az + \frac{1}{2}be^2 - \frac{1}{2}bz^2 + \frac{1}{3}ce^3 - \frac{1}{3}cz^3 + \frac{1}{4}de^4 - \frac{1}{4}dz^4\ (a + \frac{1}{2}be +$
$ae - az$ $\qquad (\frac{1}{2}bz + \frac{1}{3}cee + \&c. = q.$

$\frac{1}{2}be^2 - \frac{1}{2}bz^2$
$\frac{1}{2}be^2 - \frac{1}{2}bez$

$\frac{1}{2}bez - \frac{1}{2}bzz$
$\frac{1}{2}bez - \frac{1}{2}bzz$

The required ordinate or equation of the curve, but when z becomes $= e$, then q becomes $= y$, and this equation or quotient becomes, $a + b e + c e e + d e^3 + \&c. = y$, for the equation of that curve whose area is $a e + \frac{1}{2} b e e + \frac{1}{3} c e^3 + \frac{1}{4} d e^4 + \&c. = A$, viz. if the equation be $a = y$, the area will be $a e$, if the equation, be $a + b e = y$ the area will be $a e, + \frac{1}{2} b e e$ if the equation be $b e = y$, the area will be $\frac{1}{2} b e e$, and universally, if $p e^n = y$, the area will be $\frac{p e^{n+1}}{n+1}$, this last equation is (by theo. 60) that of a parabola, and if it be a common parabola, then $n = \frac{1}{2}$, in which case $\frac{p e^{n+1}}{n+1}$, becomes $\frac{2}{3} p e^{\frac{3}{2}} =$ (because $p e^{\frac{1}{2}} = y$) $\frac{2}{3} e y$, for the area of a common parabola. Theorem 77.

If $n = 1$, the (by theo. 52) $pe = y$ is the equation of a plane Δ, in this case $\frac{p e^{n+1}}{n+1}$ becomes $\frac{1}{2} p e e =$ (because $p e = y$) $\frac{1}{2} e y$, the same with theo. 24, from these things we may deduce the following general rule, viz.

198. Having the ordinate (y) clear on one side of the equation, if the other side be in surds or fractions &c. it must be reduced into a series, &c. Then multiply each term by the abscissa (e), which done divide each term by the index of e in that term, so have you the area or the sum of an infinite number of ordinates which compose the space, or figure, the reverse of this rule gives the curve's equation, this rule, is also fairly proved in art. 93 in prob. 172. Theo. 78.

199. In these theorems &c. I make no difference between the ordinate, semi-ordinate for (fig. 31) if you use the semi-ordinate, G C (y) with the abscissa T C (e) you'll have the semi-segment T C G, but if I G (y) be taken, you will have the whole segment I T G, for $G I = 2 C G$, &c. for others.

200. From the above rule, arises the method of fluxions and fluents for if e be a variable quantity, the fluent of $v e^n \dot{e}$ is $= \frac{v e^{n+1}}{n+1}$ &c.

201. But to apply the above rule to practice. Let z or e always denote an abscissa, to an ordinate y, A the area of the space, and S its solidity, then first let the area of that space or curve be required whose equation is $\frac{c c}{a a} \times : a a - z z : = y y$, or $\frac{c}{a} \sqrt{: a a - z z :} = y$,

which put into a ſeries (by art. 73, prob. 170) will be $y = \frac{c}{a} \times : a - \frac{zz}{2a} - \frac{z^4}{2,4a^3} - \frac{3z^6}{2,4,6a^5} - \frac{3,5z^8}{2,4,6,8a^7}$ — &c. each term in which multiplied by the abſciſſa z, gives $zy = \frac{c}{a} \times : za - \frac{z^3}{2a} - \frac{z^5}{8a^3} - \frac{3z^7}{48a^5}$ &c. and dividing each &c. as by the laſt theorem $A = \frac{c}{a} \times za - \frac{z^3}{6a} - \frac{z^5}{40a^3} - \frac{3z^7}{336a^5}$ &c. this ſeries will (by theo. 57) give the area of an elliptical ſpace intercepted between the diameter c and ordinate y. Theo. 79.

If z be taken $= a$, we'll have $A = ca \times : 1 - \frac{1}{6} - \frac{1}{40} - \frac{3}{336} - \frac{15}{3456}$ — &c. which ſeries continued will turn out $ca \times 0{,}78539$ &c. $= A$, for the area of a whole or ſemi-ellipſis (as you take c and a for the whole or half diameters &c.) Theo. 80.

If from 0,78539 &c. a c, you take the ſeries in theo. 79, its manifeſt there will leave the area of an elliptical ſegment, or by ordering $\frac{c}{a}\sqrt{2ae - ee} = y$, as you did $\frac{c}{a}\sqrt{aa - zz} = y$ (ſee theo. 54) you'll have a ſeries for the ſame ſegment, cut off parallel to the tranſverſe, when a is taken for the conjugate, but cut off parallel to the conjugate when a is taken for the tranſverſe.

202. If a be taken $= c$, the ellipſis becomes a circle, and then theſe theorems holds true for the like parts of a circle. Theo. 81.

If we take the diameter of a circle $= 1$, its periphery (by art. 136) be 3,1415926, &c. and (by theorem 28) half of 1 (0,5) multiplied by half of 3,1415926 (1,5707963) gives 0,78539815, for the area of a circle whoſe diameter is unity, or 0,7854 may ſerve as a conſtant factor, the very ſame will turn out by theo. 80, whence, $0{,}7854\,dd =$ area of any circle. Theo. 82.

Becauſe circles are like figures, therefore (by theo. 36) as the ſquare of 3,1416, the periphery is to 0,7854 its area ſo is $\square$ 1, viz. 1 to ,0795 &c. the area of a circle whoſe periphery is unity. Theo. 83.

It is plain (fig. 134) that if from the ſector C P N F, we take the $\triangle$ C P F, there will leave the ſegment P N F, that is, if $CP = r$, o N the verſed ſine $= e$, P o half chord $= C$, $Co = d = r - e$, and $a =$ $PN = FN$, the half arch, then $ra - dc$ (by theo. 29 and 24) $= A$, the ſaid ſegments area, but (by theo. 51) $a = \frac{\sqrt{2re}}{12r} \times : 12r + e :$

nearly, therefore $ra = \frac{\sqrt{2re}}{12} \times : 12r + e :$ the area of the said sector, from which take d c the Δ, and there leaves the segment $= \frac{\sqrt{2re}}{12} \times : 12r + e : - dc$, Theo. 84. nearly, or $= : \frac{160r - 39e}{32r - 3e}, \times \frac{4e}{15} \sqrt{2re}$, nearer, and if t and q, be the diameters of an ellipsis, we'll (by theo. 81) have $: \frac{160r - 39e}{32r - 3e} : \times \frac{4et}{15g} \sqrt{2re}$, for the area of an elliptical segment. Theo. 85.

203. Thus you may proceed to find the area of any space &c. if its equation be given, and having so found the areas, you may compare the figures or find what proportion one plane bears to another, as for instance, if a circle be circumscrib'd about an ellipsis and another circle inscrib'd therein, let $p = 0,7854$, a and c, the diameters of the ellipsis, which are also the diameters of the said circles, then paa and pcc, and pca, are by the foregoing theorems, the areas of these 3 figure, which are, as $paa : pcc, :: aa : cc$, also, as $paa : paa :: aa : ac$, likewise, $paa \times pcc = \square\, pca$, which shews that every ellipsis is a mean proportional, between its circumference and inscrib'd circle.

PROBLEM CLXXXVI.

To find the contents of solids, when the equations of their generating planes are given.

204. As a plane is composed of an infinite number of lines, or ordinates, so is a solid of an infinite number of planes or areas. whence it follows, that if one of these areas be multiplied by their number, the product (if the areas are equal) will be the content of the solid, therefore as an area, divided by its length gives a breadth (q) so a solid divided by its length gives an area (pqq), whence, if instead of taking $a + be + cee + de^3 + \&c. = y$, we take it $= yy$, and work as in the last problem, we'll find that when the equation of the generating curve is $ve^n = y$, or the equation of the solid generated thereby is $vve^{2n} = yy$, or $ce^m = yy$, the solidity will be $\frac{pce^{m+1}}{m+1}$, which gives this general rule, viz. if the ordinate (y) or radius of the base be clear on one side of the sign $=$, then square the whole equation, and multiply each term therein by the abscissa (e or z) then divide each term by the index of the said abscissa or axis in that term, which

multiplied by p gives the solidity, p being $= 3{,}1416$, if $y =$ radius of the base, but $p = 0{,}7854$, if $y =$ the diameter of the base (for $\square \frac{1}{2} = \frac{1}{4}$ and $3{,}1416 \div 4 = 0{,}7854$) Theo. 84. or writing yy for ce^m its equal we'll have $\frac{pyye}{m+1} = S$, which is a general theorem for all cones, pyramids and parabolic conoids, thus if $ce = yy$, then $\frac{pyye}{m+1} = \frac{1}{2} pyye = S$, for a common parabolic conoid. Theo. 87.

If $qe = y$ (the equation of a $\triangle$) or $cee = yy$, then $\frac{pyye}{m+1} = \frac{1}{3} pyye = S$, for the solidity of a cone (m in this case being $= 2$.) Theo. 88.

If y be the side of any regular polygon, and p such a number as that $pyy =$ the area of that polygon, $e =$ the axis of a pyramid whose base is the said polygon, then $\frac{1}{3} pyye = S$, its solidity. Theo. 89.

205. In the ellipsis (see theo. 54) $\frac{cc}{aa} \times : 2ae - ee : = yy$, which multiplied by ep, and each term divided by the index of e in that term, Theo. 86, gives $\frac{cc}{aa} \times paee - \frac{1}{3} peee = S$, for the solidity of a spheroid's segment, QMm (fig. 144) $=$ (because $yy = \frac{cc}{aa} \times : 2ae - ee$) $pe \times : \frac{1}{6} \frac{cc}{aa} ee + \frac{1}{2} yy$. Theo. 90.

If $\frac{cc}{aa} \times : aa - zz = yy$, then (by theo. 86) $\frac{cc}{aa} p \times : aaz - \frac{1}{3} zzz = S$, the solids of any half zone, if $z = Cn$, or of the whole zone MMmm (fig. 140) if $z = nn$, of a spheroid $= pccz - \frac{pccz\,zz}{3aa}$, whence, because $\frac{cc}{aa} \times : aa - zz : yy$, therefore $aa = \frac{zzcc}{cc - yy}$ which put in the last equation for aa gives after reduction $\frac{1}{3} pz \times : 2cc + yy : = S$, Theo. 91, for the zone of a spheroid as also of a globe, as is evident because a, the axis of the spheroid is not in the expression, if e be taken $= a$, then $\frac{cc}{aa} p \times : aee - \frac{1}{3} eee = S$, (see theo. 90) becomes $\frac{2}{3} pcca = S$. Theo. 92.

If in these theorems, you write a for c, and c for a, the same theorems will hold true in the same parts of an oblate spheroid, as is plain from theo. 57. Also, if the ellipsis become a circle viz. if $c = a$, then these theorems hold true in a globe, or sphere and its parts, viz. $\frac{2}{3}paaa$ = solidity of a half globe, $pe \times : ae - \frac{1}{3}ee$ = solidity of a globe's segment $= pe \times : \frac{1}{6}ee + \frac{1}{2}yy$: &c. Theo. 93.

206. If QVQv be a spindle formed by the revolution of the elliptic arch QVQ about QQ (fig. 140) a chord parallel to TT (a) one of the ellipsis's diameters, whose other diameter is 2OV (c) let OC = d, the distance between the center O, of the ellipsis and the axis of rotation QQ, e = Cn, A = area of the elliptic segment MVd, b = Vv, and y = Mm, then (by theo. 57) $\frac{cc}{aa} \times : aa - 4ee :$

$= \overline{2d + y}|^2 = \square Mm$, whence $y = \frac{c}{a}\surd : aa - 4ee : - 2d$, so

$yy = \frac{cc}{aa} \times : aa - 4ee : + 4dd - 4d\frac{c}{a}\surd\overline{aa - 4ee}$ which multiplied by pe, and each term divided by its index of e gives $pcce - \frac{4cce^3p}{3aa} + 4ddep - : 8dp \times$ space MqOV, which space (see theo. 79) is manifestly $= ed + \frac{1}{2}ey + A$ which being put for MqOV, and $c - : \square 2d + y$: for $\frac{4ccee}{aa}$ its equal, we'll have : $2cc + \overline{2d + y}|^2 - 12dd - 12dy : \frac{1}{3}pe - 8Ad$, = S, the solidity of VMmv a frustum or half zone of the spindle QVQv, but $d + \frac{1}{2}b = \frac{1}{2}c$, so $2cc = 8dd + 8db + 2bb$, which put for 2cc in the last equation gives $\overline{2bb + yy + 8d : \times b - y - \frac{3A}{e}} : \times \frac{1}{3}pe =$ S, the same solidity, Theo. 94. If a = c, or the ellipsis become a circle, then the segment MVd will be a circular segment whose area is A, and the last theorem will become S =

$\overline{: 2bb + yy + 8d : \times b - y - \frac{3A}{e}} : \frac{1}{3}pe$, for the frustum of a circular spindle, Theo. 95. If in theo. 94. we write $-d$ for $+d$, then that theo. will become $S = : \overline{2bb + yy - 8d \times : b - y - \frac{3A}{e}} :$

$\times \frac{1}{3}pe$, for the frustum or zone of an hyperbolic spindle, Theo. 96. A being = area of a hyperbolic segment MVd, if y = o, the Cn = e becomes = CQ, half the spindle's length, then the 3 last theorems

become $S = 2bb \pm \overline{8d} \times : \overline{b - \frac{3A}{c}} : \times \frac{1}{3} pe$, for the ſolidity of an elliptic, circular, and hyperbolic ſpindle. Theo. 97.

If $d = 0$, then theorems 94, 95, and 96, becomes, $S = : 2bb + yy : \times \frac{1}{3} pe$, for the zone of a globe or of a ſpheroid, or of an hyperbolic conoid. Theo. 98. And if $y = 0$, then $S = \frac{2}{3} pebb$, for a globe, ſpheroid, and hyperbolic conoid. Theo. 99. Theſe two laſt theorems are the ſame with theorems 91 and 92, which ſee.

If you would have the above theorems without d, you may put in its equal $\frac{c - b}{2}$ in the ellipſis, or $\frac{a + b}{2}$ in the hyperbola's fruſtum and by the equations of theſe two curves, work out a and c, but then the theorems will be very complex. Alſo, you may aproximate the area of the ſegment M V d, (ſee theo. 84 and 85,) and put it in theſe theorems for its equal A, but they are eaſieſt as they now ſtand.

207. If Q V Q v (fig. 140) be a parabolic ſpindle, form'd by turning the parabola Q V Q, round its double ordinate Q Q, let $Mm = y$ (being $\parallel$ V v) $2CV = Vv = b$, the double axis of the generating parabola, or diameter of the ſpindles greateſt circle, $Cn = e$, and $c =$ the parameter of the ſaid parabola, then (by theo. 60) $c \times Vd = \square Md$, or $c \times : \frac{b - y}{2} : = ee$, and therefore, $y = b - \frac{2ee}{c}$, ſo $yy = bb - \frac{4bee}{c} + \frac{4eeee}{cc}$, whence (by theo. 86) $S = pe \times : bb - \frac{4}{3c} bee + \frac{4}{5cc} e^4$: but $\frac{2ee}{c} = b - y$, whence $pe \times bb - \frac{4}{3c} bee + \frac{4}{5cc} eeee$: will become $ep \times : bb - \frac{2b \times : \overline{b - y}|^2}{3} + \frac{\overline{b - y}|^4}{5} : = \frac{1}{3} pe \times : \frac{8bb + 4by + 3yy}{5} : = : \frac{10bb + 3yy - 2bb + 4by}{5} : \times \frac{1}{3} pe = : 2bb + yy - \frac{2}{5} \overline{b - y}|^2 : \times \frac{1}{3} pe$, which are 4 theorems, for the fruſtum of a parabolic ſpindle. Theo. 100.

If $y = 0$, the fruſtum becomes a whole ſpindle, and then we'll have $: 2bb - \frac{2}{5} bb : \frac{1}{3} pe = \frac{8}{15} pebb = S$, for the whole ſpindle (if $e = QQ$). Theo. 101.

If U E F (fig. 139) be a conoid form'd by the revolution of the $\frac{1}{2}$ parabola E S U about its axis u P, let $uP = a$, $PE = PF = b$, the greateſt $\frac{1}{2}$ ordinate $SI = y$ another $\frac{1}{2}$ ordinate, $IP = e$, and $c =$ para-

meter, then (by theo. 60) $c \times : a - e : = yy$, whence (by theo. 86) $pce \times : a - \frac{1}{2}e = S$, but $ca - ce = yy$, so $\frac{1}{2}yy - \frac{1}{2}ca = -\frac{1}{2}ce$, whence $S = pe : \times : ca + \frac{1}{2}yy - \frac{1}{2}ca : = : \frac{1}{2}ca + \frac{1}{2}yy$: also, $ca = bb$, so $S = \frac{1}{2}pe \times : bb + yy$: the solidity of a parabolic conoid's frustum, Theo. 102. And if $y = 0$, then $S = \frac{1}{2}pabb$, for the whole conoid, as by theo. 87.

207. If A B F E (fig. 127) be the frustum of a cone, let its height $nE = e$, diameter of the greater base $AB = b$, that of the lesser base $EF = y$, put $c =$ the parameter of the $\triangle$ B v A, and $d = b - y$, then (by theo 52), $c \times : a - e : = y$ (a being = the axis or $\perp$ D v) so $cc : aa - 2ae + ee : = yy$, whence (by theo. 86) $S = pcce \times : aa - ae + \frac{1}{3}ee$: but because the $\triangle$s a E B and A v B are similar, ce will be $= d$, so $\frac{1}{3}ccee = \frac{1}{3}dd$, also, $ca = b$, or $ccaa = bb$, and $ce \times ca = ccae = db$, which put in the equal of S, gives $S = pe \times : bb - db + \frac{1}{3}dd$: the solidity of the frustum. Theo. 103.

If the frustum become a cone, then $y = 0$, $d = b$, $e = a$, and the last theo. will become $S = \frac{1}{3}pabb$, the same with theo. 88, but if $y = b$ then $d = 0$, in which case $S = pebb$, the solidity of a cylinder Theo. 104.

If $b =$ a side of a square prism's base, then $p = 1$, so $S = ebb$, Theo. 105.

If in theo. 103, you write $b - y$ for its equal d, you'll have $S = pe \times : bb - bb + yb + \frac{1}{3}dd : = pe \times : yb + \frac{1}{3}dd$: Theo. 106.

If in this last theo. you put $bb - 2by + yy$, for its equal dd, you'll have $S = pe \times : yb + \frac{bb - 2yb + yy}{3} : = \frac{1}{3}pe \times : bb + yb + yy$. Theo. 107.

And because circles are as the squares of their diameters, therefore if A and a, denote the areas of the two bases, then $3S = pe \times : A + \surd : Aa + a :$, a general theorem for all such frustums. Theo. 108.

If A B C be a cone or pyramid (fig. 133) with an oblique base A C (whose area let be = a) where the axis B D (e) cuts the plane A C of the base produced, then by what is said in this article, with theo. 34, we'll have $S = \frac{1}{3}ea$, for the solidity of this oblique solid. Theo. 109.

Or if A B C be an oblique frustum whose bases A C (a) and B (A) are parallel, then for the same reasons $S = \frac{1}{3}e \times : A + \surd Aa + a$: Theo. 110.

208. If the frustum F E m S (fig. 141) of a pyramid, or cone, E u F, be obliquely cut by a plane S A, the parts F S A and A S m E so cut

are called hoofs, or for distinction F S A is called hoof, and A S m E the complement hoof, and if the cutting plane S A, be an ellipsis, parabola or hyperbola, the hoof is called an elliptical, parabolic, or hyperbolic hoof; in any of which it is plain by the figure, that the hoof F S A, is $=$ A U F — A U S, that is put A $=$ area of the section made by the cutting plane S A, a $=$ area of the circular segment whose versed sine is A F then by theo. 109, $\frac{1}{3}$ u p $\times$ a $=$ conical part A U F, and $\frac{1}{3}$ U Q $\times$ A $=$ oblique cone A U S, so $\frac{1}{3}$ p a — $\frac{1}{3}$ q A $=$ hoof F S A, of any kind or form whatever either conical or pyramidical, (p being $=$ u p and q $=$ Q u) Theo. 111.

And this hoof taken from the frustum F E m S, leaves A S m E, or taken from S n A F, leaves comp. hoof S n A Theo. 112.

But by theo. 72, you may exterminate p and q, and so have the hoofs in terms of the frustum &c. thus by writing $\frac{GP \times SR}{FR}$ for q, and $\frac{Fp \times SR}{FR}$ for p, theo. 111, will become $\frac{a}{3\,FR} \times : Fp \times SR : - \frac{A}{3\,FR} \times GP \times SR := : Fp \times a - GP \times A : \frac{PR}{3\,FR} = P$, the same solidity. Theo. 113.

This theorem is general, whether the cone &c. be right viz. its axis fall in the middle of its base, or oblique, viz. when the axis is not in the middle of its base, by some called a pyramidic cone, such an one is the cone, A u F, or A U S, but in these theorems following, I mean the right cone, made by prob. 155, where the last theorem will become more simple, for then E p $=$ F p, and G P $=$ I P, and by similar triangles, as S A : $\frac{1}{2}$ F A : : S m ($=$ S I) : $\frac{FA \times Sm}{2\,SA} = GP$, let D $=$ E F, d $=$ S m, and h $=$ S R, the height then F p $= \frac{1}{2}$ D, F R $= \frac{D-d}{2}$, and G P $= \frac{d \times FA}{2\,SA}$, whence the last theorem will become $: D \times a - \frac{d \times FA \times A}{SA} \times \frac{h}{3D-3d} = S$, Theo. 114.

Let T m S n (fig. 136) be the frustum of a square pyramid, cut diagonally by a plane S T, each side of the greater base being $=$ D, and of the lesser $=$ d, and height S R $=$ h, here a $=$ D D A $= \frac{D+d}{2}$ $\times$ T S $=$ S A F A (n T) $=$ D, which put in the last theorem gives (for

a, A, and $FA = D = Ta)$: $DDD - : d \times D \times \frac{D+d}{2} \times \frac{h}{3:D+d:}$

$= \frac{2hDDD - dDDh - Dddh}{6D - 6d} = : 2DD + Dd : \times \frac{1}{6} h, = S,$ which taken from : $DD + Dd + dd$: (see theo. 107) $\times \frac{1}{3}$ h, leaves : $2dd + Dd : \times \frac{1}{6} h, = S,$ for these two hoofs, S T n, and S T m. Theo. 115.

Let T m S (fig. 136) be an elliptical hoof of T m S n, the frustum of a right cone, with circular bases, whose diameters are $Sm = d$, $Tn = D$, and $\perp$ height $SR = h$, put $p = 0{,}7854$. now by theo. 63, $\sqrt{Dd} = BAB$, the conjugate diameter of the ellipsis, so $p\sqrt{Dd} \times ST = A$, and $a = pDD$, which put in theo. 114, for A and a, and TS, for S A as also T n for F A, we'll have $S = : \frac{DDDp}{1} - \frac{d \times D \times \sqrt{Dd} \times p \times TS}{TS} : \times \frac{h}{3:D-d:} = : DD - d\sqrt{Dd} : \times \frac{ph}{3:D-d:}$ the whole frustum T m S n, leaves : $D\sqrt{Dd} - dd$: $\times \frac{dph}{3:D-d:} = S$, the comp. hoof T m S. Theo. 116.

In the parabolic hoof S A F (fig. 138) we have (by theo. 77) $\frac{2}{3}$ S A $\times BB =$ area parabola $BSB = \frac{4}{3} SA \times : \sqrt{d \times \overline{D-d}} : = A,$ also, $a =$ area of a circular segment whose chord is B A B, and height F A, which F A is $= D - d$ (because S A is || E m) these put in theo. 114, for A and F A gives $S = : Da - \frac{\frac{4}{3}d \times \overline{D-d} \times SA \times : \sqrt{d \times \overline{D-d}}}{SA} : \frac{h}{3:D-d:} = \frac{D}{D-d} \times a - \frac{4d}{3}\sqrt{: d \times \overline{D-d}} \times \frac{1}{3} h =$ hoof S A F which taken from the frustum (E F S m) : $DD + d \times \overline{D+d}$: $\times \frac{4}{3}$ ph, (see theo. 107) leaves, $S = : DD + d \times \overline{D+d} : \times p - \frac{Da}{D-d} + \frac{4d}{3}\sqrt{: d \times \overline{D-d}} : \times \frac{1}{3} h$ for the upper, or comp. hoof S A E m. Theo. 117.

If the cutting plane be an hyperbola (fig. 137), in this case, F A and S A must be had before the hoofs can be measured, and then D, d, A, a and h, being as before, we'll (by theo. 114) have $S = Da - \frac{dA \times FA}{SA} : \times \frac{h}{3:D-d}$ for the lower hoof S A F. Theo. 118.

But if the cutting S A, be $\perp$ E F, then (per fig.) S A $=$ h, and F A $= \frac{D-d}{2}$, by which the laſt theorem becomes, $S = \frac{Dha}{3:D-d:} - \frac{dA}{6}$ the ſame hoof. Theo. 119.

But as $\frac{D-d}{2}$, is commonly ſmall in reſpect to D, we may in ſuch a caſe, take : $\frac{47DD+23Dd}{8D+6d} - \frac{47dd+23DD}{8d+6D} : \times \frac{h}{45} \sqrt{DD-dd}$ $=$ S, $=$ hoof S A F nearly. Theo. 120.

PROBLEM CLXXXVII.

To find multipliers for reducing caſks to cylinders, &c. *with ſeveral other uſeful theorems in menſuration, and gauging.*

209. Having in the laſt prob. ſhewn how to inveſtigate theorems for the moſt uſeful ſolids, I ſhall here ſhew how a multiplier may be found for reducing a ſolid to a cylinder &c. which is of great uſe in the gueſſing way of caſk gauging. Theſe caſks are ſaid to be of four forms or varieties, viz.

the variety	1	two equal	ſpheroid	curving	moſt	between the head and bung.
	2	fruſtums	parabolic ſpindle		leſs	
	3	or zones	parabolic conoid		leaſt	
	4	of a	cone		nothing	

With theſe four varieties ſome mix two more, firſt, the fruſtum of a circular ſpindle after ſpheroid, and the fruſtum of an equilateral hyperbolic ſpindle, before parabolic conoid, but theſe four above are ſufficient.

Now, let e $=$ the caſk's length, b $=$ bung, and h $=$ head diameters, d $=$ b $-$ h, their difference, m $=$ the required multiplier, then m d, or m $\times$: b $-$ h : added to h muſt be the mean diameter, viz. that of a cylinder $=$ to the caſk and of the ſame length, whence $pe \times \overline{h+md}|^2$ $=$ S, the caſk's content, $= : 2bb + hh : \times \frac{1}{3} pe$, by theo. 91, $=$ $: 2bb + hh - \frac{2}{5} dd : \times \frac{1}{3} pe$, (by theo 101,) $= : bb + hh : \times \frac{1}{2} pe$, (by theo. 102) $= : bb + hh + bh : \times \frac{1}{3} pe$, (by theo. 107), divide each of theſe by p e, and you'll have

for variety	1	$2bb+hh = 3\times : \overline{h+dm}|^2 : = 3hh+6hdm+3ddmm$,
	2	$2bb+hh-0{,}4dd = 3hh+6dhm+3ddmm$,
	3	$bb+hh = 2hh+4dhm+2ddmm$,
	4	$bb+hh+bh = 3hh+6dhm+3ddmm$;

And by tranſpoſing 2 h h, in the third variety, and 3 h h in the other three varieties,

we'll have
1	$2bb - 2hh = 6dhm + 3ddm,$
2	$2bb - 2hh - 0{,}4dd = 6dhm + 3ddmm,$
4	$bb - 2hh + bh = 6dhm + 3ddmm,$
3	$bb - hh = 4dhm + 2ddmm.$

In these 4 equations, for bb write its equal $\overline{h + d}|^2$, viz. $hh + 2hd + dd = bb$, and in variety 4th, for hb, write its equal : $h + d : \times h$, and you'll have,

$$\left.\begin{array}{l|l} 1 & 4dh + 2dd \\ 2 & 4dh - 1{,}6\,dd \\ 4 & 3dh + dd \end{array}\right\} = 6dhm + 3ddmm,$$
$$\begin{array}{l|l} 3 & 2dh + dd = 4dhm + 2ddmm. \end{array}$$

Now by transposition, and division, these will become,

1. $\dfrac{:6m - 4: \times h}{2 - 3mm} = d$

2. $\dfrac{:-6m + 4: \times h}{-1{,}6 + 3mm} = d$

3. $\dfrac{:4m - 2: \times h}{1 - 2m} = d$

4. $\dfrac{:6m - 3: \times h}{1 - 3mm} = d$

Now to have d the greatest possible, its plain the denominator $2 - 3mm$ must be $= 0$, or $2 = 3mm$, and to have d the least possible, the numerator $6m - 4 = 0$ &c. or $6m = 4$ &c. by which the value of m, in each variety is,

least limit	greatest limit	variety.
$m = \frac{2}{3}$	$m = \sqrt{\frac{2}{3}} = ,8165$	1
$m = \frac{2}{3}$	$m = \sqrt{\frac{8}{15}} = ,7303$	2
$m = \frac{1}{2}$	$m = \sqrt{\frac{1}{2}} = ,7071$	3
$m = \frac{1}{2}$	$m = \sqrt{\frac{1}{3}} = ,5774$	4

Thus you see the least and greatest value that m can possibly have, in these 4 varieties, and that there can be no one multiplier set to any one of the said varieties, but to find (m) a multiplier for any particular case, let us suppose a cask of the most common form, bung diameter $b = 32$, head diameter $h = 26$, the $b - h = d = 6$, then by what goes before we'll have

for variety

1. $m = \dfrac{1}{d} : \sqrt{\dfrac{2bb + hh}{3}} : -h : = 0{,}6888$ &c. Theo. 121.

2. $m = \dfrac{1}{d} : \sqrt{:} \dfrac{2bb + hh - 0{,}4dd}{3} : -h : = 0{,}6753$ The. 122

3. $m = \dfrac{1}{d} : \sqrt{:} \dfrac{bb + hh}{2} : -h : = 0{,}5259$ &c. Theo. 123.

4. $m = \dfrac{1}{d} : \sqrt{:} \dfrac{bb + hb + hh}{3} : -h : = 0{,}5087$ &c. The. 124.

By putting ſquare of $\overline{h + m d} = \frac{2bb + hh}{3} = 2bb + hh - \frac{2}{3}$ $dd =$ &c. &c.

210. Let there be given a trapezia A B E F (fig. 127) with two parallel ſides A B and E F, to find the length o u = L D, of a given part to be cut off, by a line G u parallel to A B or E F; let b = E F l = n E, a = E u, S = area part G u E F to be cut off, and 2 d = A B — E F = a B. Then by ſimilar triangles, as $l : d :: a : \frac{da}{l} = \frac{1}{2}uz$ but $\frac{1}{2}uz \times uE = \frac{daa}{l} = \Delta Ezu$, which added to $\square$ F G z E ($b \times a$) gives $\frac{daa}{l} + ba = S$, whence $a = \frac{\sqrt{:ydls + llbb:} - lb}{2d}$ Theo. 125.

211. If A B E F (fig. 127) be the fruſtum of a pyramid &c. and it be required to cut it into two other fruſtums A B n G and G u E F, let h = E n, the height b = a ſide of the leſſer baſe E F, d = difference between a ſide of each baſe, = a B, c = a factor by which if you multiply the ſquare of a ſide of the pyramids baſe &c. and that product by its axis, may give the pyramids ſolidity, a = E u, the required length, and z the ſolidity of the part G u E F to be cut off, then by ſimilar Δs, as $d : h :: b :: b : \frac{bh}{d} = yv$, ſo $\frac{hb^3c}{d}$ — ſolidity top pyramid E F V, then (by theo. 37) as $\frac{chb^3}{d} : \frac{b^3h^3}{d^3}$, or in lower terms, as $c : \frac{hh}{dd} :: \frac{chb^3}{d} + z : \frac{ch^3b^3 + zdhh}{cddd} =$ cube of L V, ſo $\sqrt[3]{:\frac{h^3b^3}{d} + \frac{zhh}{cdd}:} - \frac{bh}{d} = a$, Theo. 126. or putting $q = \frac{1}{c}$ it will be $\frac{\sqrt[3]{:h^3b^3 + qzhhd:} - bh}{d} = a$.

212. If A B C D (fig. 142) be a ſtreight ſided fruſtum with parallel baſes A B and C D, and there be given the ſolidities of the three parts or fruſtums E D, D G, and D I, = A, B, C, reſpectively, to find the ſolidity of a fourth fruſtum D I = D, the perpendicular diſtances being equal to each other ſuppoſe = a (viz) a = La = 2 a, a = 2 a, 3 a = &c. let e = T L, then per ſimilar Δs, as A B : P T :: C D: $\frac{PT \times CD}{AB} = TL = e$, let $\frac{PT}{AB} = 1$, then e = C D, and for the

reason $e+a=EF$, $e+2a=GH$, $e+3a=IK$, $e+4a=Lm$, CD, EF, GH &c. being parallel to one another and may be taken for the square roots of the areas at the several distances TK, Ta, T2a, &c. and therefore (by theo. 89) $\frac{1}{3}eee=\frac{\square CD\times TL}{3}$ $=$ solidity of TCD, likewise $\frac{\overline{e+a}|^2\times:e+a:}{3}=\frac{\square EF\times Ta}{3}$ $=$ solidity TEF, now $\frac{3aae+3aee+aaa}{3}$ the difference of the two solidities is manifestly $=$ the frustum CDFE, in which expression, by writing 2a, 3a, 4a, severally for a we shall have the required values of A, B, C, D.

viz.	1	$\frac{3aae+3aee+aaa}{3}=A,$	
and	2	$\frac{6aee+12aae+8aaa}{3}=B$	The height of the frustum being L 2a
	3	$\frac{9aee+27aae+27aaa}{3}=C$	L 3a
	4	$\frac{12aee+48aae+64aaa}{3}=D$	L 4a
$1\div a$	5	$\frac{1}{3}:3ee+3ea+aa:=\frac{A}{a}$	
$2\div 2a$	6	$\frac{1}{3}:3ee+6ea+4aa:=\frac{B}{2a}$	
$3\div 3a$	7	$\frac{1}{3}:3ee+9ea+9aa:=\frac{C}{3a}$	
$7-6$	8	$\frac{1}{3}:3ea+5aa:=\frac{C}{3a}-\frac{B}{2a}$	
$8\times\overline{3}$	9	$\frac{1}{3}:9ea+15aa:=3\times:\frac{C}{3a}-\frac{B}{2a}:$	
$9+5$	10	$\frac{1}{3}:3ee+12ea+16aa:=\frac{A}{a}+3\times:\frac{C}{3a}-\frac{B}{2a}:$	
$10\times\overline{4}a$	11	$\frac{1}{3}:12eea+48eaa+64aaa:=4a:\times\frac{A}{a}+3\times:\frac{C}{3a}-\frac{B}{2a}:=D$	

as appears by the fourth step. Theo. 127.

Again, if a = L a the height d d = the area at C D, alſo, D D = the area at E F, and e = difference between the ſquare roots of theſe two area's viz. e = D — d, then e + d = D, which put in theorem 106 inſtead of b, we'll have : d d + d e + $\frac{1}{3}$ e e : × a = fruſtum C D F E, now if z a be put = any depth as L 2 a, L 3 a, &c. then by the property of the △ it will be, as a : e : : z a : z e, by which we have $\overline{\text{d d + : d} + \frac{1}{3}\text{ z e :z e}}$ × z a, = that fruſtum whoſe depth is z a. Theo. 128.

PROBLEM CLXXXVIII.

To inveſtigate theorems for the ſurfaces of ſolids.

213. To underſtand the ſurface, or ſuperficial content of a ſolid, imagine it to be ſewed in cloth &c. and then this cloth to be ripped open and ſpread out flat, and meaſured in this poſture as a plane, will give the ſuperficial content of the ſolid, from which it plainly appears that if a = the periphery of the leſſer baſe, A = that of the greater baſe, and e = the ſlant length of any ſtreight ſided fruſtum, we'll (by theo. 26) have, : a + A : × $\frac{1}{2}$ e = C, the curve, or convex ſurface, Theo. 129.

And if a = A, then e A = C, the convex ſurface of any priſm, cylinder, &c. Theo. 130.

If a = o, then $\frac{1}{2}$ e A = C, the curve ſurface of any cone, &c. Theo. 131. Convex, or curve ſurface, means the ſuperficial content of the ſolid without its baſes or ends.

214. If the ſemi-ſector, B I e, be turned about the radius B I (fig. 135) its manifeſt, the arch e I will deſcribe the ſegment of a globe or ſphere, every point in the ſaid arch, as v, e, &c. deſcribing the periphery of a circle the ſum of which peripheries are = the convex ſurface of the ſaid ſegment; therefore, any part of this arch, as e v may be taken as an abſciſſa, and that the periphery deſcrib'd be v or e, as an ordinate to that abſciſſa, let the arch v e be very ſmall, ſo as to be eſteemed a right line, which will be (by the nature of the circle) at right angles to the radius e B, ſo will the △s B a e, and v n e be ſimilar (v n being ∥ B I) and therefore as e a (y) : e B (r) : : v n (e): $\frac{er}{y}$ = v e the abſciſſa, let c = periphery to the radius r, then as r : c : : y: $\frac{cy}{r}$ = periphery to the radius y, which multiplied by the abſciſſa $\frac{er}{y}$ (ſee art. 198) gives $\frac{cyer}{ry}$ = c e = C, the curve ſurface deſcribed by the arch v e. Theo. 132.

And if $p = 3{,}1416$, and $d = e = 2r$, the diameter of the whole circle, then $c = pd$, and $ce = C$, becomes $= pdd = C$, the ſurface of a globe or ſphere. Theo. 133.

Becauſe $\frac{2}{3}pddd = S$, (ſee theo. 93) and $pdd = C$, therefore, $\frac{2}{3}Cd = S$, for the ſolidity of a globe or ſphere. Theo. 134.

It appears that $e = vn = qa$, may be taken $=$ any part of the radius IB whence $ce = pde = C =$ curve ſurface of any ſegment, fruſtum or part of a fruſtum whoſe height is e, and therefore, as $d : pdd :: e : \frac{pdde}{d} = pde$, that is, as the axis of a globe is to its ſurface ſo is the height of any ſegment &c. to its curve ſurface. Theo. 135.

The ſurfaces of the ſpheroids &c. being tedious to turn without fluxions, ſee them &c. in prob. 190.

PROBLEM CLXXXIX.

Shewing the inveſtigation of a new and ſwift converging ſeries, for the areas and ſolidities of curve-lined ſpaces, &c.

215. Let $e =$ an abſciſſa to the ordinate y, and $de^{pn-1} \times \overline{z + fe^n}|^m = y$ the equation of the curve, firſt involve $\overline{z+fe^n}$ to the mth power and it will be $z^m + mfe^n z^{m-1} + m \times \overline{\tfrac{1}{2}:m-1:}\, ffe^{2n} z^{m-2} +$ &c. which multiplied by de^{pn-1}, and by the abſciſſa e, and then each term divided by the index of e in that term (ſee art. 198) we will have $\frac{de^{pn}}{n} \times : \frac{z^m}{p} + \frac{mfe^n z^{m-1}}{p+1} + \frac{m}{1} \times \overline{\frac{m-1}{2}}\, \frac{ffe^{2n} z^{m-2}}{p+2} +$ &c. $=$ A the required area, but to make the ſeries converge faſter, let $v = z + fe^n$ then $z = v - fe^n$, which being put in the laſt ſeries inſtead of of z, and then each term involved to the mth power we will have,

$$\text{The}\ \left|\begin{array}{l} \text{1ſt} \\ \\ \text{2d} \\ \\ \text{3d} \end{array}\right|\ \text{terms} = \left\{\begin{array}{l} \frac{v^m}{p} - \frac{mv^{m-1}fe^n}{p} + \frac{m}{1} \times \overline{\frac{m-1}{2}}\, \frac{v^{m-2}ffe^{2n}}{p} - \&c. \\ \\ \qquad \frac{mv^{m-1}fe^n}{p+1} - \frac{m}{1} \times \overline{\frac{m-1}{2}}\, \frac{v^{m-2}ffe^{2n}}{p+1} + \&c. \\ \\ \qquad\qquad \frac{m}{1} \times \frac{m-1}{2}\, \frac{v^{m-2}ffe^{2n}}{p+2} - \&c. \end{array}\right.$$

Now by collecting these 3 last steps we have $\frac{v^m}{p} - \frac{m\,v^{m-1}\,fe^n}{p\cdot p+1,} +$ $\frac{m,m-1,v^{m-2}ffe^{2n}}{p,p+1,p+2,}$ — &c. where the law of continuation is plain now this multiplied by $\frac{de^{pn}}{n}$, becomes $\frac{de^{pn}v^m}{pn}\times : 1 - \frac{mfe^n}{v\cdot p+1\cdot}$ $+ \frac{m\cdot m-1.ffe^{2n}}{v^2\cdot p+1\cdot p+2\cdot}$ — &c. = A, by which the area of any known figure may be had nearly, in a few terms, as for example.

Let $\frac{cc}{aa}\times : ae \mp ee : = yy$, or $\frac{2c}{a}\sqrt{ae \mp ee} = 2y$ (see art. 189) $= \frac{2c}{a}\times\sqrt{e}\times\sqrt{: a \mp e}$: which compared with the general equation $de^{pn-1}\times\overline{z+fe^n}|^m = y$, it appears that $\frac{2c}{a} = d$, $e = e$, $n = 1$, $pn - 1 = \frac{1}{2}$, ergo, $p = \frac{3}{2}$, $a = z$, $f = \mp 1$, $m = \frac{1}{2}$, $v = a \mp e$, which put in the last series, we will have $A = 4y\times : \frac{e}{1.3} \pm \frac{ee}{1.3.5v} - \frac{eee}{3.5.7v^2} + \frac{e^4}{5.7.9v^3}$ — &c. for the area of an elliptical segment if you write + for ±, but for the area of an elliptical segment if you write − for ±. Theo. 136.

And if a = c, the elliptical segment becomes a circular one, viz. $A = 4\sqrt{dv}\times : \frac{v}{1.3} + \frac{vv}{3.5d} - \frac{vvv}{3.5.7dd} + \frac{v^4}{5.7.9d^3}$ —&c. the area of a circular segment whose versed sine is v, diameter of the circle D, and $d = D - v$. Theo. 137.

Again, for solids, let $q = 3{,}1416\frac{cc}{aa}$, then $3{,}1416\,yy = qae \mp qee$ $= qe\times : a \mp e$: which compared with the general equation de^{pn-1} &c. we'll have $q = d$, $e = e$, $n = 1$ so $p = 2$, $z = a$, $f = \mp 1$ and $m = 1$, by which the general series becomes $\frac{qee}{2}\times : a \mp e \pm \frac{e}{3} - 0 : = \frac{1}{2}qaee \mp \frac{1}{3}qeee = S$, the same in effect with theorems 98, and 90.

PROBLEM CXC.

To measure any plane or solid without its equation given.

216. If instead of taking a, b, c, d, &c. (see prob. 172) for a series of numbers, we take them for as many ordinates at right angles to an abscissa e = S C (fig. 31) and v distant from each other, viz. a a, a a, &c. = v, now if y = any of these ordinates, a = the first of them, and n = the number of ordinates between a and y, or n + 1 = the whole number of ordinates, it is manifest, from steps 10, 11, 12, of prob. 172, that $y = a + nA + n \times \frac{n-1}{2} B + n \times \frac{n-1}{2} \times \frac{n-2}{3} C$ + &c. now if e = the abscissa, or distance between the ordinates a and y, and v = the distance between each ordinate betwixt a and y, its plain that $nv = e$, therefore, $n = \frac{e}{v}$, which put in the last equation instead of n, and reduced to simple terms will be $y = a + \frac{e}{v} A + \frac{eeB}{2vv} - \frac{eB}{2v} + \frac{e^3C}{v^3} - \frac{eeC}{2vv} + \frac{eC}{3v}$ &c. which multiplied by e, and each term divided by its index of e (see art. 198) gives $A = e \times : a + \frac{eA}{2v} + \frac{eeB}{3.2vv} - \frac{eB}{2.2v} + \frac{eeeC}{4vvv} - \frac{eeC}{3.2vv} + \frac{eC}{2.3v}$ for the area of that space whose abscissa is e, and ordinates, a, b, c, d, &c. to y, let the equation of the said space be what it will, hence, if we take two ordinates, then e = v, and B, C, &c. (as appears from steps 7, 8, 9, prob. 172) are = 0, therefore, area $= e \times : a + \frac{1}{2} A$: but A = b − a (by step 6, prob. 172), therefore, area $= e \times : a + \frac{b-a}{2} : = \frac{1}{2} e \times : a + b$: again, if three equi-distant ordinates be taken then (by steps 8, 9, prob. 172) C = 0, &c. and B = a − 2b + c, and here $\frac{1}{2} e = v$, whence area $= e \times : a + \frac{eA}{v} + \frac{eeB}{3.2vv} - \frac{eB}{3.2v}, : = e \times : a + A + \frac{2}{3} B - \frac{1}{2} B : = \frac{1}{6} e \times : a + 4b + c$: and thus going on by taking e = v, $\frac{1}{2}e = v$, $\frac{1}{3} e = v$ &c. you'll get the following tables, giving the area when 2, 3, 4, 5, &c. ordinates are taken, in which (for brevities sake) I put A = the sum of the two extream ordinates B = the sum of the next two, C = the sum of the next two &c. viz. if there be taken five equi-distant ordinates a, b, c, d, f, I take A = a + f, B = b + d, and c = d + 0.

So, now when equidistant ordinates are taken, the area is

2	$A\times\frac{1}{2}e = :a+b:\times\frac{1}{2}e$,
3	$A+4B:\times\frac{e}{6} = :a+4b+c:\times\frac{1}{6}e$. Theo. 138.
4	$:A+3B:\times\frac{e}{3} =$ &c.
5	$:7A+32B+12C:\times\frac{e}{90}$
6	$:19A+75B+50C:\times\frac{e}{288}$
7	$:41A+216B+27C+272D:\times\frac{e}{840}$
8	$:751A+3577B+1323C+2989D:\times\frac{e}{17280}$ &c.

It is proved in what follows that three ordinates being taken, one at the greater end, one at the leſſer end, and one in the middle between the two ends of any ſpace to be meaſured, will by this method give the content near enough for common uſe in any figure whatever, for which reaſon the table of three ordinates is noted with theorem 138, if $a = eA$, $b = EF$ and $e = AD$, (fig. 132,) then per table of two ordinates, we have area $= :a+b:\times e$, the ſame with theo. 26, and if $a = 0$, then $\frac{1}{2}eb =$ area, the ſame with theo. 24, &c.

217. The ſame things hold alſo true in ſolidities, for its plain by the proceſs, that there is no difference whether a, b, c, d, &c. be taken as areas or as ordinates, therefore, if they be taken as ſpaces, or areas, then the above tables give ſolidities inſtead of areas, ſo that if a and c = the areas of the two baſes of any ſtreight ſided fruſtum whoſe length is e, and b an area in the middle between a and c, then $\sqrt{a}$ and $\sqrt{c}$ = a ſide of a ſquare at each end, and by the property of ſuch a ſolid $\frac{\sqrt{a}+\sqrt{c}}{2} = \sqrt{b}$, a ſide of a ſquare in the middle, whence, $4b = a + 2\sqrt{ac} + c$, which put in theo. 138 for $4b$, we get $: 2a + 2\sqrt{ac} + 2c : \times\frac{1}{6}e$, the ſame with theo. 108. If there be ſuch a priſmoid as that $T\times D = a$, and $t\times d = c$, the areas of its two ends or baſes, and if D and d be parallel as alſo T and t, then by the property of the ſolid $\frac{T+t}{2}\times\frac{D+d}{2} = b$, the area in the middle between the two baſes a and c, ſo $4b = \overline{T+t}\times\overline{D+d} = DT + Dt + dT + dt$, which values of a, 4 b, and c ſubſtituted in theo. 138

gives, $S = : 2DT + Dt + dt + 2dt : \times \frac{e}{6}$ the ſolidity of the priſmoid. Theo. 139.

Again, for the hoofs of a ſquare pyramid, let $D =$ a ſide of the greater baſe, $d =$ that of the leſſer, and $e =$ the hoofs height, then $a = DD$, $b = \frac{D+o}{2} \times \frac{D+d}{2} = \frac{1}{4} : DD + Dd$, and $c = o$, ſo by theo. 138, $S = : DD + \frac{1}{2} Dd : \times \frac{e}{3}$ the ſame with theo. 115.

If $h =$ the head, $b =$ the bung diameters, $m =$ a diameter in the middle between the head and bung, and $e =$ the length of any caſk, whatever, then $pbb = a$, $4pmm = 4b$, and $hh = c$, ſo: $a + 4b + c : \times \frac{e}{6} = : bb + 4mm + hh : \times \frac{1}{6} pe =$ the content of that caſk either true or very near, or $=$ the ullage of it, if b, h and m be verſed ſines of theſe diameters. Theo. 140.

For (by theo. 66) $4mm = 3bb + hh$, which put in the laſt theo. for $4m^2$ gives : $4bb + 2hh : \times \frac{1}{6} pe = S$, the ſame with theo. 91, alſo, (by theo. 64) $4mm = 2bb + 2hh$, which put in theo. 140, for $4mm$ gives : $3bb + 3hh : \times \frac{1}{6} pe = S$, the very ſame with theo. 102. Again (by theo. 65) $4mm = \overline{\frac{3b+h}{2}}\Big|^2 = \frac{1}{4} : 9bb + 6bh + hh$: by which theo. 140, becomes : $13bb + 6bh + 5h^2 : \times \frac{1}{24} ep = S$ nearly, but (by theo. 100) $S = \frac{1}{3} pe \times : \frac{8bb + 4bh + 3hh}{5} : = \frac{pe}{120} \times : 64bb + 32bh + 24hh$: this taken from $\frac{1}{24} pe \times : 13bb + 6bh + 5hh$, viz. from $\frac{1}{120} pe \times : 65bb + 30bh + 25hh$: leaves $\frac{1}{120} pe \times : bb - 2hb + hh : = \frac{1}{120} pe \times \overline{b-h}\big|^2$, too much which is all that any of theſe theorems differs from truth, and is ſo ſmall, that in gauging, the error cannot exceed $\frac{1}{10}$ part of a gallon, but if inſtead of taking h, m, and b, as three equi-diſtant ſpaces, we work with them as 5, viz. $h + h = A$, the ſum of the two extream ones, $m + m = B$, the ſum of the next two, and $b = C$, the middle one, then by table 5, we have : $7A + 32B + 12C : \times \frac{ep}{90} = S$, wherein $A = phh + phh = 2phh$, $B = pm^2 + pm^2 = 2pmm = p \times : \frac{\overline{3b+h}\big|^2}{8}$, and $C = pbb$, which put for A, B and C, in the ſaid

table, becomes : $14hh + {}^{3}\frac{2}{8}\,\overline{3b+h|}^{2} + 12bb : \times \frac{ep}{90} = : \frac{ep}{90} \times : 48 bb + 24bh + 18hh : = S$, the very same with theorem 100.

If enne (fig. 31) be a part of a spheroid's frustum, let $c = CI$, $a = CT$, $b = na$, the radius of the greater base, $h = na$, that of the lesser and $e = aa$, the frustum's length, put $z = Ca$, then by the property of the ellipsis, (putting $n = \frac{cc}{aa}$).

we have	1	$2n \times : aa - zz : = 2bb$
	2	$en \times : aa - (\overline{z + e|}^{2})\, zz - 2ze - ee : = 2hh$
	3	$4n \times : aa - (\overline{z + \frac{1}{2}e|}^{2})\, zz - ze - \frac{1}{4}ee : = 4mm$
1+2	4	$4n \times : aa - zz : - 4ze - ee : = 2bb + 2hh$
3−4	5	$nee = 4mm - 2bb - 2hh$, or $4mm = nee + 2bb + 2hh$.
but	6	$: bb + hh + 4mm : \times \frac{1}{6} ep = S$ by theo. 138,
5 : 6	7	$: 3bb + 3hh + nee : \times \frac{1}{6} ep = S = : \frac{bb+hh}{2} + \frac{nee}{6} : \times pe$. Theo. 141.

Or putting h and b = whole diameters, m = 1 in the middle between them, then p must be $= ,7854$ instead of $3,1416$, and then $S = : \frac{bb+hh}{2} + \frac{2nee}{3} : \times pe$, for the same solidity true as found by other methods.

218. By this method we may also approximate the center of any curve, or space of a given equation and that in a few terms, by what any common series can do, as for example. Required A the area of a curve whose equation is $\frac{1}{1+e} = y$, let the abscissa e be divided into $6 =$ parts, and then we will have these 6 equi-distant ordinates, viz. $\frac{1}{1+\frac{1}{6}}$, $\frac{1}{1+\frac{2}{6}}$, $\frac{1}{1+\frac{3}{6}}$, $\frac{1}{1+\frac{4}{6}}$, $\frac{1}{1+\frac{5}{6}}$, $\frac{1}{1+\frac{6}{6}}$, which reduced are $1.\ \frac{6}{7}, \frac{6}{8}, \frac{6}{9}, \frac{6}{10}, \frac{6}{11}, \frac{6}{12}$, so by table 7, we have $A = 1 + \frac{6}{12} = 1,5$, $B = \frac{6}{7} + \frac{6}{11} = 1,4025974$, $C = \frac{6}{8} + \frac{6}{10} = 1,35$, $D = \frac{6}{9} + 0 = 0,6666666$, whence $41\,A + 216\,B + 27\,C + 272\,D = 582,2443717$, which multiplied by $\frac{e}{840}$, viz. by $\frac{1}{840}$, gives $0,6931479 = A$, which by article 113, is the hyperbolic logarithm of 2, (for $\frac{1}{1+e} = e - \frac{1}{2}ee$ &c. the same series as you'll see there) and would

require 1000000 terms of that series, to get 6 places of figures true as is here done by only 7 terms, and by taking $e = 2, 3, 4,$ &c. you may have the hyperbolic log. of 3, 4, 5, &c.

Note. What is here said of areas, is true of convex surfaces.

PROBLEM CXCI.

An equation being given, to find the greatest or least of its values possible, called maxima and minima.

219. If a be an indefinitely small quantity, and $b + a > e$, as also $b - a < e$, I say that $b = e$, for by transposition, $b > e - a$, and $b < e + a$, but a being indefinitely small, can by adding or by subtracting make no alteration, and therefore, in such cases may stand as 0, viz. $b > e - 0$, and $b < e + 0$, consequence $b = e$.

220 In the equation $pq - t + be^n - e^v = m$, wherein p, q, t, n, b, v, m, are known quantities, and e a variable quantity, let the value of e, be required when m is a maximum, or the greatest possible solution, let $a =$ an indefinitely small quantity, and put $f = pq - t$, and $z^n = be^n$ then the equation will be $f + z^n - e^v = m$, and let z and e, each have a added, then $m = f + \overline{z + a}|^n - \overline{e + a}|^v = $ (by evolution) $f + z^n + naz^{n-1} + n \times \frac{n-1}{2} aaz^{n-2} +$ &c. $- e^v - vae^{v-1} - v \times \frac{v-1}{2} aae^{v-2} -$ &c. but the former part of this equation being a maximum is greater than the latter part thereof, that is $f + z^n - e^v > f + z^n + naz^{n-1} + n \times \frac{n-1}{2} aaz^{n-2} +$ &c. $- e^v\ vae^{v-1} - v \times \frac{v-1}{2} aae^{v-2} -$ &c. (which by transposition and division becomes) $ve^{v-1} > nz^{n-1} + n \times \frac{n-1}{2} a$ &c. $- v \times \frac{v-1}{2} a$ &c. Again, if you take $z - a$ and $e - a$, for $z + a$ and $e + a$, and proceed in the same manner, you will find $ve^{v-1} < nz^{n-1} - n \times \frac{n-1}{2} a$ &c. $+ v \times \frac{v-1}{2} a$ &c. but a being small, all the terms multiplied thereby, may be rejected, as being $= 0$, and then we'll have $ve^{v-1} > nz^{n-1} + 0$, and ve

$^{v-1} \angle n z^{n-1} - o$, whence by art. 219, $v e^{v-1} = n z^{n-1}$, or $n z^{n-1} - v e^{v-1} = o$, viz. $n b e^{n-1} - v e^{n-1} = o$, whence we have.

221. This rule viz. having got m the maxima &c. on one ſide of the equation, and the other ſide conſiſting of one variable quantity e, and its powers, in at leaſt two terms, then multiply each term by its index of e, and from the ſaid index of e in every term take unity, which done for m write o, ſo you'll find e, and if there be any terms in the equation not engaged with e, ſuch terms vaniſh, as is plain by the above proceſs; by the ſame rule you find a minimum, which is only known from a maximum by the nature of the queſtion, the one being contrary to the other. The above proceſs alſo demonſtrates the method of fluxions, and by the rule ſuch expreſſions may be fluxed, thus, the fluxion of $f + e^v$ is $= v e^{v-1} \dot{e}$, that of $\sqrt{ae}$, is $= \dfrac{a\dot{e}}{2\sqrt{ae}}$, for $v\,\overline{ae}^{v-1} = \frac{1}{2}\,\overline{ae}^{\frac{1}{2}-1} = \frac{1}{2}\,\overline{ae}^{-\frac{1}{2}} = \frac{1}{2} a \div \sqrt{ae}$, to which join e, and you have the fluxion of $\sqrt{ae}$, hence, the fluxion, of any expreſſion made $= o$, gives the maximum or minimum, of that expreſſion.

222. Let it be required, to divide the quantity b into two ſuch parts as being multiplied together, may produce a maxima, let $e =$ one of the parts, then $b - e$ muſt be $=$ the other part, ſo $e \times : b - e : = eb - ee = m$ and per rule, $b - 2e = o$, whence, $e = \frac{1}{2} b$. Theo. 142.

Alſo, if $e \times : b - e :$ were to be a minima, we would by the ſame proceſs have $e = \frac{1}{2} b$, but then its plain, the leſs we take e, the leſs will be the product, ſo in this caſe e muſt be indefinitely ſmall, and thus it may ſometimes happen that a maximum or minimum, may be had by reaſoning from the nature of the problem. Again, if b is to be divided into 3 ſuch parts as that their product may be the greateſt poſſible, let $b - e$, $=$ one of theſe parts, then will $e =$ ſum of the other two parts, but (per laſt theo.) the greateſt product that can be made out of the ſum of any two parts of e, is $\frac{1}{2} e \times \frac{1}{2} e$, ſo $\frac{e}{2} \times \frac{e}{2} \times : b - e : = \dfrac{bee - eee}{4} = m$, a max. ſo per rule, $\dfrac{2be - 3ee}{4} = o$, or $2be - 3ee = o$, or $2be = 3ee$, or $2b = 3e$, whence, $e = \frac{2}{3} b$. Theo. 143.

Required e, the axis of the greateſt cone, that can be cut out of the ſpheroid whoſe axis is a, and radius of its greateſt circle c.

1. By the property of the ellipsis we have $\frac{4cc}{aa} \times ae - ee := yy$, which multiplied by pe gives $\frac{4ccp}{aa} \times : aee - eee := peyy$, the cone $= m$, so per rule, $\frac{4ccp}{aa} \times : 2ae - 3ee := 0$, whence $e = \frac{2}{3}$ a, but if $e =$ the axis of the greatest inscribed cylinder, then per said property, $\frac{cc}{4aa} \times : ea - ee := yy = \square$ radius cylinder's base, which multiplied by pe gives $\frac{ccp}{4aa} \times : aae - eee := peyy =$ its solidity $= m$, so per rule, $\frac{ccp}{4aa} \times : aa - 3ee := 0$, whence $e = a\sqrt{\frac{1}{3}}$, these also hold true in a globe, because c is out of the equation. Theo. 144.

Required En (h) the axis of the greatest cylinder, that can be cut out of a given cone AvB (fig. 127) whose axis $vD = a$, diameter AB of its base $= d$, let $e =$ EF the diameter of the cylinders base, then per similar $\triangle$s, as $a : \frac{1}{2}d :: h : \frac{d-e}{2}$, ergo, $h = \frac{ad - ae}{d}$. Let $\frac{a}{d} = c$, then $c \times : d - e := h$, then $peeh = cp \times : dee - eee = S$ the cylinders solidity, which ordered by the rule, will give $e = \frac{2}{3}d$, whence $\frac{a}{d} \times : d - e := \frac{a}{\frac{3}{2}e} \times : \frac{3}{2}e - e = \frac{1}{2}ae \div \frac{3}{2}e = \frac{1}{3}a = h$. Theo. 145.

Required e, the axis of the greatest cylinder, whose diameter is d, and diagonal a, here, $aa - ee = dd$, and $pedd = p \times : aae - eee := m$, so per rule, $aa - 3ee = 0$, whence $e = a\sqrt{\frac{1}{3}}$, which is also $=$ the axis of the greatest cone that can be made out of a slant side a, and bases diameter d. Theo. 146.

Required e the diameter of the greatest cylinder (open at one end) made out of a given superficies c, let $S =$ its solidity, $a = ,7854$, then $\frac{4S}{e} =$ curve superficies, and $aee =$ area base, so $\frac{4S}{e} + aee = 4Se^{-1} + aee = c, = m$, a minimum, or the least surface out of the greatest solidity, so per rule, $-4Se^{-2} + nae = 0$, or $2ae \frac{4S}{ee} = 0$, or $2ae = \frac{4S}{ee}$, whence $e = \overline{\frac{2S}{a}}\Big|^{\frac{1}{3}}$, Theo. 147.

223. If q be a variable quantity and $q^{v} \times q^{n} = q^{v+n} = m$, then per rule, $\overline{n+v} \times q^{v+n-1} = 0$, and if for q^{n} we take z^{n} then $\overline{n+v} \times q^{v+n-1} = n z^{n-1} q^{v} + v q^{v-1} z^{n} = 0$. Theo. 148.

Again, if $\overline{d+z^{n}}|^{v} = m$, first (by art. 220) $d+z^{n}$, becomes $0 + n z^{n-1}$ which call t, also $\overline{d+z^{n}}|^{v}$ becomes $v \times \overline{d+z^{n}}|^{v-1}$, which call w, then $tw = n z^{n-1} \times v\, \overline{d+z^{n}}|^{v-1} = 0$. Theo. 149.

By these two last theorems you may find the maximum of a surd, or of a fraction &c. thus if $\frac{eSQC}{:e+C \times e+Q:} = m, = eSQC \times \overline{ee+eC+eQ+QC}|^{-1}$. Here $t = 2e+C+Q$, and $w = \overline{ee+eC+eQ+QC}|^{-2} \times -1$ so (by theo. 148) $tw \times eSQC + SQC \times :\overline{ee+eQ+eC+QC}|^{-1} = 0 = \frac{2ee+eC+eQ: \times -SQC}{\overline{ee+eC+eQ+QC}|^{2}} + \frac{SCQ}{ee+eQ+eC+QC}$, whence $-2ee-eC-eQ = ee+eC+eQ+QC$, and so $QC = ee$.

Again, if $\frac{ee-eee}{1+e} = m, =: ee-eee: \times \overline{1+e}|^{-1}$, here in $\overline{1+e}|^{-1}$, $t = 0+1$, and $w =: \overline{1+e}|^{-2} \times -1$, so (per theo. 148) $tw \times :ee-eee: + :2e-3ee: \times \overline{1+e}|^{-1} = 0 = -\frac{ee-eee}{\overline{1+e}|^{2}} + \frac{2e-3ee}{1+e}, = \frac{:1+e: \times :2e-3ee: -ee+eee}{\overline{1+e}|^{2}} = 2e-2ee - 2eee = 0$, whence $ee+e=1$, so (by art. 81) $e = \frac{-1+\sqrt{5}}{2} = 0{,}6180$ &c. for any such like.

PROBLEM CXCII.

This and the 8 following problems, contain the theory of mechanics, but first of definitions.

224. By mechanics, is meant the forces, velocities, motions, actions &c. of bodies one upon another.

225. Motion is a restless state, or continual change of place of a body.

226. Uniform motion, is when a body moves over equal spaces in equal time.

227. Velocity, or ſwiftneſs, is that affection of motion by which a moving body runs over more or leſs ſpace in equal times.

228. Accelerated motion, is when the velocity continually increaſes but if the velocity continually decreaſe, its called retarded motion, if it increaſes or decreaſes uniformly, it is equally accelerated or retarded. Alſo, if its motion be conſidered in regard to ſome other body at reſt, its called abſolute motion, but if the motion of a body be conſidered with reſpect to ſome other bodies alſo in motion, its called relative motion, and that way it moves is its direction.

229. Momentum, quantity of motion, or impetus, is the power or force, with which a moving body ſtrikes another body at reſt, or is the motion a body has both in reſpect of its velocity and matter.

230. Celerity of motion is that affection by which a body paſſes over a given ſpace in a given time, or what is called the ſwiftneſs or ſlowneſs of motion.

231. Compound motion, is that which is produced from different powers acting in different directions, Sir I. NEWTON has eſtabliſhed and divided motion into theſe three general laws, viz.

232. Law 1. All bodies continue their ſtate of reſt, or of uniform motion in a right line, till they are made to change that ſtate by ſome external force impreſſed on them.

233. Law 2. The change of motion produced in any body is always proportional to the force, whereby it is affected, and in the ſame direction wherein that force acts.

234. Law 3. Re-action is always equal and contrary to action, or the action of two bodies upon each other are equal, and in contrary directions.

235. Body is the maſs or quantity of matter, if a body yields to a ſtroke and recovers its former figure again, its called an elaſtic body, if not its in-elaſtic, or a non-elaſtic body.

236. Denſity of a body, is the ratio of the quantity of matter it contains, to that in another body of the ſame bulk.

237. Force is a power exerted on a body to move it, if it act but for a moment it is called the force of percuſſion, or impulſe, if it act conſtantly, its called the accelerative force, if conſtantly and equally, it is called a uniform accelerative force.

238. Gravity, is that force wherewith a body endeavours to deſcend towards the earth's center. Abſolute gravity, is when the body falls in free ſpace, and relative gravity when it deſcends in a fluid. Alſo, ſpecific gravity, is the greater or leſſer weight of bodies of the ſame bulk.

239. Center of gravity of a body is ſuch a point in it, as that the body being freely ſuſpended on that point, would reſt in any poſition.

240. Center of motion of a body is a fixt point about which a body is moved, and axis of motion is a fixt line about which a body moves.

241. Center of oſcillation, is that point in any pendulum into which if its whole weight or gravity be collected, the time of its vibrating will not be altered thereby, and is the ſame with the center of percuſſion in bodies that move about a fixt point, but when the body moves in a parallel direction, the center of percuſſion is the ſame with the center of gravity, and in either caſe, is that point in which the force of the ſtroke is greateſt.

242. Friction, is the reſiſtance ariſing from the rubbing of bodies againſt one another.

243. The length of pendulums &c. are meaſured between the center of oſcillation, and the axis of motion, or pin on which they hang, the pin being very ſmall.

244. It is manifeſt from art. 239, that the whole weight or force of a body may be conſidered, as acting, or contracted into its center of gravity, and though lines and ſurfaces are conſidered, as having no weight, yet if they are taken with any thickneſs as the baſes of ſolids &c. they muſt have weight and conſequently centers of gravity.

245. If a right line be ſo drawn thro' any plane or ſolid, as to biſect all the ordinates or ſpaces which compoſe that plane or ſolid, this line is called the diameter of gravity, becauſe the center of gravity is always in ſome point thereof, ſo if any rectangle have drawn within two diagonals, the interſection of theſe diagonals is its center of gravity &c.

246. Axis of ſuſpenſion, a line at right angles to the axis of motion.

247. Equilibrium, the equality of weight, of bodies keeping one another at reſt.

248. In the following theory, all planes are ſuppoſed to be perfectly even and ſmooth, all bodies perfectly ſmooth, except it be mentioned otherwiſe.

249. All lines ſtreight, levers, inflexible, chords or ſtrings, pliable and without weight, unleſs expreſſed to the contrary.

250. Rare, or lightneſs, little matter under much bulk, and is oppoſite to denſe.

PROBLEM CXCIII.

To demonstrate the universal laws of motion.

251. Let q = the body, or quantity of matter in the body to be moved, f = force acting on the body q, m = momentum, generated in q, v = velocity, generated in q, S = space described by the body q, in the time t, with the velocity v. Then. first in all kinds of motion m is proportional to q v, for its plain, if the velocity be double, the quantity of motion m will be as 2 q, if the velocity be treble it will be as 3 q &c. consequently, if the velocity be v, m will be as q v; otherwise, since v acts equally on every particle of matter in q, its plain that m will be as 2 v, 3 v, 4 v, &c. if the particles of matter be 2, 3, 4, &c. consequently, if the particles of matter be q, m will be as q v, Secondly, in all kinds of motion S is, as t v (252) for if the velocity be uniform, viz. if a body moves v miles every hour, its manifest, that in 1, 2, 3, --- t hours, it will move 1 v, 2 v, 3 v, --- t v miles, or the space S, but if the velocity be uniformly accelerated viz. be 1, 2, 3, --- t, in the 1st. 2d. 3d. --- t th equal parts of time, then S = 1 + 2 + 3 --- + t, = (by theo. 24) $\frac{1}{2}$ t t, but, as the time increases, the velocity also increases, therefore, t is as v, so S = $\frac{1}{2}$ t v, or S is as t v, as before. Also, because v is as t, therefore, in this case S = $\frac{1}{2}$ t t, becomes, S = $\frac{1}{2}$ v v, or S $\propto$ v v, (253) hence it appears that in uniform motion, if the time be given, or constant, S $\propto$ v and if the velocity be constant, then S $\propto$ t, but in uniformly accelerated motion, S $\propto$ v v, when the time is given, and S $\propto$ t t, if the velocity is given, but in both motions we have S $\propto$ t v.

254. In any uniform motion, m is as f, for if no force act on the body to put it in motion, it will have no momentum, therefore the greater the quantity of motion, and because the velocity is still the same, we must always have m $\propto$ f, let the time be what it will, but in uniformly accelerated motion, where the velocity, and consequently the force, increases with the time, we must have m $\propto$ t f.

255. From these articles, by using the sign $\propto$ as it were the sign =, we'll have the following theorems, from m, $\propto$ q v, S $\propto$ t v, f $\propto$ m, and m, $\propto$ t f, viz.

$q \propto \frac{m}{v} \propto \frac{f}{v} \propto \frac{mt}{S} \propto \frac{St}{S}$. Theo. 150, q $\propto$, &c. as before.

$t \propto \frac{S}{v} \propto \frac{Sq}{m} \propto \frac{Sq}{f}$. Theo. 151, $t \propto \frac{qb}{f} \propto \sqrt{\frac{qS}{f}} \propto \frac{m}{f}$.

$v \propto \frac{S}{f} \propto \frac{m}{q} \propto \frac{f}{q}$. Theo. 152, $v \propto \frac{ft}{q} \propto \sqrt{\frac{fS}{q}}$.

$S \propto tv \propto \frac{tm}{q} \propto \frac{tf}{q}$. Theo. 153, $S \propto \frac{qvv}{f} \propto \frac{ftt}{q}$.

$m \propto f \propto vq \propto \frac{qS}{t}$. Theo. 154, $m \propto ft$.

$f \propto m \propto vq \propto \frac{qS}{t}$. Theo. 155, $f \propto \frac{qv}{t} \propto \frac{qS}{tt} \propto \frac{qvv}{S} \propto \frac{m}{t}$.

These on the left side of the theorems, are for uniform motion, and those on the right for accelerated motion, and if m $\propto$ f, or m $\propto$ tf, were alike, (see art. 254) these theorems would be the same in both motions, and therefore, if what arises from art. 254., be entirely left out, the proportions on the left hand side of the theorems, will hold true in both motions.

256. Of compound motion, let us suppose a body to be moving from A towards B, (fig 143) in the direction A B. but by some force acting thereon, is compelled to move in the direction A D, or which is the same thing, if while a body would move from A to B, the □ A B D F, would move from A B to D F in the direction A F = B D, its plain the body by this compound motion, would in the same time, describe the diagonal A D, that it would describe the line A B, in by its own motion, that is, while the body by its own force, would be carried from A to D. Hence, if A B express the force of the body, then B D = A F, will express that of the impressing body, and A D, the joint force or effect of both A B and B D (= A F) viz. force A D = force A B + force B D. Theo. 156.

Hence, if a body at D, acts on an obstacle, or plane A B, in the direction D B, and with the force D B, let fall the ⊥ D C, and compleat the rectangle B C D E, so is force D B — force D E = force E B, (by the last theo.) whereof force E B = D C, acts perpendicularly against the plane A B, but force D E acts parallel thereunto, and so can avail nothing, so in this case force D E = 0, and so force D C = the whole, or greatest force possible of D B, against the plane whence the greatest force impress'd, is in a line D C, perpendicular to the plane of the obstacle. Hence appears, the method of dividing one force into two or more forces, for if D A express the force of a body at A, in direction D A, and it be required, what part of that force acts in any other direction B A, then if from D you let fall the ⊥ D C upon the said B A, you'll have C A for the required force, viz: as D A to C A so is radius to co-sine ∠ A D C so is force in direction D A to force in direction D A to force in direction B A. Theo. 157.

Also, if a body at D acts on a plane B A in direction D B, and it be required what impression it makes on the said plane, first let fall the ⊥ D C, then (per last theo.) as D B : the bodies whole force : : D C :

that part of its force which acts on the plane B A, and is as radius : the sine of the incident ∠ C B D. Theo. 158.

If there be any number of forces, suppose 3, A, B, C, (fig. 144) acting against one another in the point D, so as to keep one another in equilibrio, viz. force of C = both the forces of A and B, first produce A D and B D, and compleat the □ D I C H, of which D C is a diagonal.

Now its plain, if a body be kept in equilibrio, the contrary forces in any one line of direction must be equal, viz. force in direction A D = that in direction I D, &c. or the body cannot be in equilibrio but will move, if these contrary or opposite forces do not destroy each other, which they cannot do unless they be equal, whence if D I = C H, express the force of A, then D H = C I, will express the force of B, which two forces (by theo. 156) are = C D; but the force of C is also = these two forces, so C D is also = the force of C, whence the forces of A, B, C, are as D I, I C, C D, or before the sides of △s, are as the sines of their opposite ∠s, there also holds true viz. as D I : C I (A : B) : : S. ∠ D C I, or C D B : S. ∠ C D I, or C D A, and as C I : C D (B : C) : : S. ∠ C D I or C D A : S. ∠ C H D, or H D I, or A D B. If there be ever so many forces acting against a point D, and keep on another in equilibrio, they may be reduced to the action of two equal and opposite forces by the same method, for if H D and I D, be two forces they are = the single force D C, &c. Theo. 159.

257. The meeting of hard non-elastic bodies, if a moving body A, whose velocity is V, momentum M, and weight or quantity of matter Q, strike another body B, whose velocity is v, momentum m, and weight q, and if e, be put = the velocity of both bodies after the stroke then (by art. 234) we'll have see Theo. 154.

1. $QV + qv = Qe + qe$, so $e = \dfrac{QV + qv}{Q + q}$ when the bodies move both the same way.

2. $QV - qv = Qe + qe$, so $e = \dfrac{QV - qv}{Q + q}$ when the bodies move to meet each other.

Whence in both cases it will be $e = \dfrac{QV \pm qv}{Q + q}$. Theo. 160.

For its plain if the bodies both move the same way, B will gain what A loses by the stroke, but if they move the contrary way, or meet each other, then the greater momentum over powers the lesser, and the bodies must both move in direction of that which had the greater momentum and with the difference of their momentums, if B be at

rest before the stroke, then $v=o$, and so $e = \frac{QV}{q+Q}$. Theo. 161,

Again, because the quantity of matter multiplied by the velocity (by theo. 154) gives the momentum, we'll have $Qe = \frac{QQV \pm Qqv}{Q+q}$ for the momentum of A after the stroke, and $qe = \frac{QqV \pm qqv}{Q+q}$ for the mom. of B, after the stroke, whence $QV - \frac{QQV + Qqv}{Q+q} = \frac{Qq}{Q+q} \times \overline{V \pm v} =$ the momentum of A lost in the stroke, and consequently = to that gained by B, but this loss or change of motion in either body measures the magnitude of the stroke, wherefore, A and B strike each other with a stroke always $= \frac{Qq}{Q+q} \times \overline{V+v}$, or proportional to $V+v$ if they meet, but to $V-v$, if they move the same way, and if B be at rest before the stroke, then $v=o$, and the magnitude of the stroke will be $\frac{QqV}{Q+q}$, or as V (because $\frac{Qq}{Q+q}$ is constant). Theo. 162.

If we put $z =$ the velocity of A, lost in the stroke, and $y =$ that of B, gained in the stroke, then, because (by theo. 162) $\frac{QV \pm qv}{Q+q} = e$, the common velocity after the stroke, we'll have $V - \frac{QV \pm qv}{Q+q} = \frac{qV + qv}{Q+q} = z$, and $\frac{QV \pm qv}{Q+q} \mp v = \frac{QV \mp Qv}{Q+q} = y$, which two equations turned into analogies gives, $Q+q : q :: V \mp v : z$, and $Q+q : Q :: V \mp v : y$, in which for v write o, if B be at rest before the stroke. Theo. 163.

258. Of elastic bodies, or such as give way when pressed by the stroke &c. but after the removal thereof, restore their form again now if this restoring force is equal to the force of compression, they are said to be perfectly elastic, but if these forces are unequal, their ratio is called the elastic force, and observe, that all homogenious bodies of the same kind have their elastic ratio invariable, for it must be the same in all bodies whose constituent parts are the same, now in these sort of bodies the velocity lost and gained must consist of two parts, viz. one from the force of compression, and the other from that with which

the parts restore themselves, which parts are equal in perfect elastic bodies, and in a given ratio in non-perfect elastic ones, now if a and b express the velocities of A and B after the stroke, and $1 : r$ as the compressing force is to the restoring force, then by the last theo. we will have

1. $\frac{q\times:V\mp v}{Q+q}$ = the velocity of A, lost by the compressing force,

2. $\frac{Q\times:V\mp v}{Q+q}$ = the velocity of B, gained by that force.

$1\times r$	3	$\frac{qr\times:V\mp v:}{Q+q}$ = the velocity lost by A in the restoring force,
$2\times r$	4	$\frac{Qr\times:V\mp v:}{Q+q}$ = the velocity of B, gained by that force,
$1+3$	5	$\frac{:1+r:\times q\times:V\mp v:}{Q+q}$ = the total velocity lost by A,
$2+4$	6	$\frac{:1+r:\times Q\times:V\mp v:}{Q+q}$ = the total velocity gain'd by B
$V-5\ 7$	7	$V-\frac{:1+r:\times q\times:V\mp v:}{Q+q}=a=\frac{-2q\times:V\mp v:}{Q+q}+V$
$6\pm v\ 8$	8	$\pm v+\frac{:1+r:\times Q\times:V\mp v:}{Q+q}=b=\pm v+\frac{2Q\times:V\mp v:}{Q+q}$ when $r=1$ or the bodies are perfectly elastic.

When the bodies move the same way, you must take the upper sign before v, but if they tend to meet, you must take the lower sign, and if B be at rest before the stroke, then $v=0$, also, if we suppose the bodies void of spring, or elasticity, these two last steps will be the same with theo. 160, whence they are a general theorem in all cases that can happen in the collision of elastic or non-elastic bodies. Again if A strikes B at rest, and a causes B to strike another body C at rest with the velocity b, and if c express the velocity of C acquired by the stroke we'll (by theo. 161) $c=\frac{Sqb}{q+C}$ (putting $r+1=S$) or because (by foregoing 6th step) $b=\frac{SQV}{Q+q}$ it will be $c=\frac{SSqQV}{:q+C:\times:Q+q}$ which multiplied by C gives $\frac{SSqQVC}{:q+C:\times:Q+q:}$ = the momentum of

C after the ſtroke, and if this momentum be taken a maxima, wherein q is variable, we'll (by theo. 149) have $QC = qq$, viz. $AC = BB$ hence as A : B : : B : C, that is B muſt be a mean proportion between A and C, that the velocity or momentum of C after the ſtroke may be the greateſt poſſible. Whence, if it be required to find (n) any number of bodies ſuch, that by the firſt ſtriking the ſecond, the ſecond ſtrikes the third, the third the fourth, and ſo on to the laſt which being ſtruck by the laſt but one, may have the greateſt velocity, and (conſequently the greateſt momentum becauſe the weights are conſtant) it is manifeſt theſe bodies muſt be in geometrical progreſſion, they being all at reſt before the ſtroke of the firſt body) whoſe ratio let be e, and Q the firſt body V its velocity, then the ſecond body will be eQ and $\frac{SQV}{Q+q}$ its velocity will become $\frac{SQV}{Q+eQ} = \frac{SV}{1+e}$, whence $\frac{SQeV}{1+e}$ will be its momentum, conſequently the laſt body will be $e^{n-1}Q$, its velocity $\left|\frac{S}{1+e}\right|^{n-1} V$, and its momentum $\overline{\frac{S}{1+e}}^{\,n-1} QV$.

In all theſe expreſſions, if the bodies are perfectly elaſtic, then $r = 1$, or $r + 1 = S = 2$, and if they are hard non-elaſtic bodies S goes out of the expreſſion.

259. In all theſe caſes the bodies are ſuppoſed to impinge on, or ſtrike one another perpendicularly, but if they ſtrike one another obliquely the ſame things may be found after the manner of theo. 157, and 158. Thus ſuppoſe the body A, (fig. 145) to be moving in the line A C, with the velocity A C and the body B, to be moving in the line B C, with the velocity B C, and let the plane or line E F C, be that where the bodies touch each other when they ſtrike, or meet in the point C, on which E F C, let fall the perpendiculars A E and B F, which (by theo. 158) expreſſes the velocities wherewith A and B approach each other, compleat the rectangles E G and F H, ſo A C the motion of A divided into two others A G and A E, to which the motion in A C, is as A C to A G and A E reſpectively, in like manner the motion of the body B is reſolved into two other motions B F and B H, to which the motion B C is as B C to B F and B H, reſpectively. But ſince A G and B H are parallel, the bodies by the velocities in theſe directions cannot ſtrike each other, ſo that velocity with which A, comes directly againſt B is = G C, and that wherewith B comes againſt A, is = H C, now let C L, be the velocity of the body A, from C towards L after the ſtroke (which C L may be found by the foregoing theorems whether the bodies be elaſtic or non-elaſtic,) and becauſe the

velocity of A in direction A G, is not altered by the stroke (as aforesaid), it must therefore be the same both before and after the stroke; so make C M = A G, and compleat the rectangle C L M N, and draw the diagonal C N, which C N will (by theo. 158) express the velocity of A, after the stroke, in like manner you may find that of B after the stroke.

260. In like manner it may be proved, that if a perfect elastic body u, be thrown obliquely in the direction and with the velocity u C against a firm obstacle G H, the body will be so reflected from the obstacle as to make the angle of reflection n C L = the angle of incidence u C L, and the velocity C n = the velocity n C, for by letting fall the ⊥ u L, the motion u C is resolved into two motions u L directly against the obstacle, and L C parallel thereunto, which L C being parallel can avail nothing, nor be any way obstructed by the stroke, and will be the same after as before the stroke, so make C L = L C, and compleat the rectangle C n, then because by the nature of elastic bodies, if the body is thrown with the ⊥ velocity u L, it will be reflected with the same or an equal ⊥ velocity L n, whence the reflected velocities are C L and L n: (= L C and u L) = the one velocity C n (see theo. 258) = u C; but since no bodies in nature are perfectly elastic, they must be something longer in restoring their forms, and therefore, the ∠ n C L will be somewhat less than ∠ U C L. Theo. 164.

PROBLEM CXCIV.

Of the descent of heavy bodies.

261. If from any part in or on the earth, a heavy body be thrown upwards it will fall down in direction to the earths center, as is seen by trial from whence it appears that this center is the center of gravitation, and that if a heavy body could arrive there it would be at rest. Also, the nearer this center any body is let fall, the greater must be its velocity, but the height to which we can project any, bodies being but small in respect of the earths axis, the force of gravity may any where on the earths surface be looked upon as equal at equal heights, and to descend in parallel direction with uniformly, equally accelerated velocities, this kind of motion is treated of in the last problem which may be done by lines thus, let A E (fig. 145) denote time and E C, velocity acquired by falling in that time draw D v and v F parallel to E C and A E, then by similar △s, as A D : D v :: A E : E C, viz. as any time A D is to D v the velocity required in that time, so is any other time A E, to E C, the velocity acquired by falling in that time, which times and velocities constitute the similar spaces or △s A D v and A E C, which spaces are as A D × D v to

A E × E C, and ſo is the diſtance deſcended in the time A D to that deſcended in the time A E. Hence, the diſtances deſcended, or fallen through, are as the times and velocities conjointly. Theo. 165.

But (by theo. 36) as △ A D v : △ A E C : : □ A D : □ A E, or : : □ D v : □ E C, that is the diſtances deſcended, are as the ſquares of the times, or as the ſquares of the velocities. Theo. 166.

If you compleat the parallelogram A E C G, uniformly deſcribed with the velocity A G = E C in the time A E, to acquire the velocity A G = E C. Theo. 167.

If a body in deſcending the time A E, acquire the velocity E C, it muſt be thrown up with the ſame velocity E C, to come to the place from whence it began to fall in the ſame time A E, by the third law of motion. Theo. 168.

At the point D, the riſing body will have the velocity D v equal to that which a body would acquire by falling in the time A D, the difference between the times of the deſcent A E, and aſcent E D. Theo. 169.

All bodies of the ſame kind whether great or ſmall falling from a ſtate of reſt, will acquire equal velocities in equal times, for its evident that gravity acts equally on every particle of matter, and were it not for the reſiſtance of the air, which acts on all bodies according to their denſities, all bodies whether light or heavy, would from a ſtate of reſt fall equal ſpaces in equal times, in which they would acquire equal velocities, from what is ſaid in this prob. it is evident, that the theorems hold true, whether the body deſcend perpendicularly or obliquely, by uſing the oblique deſcent inſtead of the ⊥ deſcent.

PROBLEM CXCV.

The theory of pendulums and vibrating chords.

262. Let A H B (fig 146) be a half circle, with its diameter A B perpendicular to the horizon, and ſuppoſe a heavy body falling along A B, put v = the velocity acquired at D, and V = that at B, then (by theo. 166) as A D : v v : : A B : V V, or a √ A D : √ A B : : v : V, and per ſimilar △s, as A D : A B : : A E : A C, whence, as √ A E : √ A C : : v : V, that is, if the ⊥ heights be equal, the acquired velocities will alſo be equal, whether the body deſcend perpendicularly down A B, or obliquely along A E C. Theo. 170.

Alſo, if T = the time of ⊥ deſcent from A to B, and t = that from A to D, then becauſe the velocities are as the times, it will be as √ A C : √ A E : : T : t, or as A C : A E : : T T : t t, alſo if inſtead of an inclined plane A C, you take A C to be any curve, this and the laſt theo. will hold true, as will appear by dividing the ſaid curve A C into

an infinite number of equal parts which equal parts may be taken as right lines, and the velocities will be equal at equal heights &c. Theo. 171.

Again, becauſe an angle A H B, in a ſemicircle is a right angle, we'll by ſimilar Δs, have as $AB : AH :: AH : \frac{\square AH}{AB} = AI$, whence $\frac{AH}{\sqrt{AB}} = \sqrt{AI}$, = the velocity acquired at H, by falling thro' the curve or inclined plane or chord A H, and becauſe (by theo. 165) the diſtance fallen thro' divided by the velocity gives the time, therefore $AH \div \frac{AH}{\sqrt{AB}} = \sqrt{AB}$ = the time of falling from A to H. Whence the times of deſcent along the diameter of a circle perpendicular to the horizon, and any chord A H or H B, of the ſame circle are equal. Theo. 172.

Let d = the length of a pendulum, or $2d$ = the diameter of a circle, c = any chord thereof, t = time of a heavy body freely falling thro' the chord c, v = the velocity acquired at the end of that fall, and let D, C, T, V, be the like things of another circle &c. then per laſt theo. as $\sqrt{2D} : \sqrt{2d} :: T : t$, or as $2D : 2d :: TT : tt$, and if the chords be very ſmall, C and c, may expreſs the arches themſelves, and then, the times of vibrations of pendulums in very ſmall arches are as the ſquare roots of their lengths. Theo. 173.

Alſo, by theo. 170, $\frac{C}{\sqrt{2D}} = V$, and $\frac{c}{\sqrt{2d}} = v$, whence as $\frac{CC}{2D} : \frac{cc}{2d} :: VV : vv$, and if $C = c$, then as $\frac{1}{2D} : \frac{1}{2d}$, or as $2d : 2D$, or as $d : D :: VV : vv$, that is the lengths of pendulums are inverſely as the ſquares of their vibrations. Theo. 174.

If $p = 3,1416$, then in article 424, it is proved that $\frac{1}{2} p \sqrt{2d} \times : 1 + \frac{a}{2,2,2d} + \frac{3,3aa}{2,2,4,4,4dd} + \frac{3,3,5,5aaa}{2,2,4,4,6,6,8ddd} + \&c. =$ the time of falling thro' an arch whoſe verſed ſine is a, diameter d, which when the arch is very ſmall, may be taken $= \frac{1}{2} p \sqrt{2d}$, and (by theo. 172) $\sqrt{2d}$, is as the time of falling down the diameter, as alſo down the chord, ſo as $\sqrt{\overline{2d}} : \frac{1}{2} p \sqrt{\overline{2d}}$, or as $1 : \frac{1}{2} p$, or as $2 : p ::$ the time of falling thro' the chord to the time of falling thro' the arch, :: the diameter to half the periphery, whence, as $2 : p :: t : \frac{tp}{2}$ = the time of vibrating in a circle, down whoſe diameter $2d$, a

heavy body would fall in the time t, the arch of the circle in which the pendulum (whose length is d) vibrates being small then as $d : \overline{\frac{1}{2} t p}|^2 :: D : \frac{tt pp D}{4d} = tt$, whence $d = \frac{pp D}{4}$, or as $1^2 : \frac{1}{4} pp :: D : d$, that is, as the square of a circles diameter is to the square of half its periphery so is the length of a pendulum vibrating in a small arch, to the distance which a heavy body will perpendicularly descend in the time of one oscillation. Theo. 175.

Again (by theo 173) as $\sqrt{d} : t :: \sqrt{D} : T = t\sqrt{\frac{D}{d}}$, but if the pendulum d, be made to describe a greater arch of the same circle, then from the above series, (by putting $n = \frac{a}{d}$), we'll have $T = t \sqrt{\frac{D}{d}} \times : 1 + \frac{n}{2,2} + \frac{3,3nn}{2,2,4,4,4} + \frac{3.3.5.5nnn}{2,2,4,4,6,6,8}$ &c. and if t = one second of time, and d = D, then the last series becomes, $T = 1 + \frac{n}{2,2} + \frac{3,3nn}{2,2,4,4,4} + \frac{3,3,5,5nnn}{2,2,4,4,6,6,8}$ &c. that is, if a pendulum oscillate once in one second of time in a very small arch, it will oscillate once in T seconds in a greater arch of that circle. Theo. 176.

It may also be proved, the vibrations being small, that if b = the degrees in an arch in which a pendulum measures equal time, the number of seconds lost per day in another arch whose degrees = c, will be $\frac{1}{7} : cc - bb$: nearly. Also, if n = the number of threads in an inch of the screw at the lower end of the pendulum, y = the time in minutes that the clock gains or loses in 24 hours, then the number of threads that the bob is to be let down, or raised up to beat seconds will be $= \frac{3}{17} ny$. Theo. 177.

262. To find the vibrations of an elastic string or musical chord, A B (fig. 147) whose length A B, let be = l, and its diameter = d, to make this plain, let us suppose the given chord A B to be stretch'd by a weight f, put over a pulley at B, and to be struck in the middle C, by another force or weight e, which puts the chord A B into the oblique position A D B, now its plain the whole force of f upon the string A B, is in the direction B D, which force may be expressed by B D, and divided into the two forces B C, tending to pull the string streight and D C tending to pull it directly upwards from D to C, whence as $DB : f :: CD : \frac{f \times CD}{DB}$ = the force of f in direction D C, which must

be = e, becauſe they ballance each other and becauſe C D is but very ſmall we may take C B = D B, and then $\frac{f \times CD}{DB} = \frac{f \times CD}{CB} = e$, = $\frac{f \times CD}{2l}$, or $\frac{f \times CD}{l} \propto e$, now becauſe the ſpace A D B, is but ſmall it may be taken as a plane △, and then its ⊥ C D will always be proportional to it, viz. C D ∝ S, whence $\frac{fS}{l} \propto e$, theſe ſpaces A D B &c. made by the vibrations of the ſtring A B, being as aforeſaid, but ſmall, may be looked upon as uniform, and then (by theo. 154) m ∝ et, or $\frac{m}{t} \propto e$, whence $\frac{fS}{l} \propto \frac{m}{t}$, or S f t ∝ m l, but (by theo. 154) m ∝ q v and (by theo. 153) S ∝ t v, whence f v t t ∝ l q v, or f t t l q ∝ l l d d, becauſe l d d is as q, the ſtrings weight, or ſolidity, whence $t \propto \frac{ld}{\sqrt{f}}$, that is, the time of one vibration is as the diameter and length of the ſtring directly, and as the ſquare root of the tenſion (f) inverſely, and becauſe C D, nor S is not in the expreſſion, it follows, that the time of a vibration is the ſame, whether the cord vibrate thro' a greater or leſſer ſpace. Theo. 178.

264. Let a = the ſpace perpendicularly deſcended by a heavy body in the firſt ſecond of time, and w = the cords weight, then w ∝ l d d ſo l w ∝ f t t, and becauſe the ſpaces fallen thro' are as the ſquares of the times, and t being as the time of one vibration and alſo as $\sqrt{\frac{lw}{f}}$, its plain, that to cauſe a vibration of the ſtring A B, the point D, muſt fall thro' a ſpace $\propto \frac{lw}{2f} \propto \frac{1}{2}tt$, whence as a : □ 1 ſecond : : $\frac{lw}{2f}$: $\frac{lw}{2af}$ = the ſquare of the time of one vibration in ſeconds, ſo $\sqrt{\frac{lw}{2af}}$ = t, the time of one vibration in ſeconds, whence $\sqrt{\frac{2af}{lw}}$ = number of vibrations in one ſecond. Theo. 179.

PROBLEM CXCVI.

The theory of wheel carriages, and of the mechanic powers; viz. the wheel, pulley, ſcrew, ballance, leaver and wedge.

265. Let A P F M, be a wheel whoſe weight is w, radius C A = r (fig. 147) and E F = h, the perpendicular height of an obſtacle E,

to the horizon N D, over which the wheel is to be drawn, let O K be a tangent to the wheel at the point E the top of the obſtacle meeting the diameter A G, produced in O, then the wheel coming to the point E, ſtands upon G, preſſing there with its whole weight, let therefore, C O, the direction in which it gravitates expreſs its whole weight which (by theo. 158 is reſolved into the two forces C E and O E, of which C E preſſes directly againſt the top E of the obſtacle, and ſo is deſtroyed by the re-action of the ſaid point E, therefore, the only force to be overcome is the other force O E, whence becauſe wheels are drawn by axle-trees thro' their centers, it follows that a force = O C, in direction C M parallel to E K, will hold the wheel in equilibrio on E, ſo by ſimilar Δs, as E O : C O : : H E : E C, whence E O $= \frac{w}{r} \times$ H E, but by the property of the circle $\square$ H E = A H $\times$ H G $= 2rh - hh$, therefore E O $= \frac{w}{r}\sqrt{2rh - hh}$, the wheel in direction C M parallel to o E K, is drawn with the greateſt eaſe, for ſuppoſe it drawn in any other direction C M, then this force C M being = the two forces C r and m r, of which C r draws the wheel directly againſt E, and ſo is deſtroyed by equal re-action of the point E, what therefore remains to draw it up in direction parallel to o K is m r, now if S = ſine $\angle$ m C r, (which C m the direction of the force called the line of traction makes with E C) and radius = 1, then as 1 : r : : S : rm, or as S : r : : r m : $\frac{r}{S} \times$ r m = 1, viz. when r m becomes $\frac{r}{S} \times$ r m, then C M will become $\frac{r}{S} \times CM = \frac{r}{S} \times \frac{w}{r} \times \sqrt{:2rh - hh:} = \frac{w}{S}\sqrt{2rh - hh} = f$, the force fit to ſuſtain the wheel on E, in any direction C M of the force, (C m being to r m as r to S). Theo. 180.

If E F (h) be very ſmall, then h h may be rejected and then $f = \frac{w}{r}\sqrt{2rh}$. $=$ (if w be given) $\frac{\sqrt{2rh}}{r} = \frac{1}{\sqrt{r}}$ (2 h being conſtant) that is in rough uneven ſurfaces, the force to draw the wheel will be inverſely as the ſquare root of the wheels radius or diameter, if h = 0, then f = 0, which ſhews that no force is required to move a heavy body on an horizontal plane perfectly ſmooth. Theo. 181.

If r $\propto$ h, and the line of traction parallel to O K, then $f = \frac{w}{r} \sqrt{:2rh - hh:} = \frac{w}{r}\sqrt{:2rr - rr:} = \frac{w}{r}\sqrt{rr} = w$, that is,

the force will be proportional to the weight of the wheel, if C m the line of traction be parallel to the horizon N D, then $\angle$ m C E $= \angle$ C E H for m $=$ C H, $=$ S $=$ r $-$ h, whence, $f = \frac{w}{r-h} \sqrt{: 2rh - hh :} = \frac{w}{r-1} \sqrt{: 2r - 1 :}$ when h is constant. Theo. 182.

266. If the obstacle E F (h) be such, as that it may be depressed or broken down by the wheel, then C E denotes the whole force with which the wheel bears upon the obstacle, and (by theo. 158) is $=$ the two forces H E, parallel to the horizon which must tend to drive the obstacle before it, and C H perpendicular to the horizon and so tends to press it down, but C H $=$ r $-$ h, the pressing force, whence the larger the wheel, the greater will be the force to press down the obstacle.

267. If the obstacle be driven forward before the wheel, then H E $= \sqrt{: 2rh - hh :}$ the driving force, whence the lesser (r) the wheel the greater the advantage, in pushing the obstacle forward.

268. The line of traction C M being parallel to O K, its plain that if E F $=$ o, the line of traction must be C m parallel to the horizon N D, for if any other line C E, be this line of traction, it is resolvable into two other forces C H, drawing the wheel downwards, and so increases the loads weight, and H E parallel to N D, whence H E, is the only force that tends to draw the wheel forwards; again, if C M be the line of traction, it is $=$ the two forces, C B, drawing the wheel upwards, and so diminishes the loads weight, and B M $\parallel$ N D, so B M is the only force that tends to draw the wheel forwards. Hence, it follows, that if the radius of a carriage wheel be $=$ the height of a horse's breast, or the traces parallel to the plane on which he draws, the carriage will go with the most ease, and wheels whose radius exceed that, are worse than those which are less, in which the traces are a little elevated, in which position of the traces, the horse on his account draws easiest, as well as on account of the wheels.

269. Of leavers there are 4 kinds, first, is when the prop, or fulcrum C (fig. 148) is between the power P and weight w. Second kind, is when the weight C is between the prop w and the power P. Third kind, is when the power C is between the weight and prop w and P, the fourth kind is a bended leaver as B C w, or B C A &c.

270. If w C P (fig. 148) be a streight line or leaver at rest parallel to the horizon with a weight w at one end, and another P at the other

end the lever being at rest over a prop C, if it be made to move into the position B C D, the weight w, will describe the arch w D, and P, the arch P e, which arches are plainly equal the velocities of these weights, and because these weights or bodies w and P are in equilibrio, the force on each side of C, must be equal, that is, (by theo. 155,) $w \times w D = P \times e P$, (for $f \propto q v$) but the arches W D and P e, are as their radii C w and C P, so $w \times w C = P \times P C$. From the first of the equations, it follows that the product of the power and its distance moved is = the product of the weight and its distance moved in the same time, so if a weight be raised by any engine whatever you may by observing the distance that it and the power moves in the same time, know the ratio of their weight, which is of vast use in mechanics, as you'll find the questions. Theo. 183.

271. Since weights hang perpendicular to the horizon, and B P is $\perp$ C P as also A w $\perp$ C w, it must be the same thing whether P act at P, or at B, and w at w, or at A, it will still be $w \times w C = P \times P C$, and if $\angle A C w = \angle P C B$, it will be $w \times A C = P \times B C$, or $w \times A w = P \times P B$. Also it is the same whether you suppose P to press or lift and therefore, if w be the prop, it will also be $w \times w C = P \times P C$, or as $P C : w C ::$ weight upon w, to the weight upon P. Theo. 184.

The other mechanic powers are dependant on theo. 183.

PROBLEM CXCVII.

Of the centers of gravity, percussion, oscillation, and gyration, Fig. 149.

272. If there be any number of bodies a, b, c, d, suspended on the same line or streight lever A B at the points A, C, D, B, and be in equilibrio upon a point G, this point G, is the center of gravity of all the bodies, then by theo. 184, we'll have $C G \times b + A G \times a = D G \times c + B G \times d$, but per fig. $C G = A G - A C$, and $D G = A D - A G$, also, $B G = A B - A G$, so $\overline{AG - AC} \times b + A G \times a = \overline{AD - AG} \times c + \overline{AB - AG} \times d$, which by transposition and division gives $A G = \frac{A D \times c + A C \times b + A B \times d + a \times 0}{a + b + c + d}$ the distance of G the common center of gravity from A. Also, by reduction $c = \frac{A G \times : a + b + d : - A C \times b - A B \times d}{A D - A G}$, any of the bodies as c, in like manner, we may get $A C = \frac{A G \times : a + b + c + d : - A D \times c - A B \times d - a}{b}$, the distance

between a and b any two of the bodies, but if the lever A B have any weight, ſuppoſe = that of the body c, and D its center of gravity, theſe equations will be the ſame, becauſe the weight of every body acts in its center of gravity. Theo. 185.

If the ſtreight beam or ballance A B (fig. 149) ſuſpended on G, be in equilibrio with the bodies a, and d, (b and c being = o) but if the body a be removed to B, it will be in equilibrio with a body w, ſuſpended at A, then (by the laſt theo.) $AG = \frac{BG \times d}{a}$, and $AG = \frac{BG \times a}{w}$, whence $BG \times d \times w = BG \times a \times a$, that is, $dw = aa$, ſo $\sqrt{dw} = a$, which put in the laſt equation for a, gives $BG \times \sqrt{dw} = BG \times dw$, or, $\frac{AG \times w}{BG} = \frac{BG \times d}{AG}$ (each = a) ſo as $w : d :: \square BG : \square AG$. Theo. 186.

If there be any irregular ſolid A c G B, whoſe weight is w, (fig. 150) and its center of gravity at G, and if by a body c, laid on the point d, the ſolid be in equilibrio upon the prop D, and alſo by a body a, laid upon q, it be in equilibrio upon the prop C, then becauſe (by art. 244) w acts at G, we'll (by theo. 185) have $w = \frac{Dd \times c}{DG}$ and $w = \frac{qC \times a}{CG}$ whence, $Dd \times c \times CG = qC \times a \times DG$, but $CG = CD + DG$, ſo $Dd \times c \times \overline{CD + DG} = qC \times a \times DG$, and therefore $\frac{Dd \times c \times CD}{qC \times a - Dd \times c} = DG$. Theo. 187.

273. To find the center of gravity of any ſurface or ſolid, as A B C (fig. 150), let G = its center of gravity A B, the diameter of gravity biſecting all the particles of matter b, c, d, g, h, i, l, &c. in the ſaid body, and then by theorem 185, we have $AG = \frac{Ab \times b + Ac \times c + Ag \times g + Ah \times h + \&c.}{b + c + g + h + \&c.}$, now if any one of theſe variable quantities A b, A c, &c. ſuppoſe A b, be called e, and the body S, and if $a + eb + eec + eeed = y$, or $= yy$, the equation of the body &c. then (by prob. 185, or 186) $b + c + d + g + \&c.$ the ſum of the particles of matter in the body will be $= ae + \frac{1}{2}eeb + \frac{1}{3}eeec + \frac{1}{4}e^4d$, and $Ab \times b + Ac \times c + Ag \times g + \&c. = \frac{1}{2}aee + \frac{1}{3}beee + \frac{1}{4}ce^4 + \frac{1}{5}de^5$, whence $AG = \frac{\frac{1}{2}ae + \frac{1}{3}bee + \frac{1}{4}ceee + \frac{1}{5}de^4}{a + \frac{1}{2}be + \frac{1}{3}cee + \frac{1}{4}deee}$. Theo. 188.

This general theorem may be laid down in a rule, and applied universally to the centers of gravity, as those in prob. 185, and 186, are to contents.

274. If e be the diameter of gravity, y an ordinate &c. bisected thereby, and if it be $ce^n = y$ or $= yy$, then by the last theo. $AG = \frac{ce^{n+2}}{n+2} \div \frac{ce^{n+1}}{n+1} = \frac{n+1}{n+2}e$, and by taking $n = 2, 1, \frac{1}{2}$, respectively, we'll have

In the common parabola in the conoid of this parabola in the cone or pyramid in a plane triangle	$\frac{n+1}{n+2}e =$ $AG =$	$\frac{3}{5}e$, $\frac{2}{3}e$, $\frac{3}{4}e$, $\frac{2}{3}e$,

In the Δ A C D, (fig. 151) if you draw a line from any of the Ls, A, C, D, to the middle of its opposite side, $\frac{2}{3}$ of that line (by what is done above) must fall at G, the center of gravity of the Δ, whence it follows, that if the 3 sides of any plane Δ be bisected by lines drawn from their opposite angles, that any two of these lines will meet in the Δs center of gravity, which will be $\frac{2}{3}$ of any bisecting line distant from its angular point, a hollow cone has the same equation with a Δ, and therefore its center of gravity will be at $\frac{2}{3}$ of its slant length, distant from its vertex, in like manner you may compute the center of gravity of any other hollow solid.

275. If G, be the center of gravity of the sector S A B, let $SA = r$ the radius $a =$ the chord A B, $e =$ any abscissa S a, $c =$ the arch A Q B (fig. 151) then as $r : e :: a : \frac{ae}{r} =$ chord a b, and as $r : e :: c : \frac{ce}{r}$ $=$ arch a q b, now (per theo. 188) $e \times \frac{ae}{r}$, and divided by the index of e plus 1, gives $\frac{eea}{3r}$ for the dividend, and $\frac{ce}{r}$, divided by the index of $e + 1$, gives $\frac{ce}{2r}$ for a divisor so $\frac{eea}{3r} \div \frac{ce}{r} = \frac{2ae}{3c}$ (when $r = e$) $= \frac{2ra}{3c} = SG$, and $a = 2r$, then $\frac{2ra}{3c} = \frac{aa}{3c} = \frac{4rr}{3c} =$ the distance of the center of gravity of a semi-circle from the circles center.

Note. The reason for using two equations $\frac{ea}{r} = ab$ and $\frac{ce}{r} =$ a q b, in this art. is clear from the demonstration of theo. 188, where all the particles of matter are supposed to be reduced to the diameter

of gravity in ſtreight or ⊥ lines. but the area of the ſector is compoſed of an infinite number of arches ſimilar and parallel to A Q B &c. this obſerve in any figure whoſe baſe is a curve.

276 If the ſemi-parabola S A D (fig. 152) be ſuſpended upon the ſemi-ordinate A D = a, d a = y, d D = a B = e, and parameter = unity, then per conicks, a a = S D and y y = S d. but per fig. SD — Sd = a B, i. e. aa — yy = e, which (by theo. 188) multiplied by y, and divided by the index of y plus 1, gives $\frac{2aay - yyy}{4}$, for a dividend, and divided by the index of y plus 1, gives $\frac{3aa - yy}{3}$, for a diviſor, whence $\frac{2ayy - yyy}{4} \div \frac{3aa - yy}{3} = \frac{6aay - 3yyy}{12aa - 4yy} = GD$, the center of gravity of the parabolic trapezia a d D A, and becomes $= \frac{3}{8}y$, when a = y, the center of gravity of the $\frac{1}{2}$ parameter S A D.

277. If A Q B, the ſegment of a circle, be ſuſpended upon its verſed ſine Q C, you may find G its center of gravity, by having given R and C, theſe of the ſector S A Q B, and △ S A B, (fig. 153), thus, let D = the area or weight of the △, E = that of the ſector, F = that of the ſegment, c = arch A Q B, a = chord A B, r = radius S A, e = ⊥ S G of the △, z = G C, the diſtance between the centers of gravity of the ſector and ſegment, then by art. 244, and theo. 184, $D \times CR = F \times CG$, or $z = \frac{D \times CR}{F}$, but E — D = F, ſo $z = \frac{D \times CR}{E - D}$ alſo, $D = \frac{1}{2}ae$, $E = \frac{1}{2}rc$ and $\frac{2ra}{3c} - \frac{2}{3}e = CR$, by art. 275, and 274. which put in $z = \frac{D \times CR}{E - D}$, inſtead of D, E and C R, gives $z = \frac{2raae - 2acee}{3rcc - 3aec}$, from which take the ⊥ (S G) e, and there leaves $\frac{2raa - 3rce + aee}{3rc - 3ae}$, for the diſtance of G, from the middle of the chord A B. After the ſame method, may be had the center of gravity of other compound figures, or of parts of figures.

278. If we take b = an ordinate or ſpace in the middle of the body, a = one at the leſſer, and c = one at the greater baſe, each equally diſtant from b, and all 3 biſected by e, the diameter of gravity, we'll have the diſtance of the center of gravity from the leſſer baſe equal $\frac{ec + 2eb}{c + 4b + a}$ theo. 189, as will appear by applying prob. 190 to theo.

188. This theo. 189 anſwers for the centers of gravity as theo. 138, doth for contents, as for example. Let the center of gravity of the ſolid in theo. 139, be required, when it is ſuſpended upon e its axis. Here $a = dt$, $b = \frac{1}{4} : TD + dT + dt + Dt :$ and $c = TD$, which ſubſtituted in theo. 189, for a, b and c, gives $: \frac{3TD + Td + td + tD}{2TD + Td + 2dt + tD} : \times \frac{1}{2} e = AG$ (fig. 150) and if this ſolid becomes the fruſtum of a cone or pyramid, then $T = D$ and $t = d$, and then $AG =: \frac{3DD + 2Dd + dd}{DD + Dd + dd} : \times \frac{1}{4} e$, and if a whole cone or pyramid then $d = o$, and then $AG = \frac{3DDe}{4DD} = \frac{3}{4} e$, if there be a plank of equal thickneſs length e, breadths at the two ends $b = a$ and c, then $b = \frac{a + c}{2}$ which put in theo. 189 for b, gives $AG = \frac{2ae + ce}{3a + 3c}$, which is the ſame thing as if e be the diameter of gravity, and ſlant length of a hollow fruſtum of a cone or pyramid, whoſe peripheries at its two parallel baſes is a and c, and if $c = o$, it becomes $AG = \frac{2ae}{3a} = \frac{2}{3} e$, for the hollow cone or pyramid &c. for other figures.

279. If the diameter of gravity be the ſame with the axis of the ſolid viz. each $= e$, then by theo. 189, $AG \times : 4b + c + a : = ec + 2eb$, whence $: a + 4b + c : \times \frac{e}{6} =: \frac{c + 2b : \times ee}{6AG}$: but (by theo. 138) $: a + 4b + c : \times \frac{e}{6}$ is = the body, ſo that if either the body or center of gravity be given, the other of them may be found. Theo. 190.

280. If P C (fig. 148) one part of a lever, fixt in a wall at C, the force to move the ſaid part will be $P \times PC$, by theo. 184.

281. It is known, that a line and plumet is only at reſt, when the line is ⊥ to the horizon, whence, and by art 244, if the beam B C (fig. 154) be ſuſpended by a cord F H, thro' its center of gravity G, the beam will reſt in any poſition when the cord F H, is perpendicular to the horizon, therefore, if the chord F H be taken away and the beam hung by the cords F C and F B, or by any two cords E B and I C, in the ſame directions, it muſt alſo be at reſt, the ſame holds if it be hung by the cords E B and h C, becauſe theſe cords meet at H in the ſaid ⊥ line, whence, and by art. 234, whether a body be

ustained by two ropes, F B and I C, or F B and F C, or e B and h C, or by two posts A B, and D C, or q B and p C, or by two planes ⊥ to A B and D C, the body can only be at rest, when these intersect in the plumb line F H, passing thro' the center of gravity G of the body, if from any point H in the line F H, we draw H I parallel to F B then the whole weight the pressure at C, the thrust or pressure at B, are respectively as F H. F I and I H and in these directions, for the body may be supposed to be supported by two planes q B and p C ⊥ to B A and C D, which planes re-act against the body in these directions, and so is the same thing as if the body was sustained by the two ropes B F and C F, and in either case, the said ratio is true, by art 256.

282. From what is said above, it follows, that a body so sustained cannot be at rest, if the ropes F B and F C, &c do not intersect in the said ⊥ line F H, and that a body will always be at rest, when hung or supported by a line ⊥ the horizon, passing thro' its center of gravity.

283. If there be any number of beams AB, BC, CD, DE, &c to be kept in equilibrio by their own weight, when set one to the end of another upon the horizon A C, as you see in fig. 155, whose centers of gravity are at the points a, b, c, d, e, &c. and weights q, r, s, t, &c. the weights upon the angles, B, C, D, with which these beams press each other, are $A = \frac{Aa \times q}{AB} + \frac{bC \times r}{BC} = B$, $C = \frac{Bb \times r}{BC} + \frac{cD \times S}{CD}$, $D = \frac{cC \times S}{CD} + \frac{dE \times t}{DE}$, &c. for (by theo. 184) as $AB : Aa :: q : \frac{Aa \times q}{AB}$, the weight of the beam A B, against the ∠B, and as $CB : Cb :: r : \frac{Cb \times r}{CB}$ the weight of the beam C B, against the said ∠ B, &c. whence, B, C, D, &c. are = the weights against their respective angles, thro' these ∠s, let r l, S m, t p &c. be drawn perpendicular to the horizon A F, and produce D C to meet l r in r, then (by art. 256) as S∠ A B C : S ∠A B r :: weight B : force in direction B C equal $\frac{B \times S.\ ABr}{S.\ ABC}$ in like manner force in direction $CB = \frac{C \times S.\ DCS.}{S.\ BCD}$, but to preserve the equilibrium, the force in direction C B, must be = that in direction B C, viz. $\frac{C \times S.\ DCS}{S.\ BCD} = \frac{B \times S.\ ABr}{S.\ ABC}$, whence, as $B : C :: \frac{S.\ ABC}{S.\ ABr} : \frac{S.\ BCD}{S.\ DCS}$, and by the same way of reasoning as C

; D : : $\frac{S.BCD}{S.BCS}$: $\frac{S.CDE}{S.EDt}$, therefore, as weight B : weight D : : $\frac{S.ABC}{S.ABr \times S.BCS}$: $\frac{S.CDE}{S.DCS \times S.EDt}$: : $\frac{S.ABC}{S.ABl \times S.CBl}$: $\frac{S.CDE}{S.CDp \times S.EDp}$, that is, if the beams be in equilibrio, the weight upon any angle C, must be as $\frac{S.BCD}{S.mCB \times S.mCD}$, so if the positions of all the beams be given, the ratios of the weights upon the ∠s may be found, and if one weight be given, all the other weights may be found.

284. Also, (by art. 256) the weight C, the forces in directions CB and CD, are as rB, CB, and Cr respectively, but as CB to Cr : : S. ∠ CrB, or SCD, or rCm : S. rCB : : co sine elevation of CD : co-sine elevation of CB, : : secant elevation of CB : secant elevation of CD, because the secants are inversely as the co-sines, whence, the force or thrust at C in direction CB, or at B in direction BC, is as the secant of the elevation of the beam or line BC above the horizon.

285 Draw Cq and Dn parallels to DE and CB, and let CD express the force in direction CD, or DC, then Cq is the force in direction DE, and Dn, the force in direction CB, but by theo 156, Dq, or the weight D, is = the two forces DC and Cq, also Cn, or the weight C is = the two forces CD and Dn i. e. the weights on C and D to preserve the equilibrium will be as Cn to Dq, whence if all the weights are given with the position of two lines, (CD and DE) the position of all the other lines CB, BA, &c may be found.

286. If the weights were to act upwards in the direction mC, pD, &c. or the figure AB . . DEF, turned upside down, and the weights remain the same, and the points A and F be fixt as before, its plain (by art. 281) that the whole figure will be exactly as before it was so turned, and that whether the lines AB, BC, CD, &c. be flexible, or inflexible chords or beams.

287. Whence it follows, that if any chords of = lengths be stretched to the same degree of curvature, the stretching forces as the weights of the chords.

288. If Ba be an inclined beam or wall (fig. 156) supported by a prop Aa ⊥ the horizon, let C be the beams center of gravity, w= its weight, e = sine ∠ BaP, or co-sine ∠ aBP, y = the weight borne by aP, then w − y = the weight at B, draw CmM, aP, now (by theo. 184) mP × y = Bm × w − y : so ; mB + mP ; (BP) × y =

$Bm \times w$, but by similar Δs, as $Bm : BP :: BC : Ba$, whence $BC \times w = Ba \times y$, and (by theo. 48) as radius $1 : Ba :: e : BP$, ergo, $Ba = \frac{BP}{e}$, whence $\frac{BP}{e} \times y = BC \times w$, or $y = \frac{BC \times w \times e}{BP}$, which will be the least, when the divisor BP is greatest, and because BP is always $\perp$ the prop it must (by prob 15) be greatest when $\angle BaA = \angle BAa$, and then it becomes $BQ \perp$ the prop aA. Theo. 191.

Again, let $c = BC$, $v =$ prop aA, then because BQ is $\perp aA$, and $Ba = BA$, therefore, aA and $\angle aBA$ are each bisected by BQ, now, suppose $mA = mB$, then $\angle amA = 2\angle aBA$, and per similar Δs, as $BP : Pa :: Bm : Cm$ (viz.) as $1 + z : \sqrt{:1 - zz} :: 1 : \frac{\sqrt{1-zz}}{1+z} = \sqrt{:\frac{1-z}{1+z}} := Cm, =$ tangent $\angle CBm$, (z being $= mP =$ co-sine $\angle amP$, radius $= 1$) so if $e =$ co-sine of any $\angle$, the tangent of $\frac{1}{2}$ that $\angle$ is $= \sqrt{:\frac{1-e}{1+e}}$: so (by theo. 47) as $\sqrt{:\frac{1-e}{1+e}} : \frac{1}{2}v :: 1 : \frac{v}{2}\sqrt{:\frac{1+e}{1-e}} = BQ$, which written in $\frac{BC \times w \times e}{BQ}$, for BQ, we get $y = \frac{2cew}{v}\sqrt{:\frac{1-e}{1+e}}$: now if y be a maximum yy, will also be one two, viz. $yy = \overline{\frac{2cw}{v}}\Big|^2 \times \frac{ee - eee}{1+e} = \frac{See - Seee}{1+e} = m$, (putting $S = \square \frac{2cw}{v}$ which (by theo. 149) is already found $= 0{,}6180 = e$. Theo. 192.

288. If CDE (fig. 157) be an upright wall whose weight is $= y$, and ABC, another wall &c. of the same height AB, with a slope side BC, next to the wall of equal thickness CDE, and the vacant ΔBCD between these two walls be filled with stones, earth, sand &c. of w weight, let P be that part of w which is sustained by the wall CDC, then its evident the ΔBCD, as a solid body would slide down the inclined and upright planes BC and DC in the same time, so (by theo. 183) as $BC : w :: DC : \frac{DC \times w}{BC} = Q$, the weight sustained by BC, and therefore, $w - Q = w - \frac{DC \times w}{BC} = P$, that sustained

by D C, but as $BC : \frac{DCcw}{BC}$ (the force in direction B C) : : A C ; $\frac{AC \times DC \times w}{\Box BC}$ the force in direction A C ⊥ to the wall C D E and may be considered as acting against the point L in direction G L || A C because G is the △s center of gravity (C L being $= \frac{2}{3}$ C D and G L $= \frac{1}{3}$ D B) now the sides of the wall C D E, being parallel its center of gravity is at R in the middle of C E, make R F = C L, then its plain, that a power lifting at F, the end of the lever F R, upon the prop R is = the same force acting at L in the direction G L, to overturn the wall E D C upon E, so F R × y, the force with which the wall resists is $= AC \times D \times w \div \Box BC$, the ⊥ force of the △ B C D against the wall C D E, but $P = w - \frac{DC \times w}{BC}$ is the force or weight sustained by the wall C D E in direction D C, that is y is the pressure upon R and P that upon C, let the point e, be the center of gravity of these two pressures, then, $Ee \times \overline{y + P} = AC \times DC \times w \div \Box BC$ in a case of equilibrium. Theo. 193.

But if the wall be only pressed in direction G L, then $RE \times y = \frac{AC \times DC \times w}{BC}$.

289. To find Q, the center of gyration of any system of bodies (fig. 158) A, B, C, connected together, and revolving about a center S. 1. Let m = the force or momentum of the whole system, at any point O, or Q, v = the velocity of any point in it at 1 distant from S, and A, B, C, the weights of their respective bodies, A, B, C, now its evident the velocity of any point in the system, will be as the distance of that point from the point S, so the velocities of the points A, B, C, will be v × S A, v × S B, or v a, v b, v c (putting a = S A, b = S B, c = S C), but c must be negative, because it and the common center of gravity of the system are on different sides of S, then (by theo. 155) v a A, v b B, — v c C, will be the forces of these bodies acting at A, B, C, and (by art. 280) as SQ : a : : v a A : $\frac{vaaA}{SQ}$ = the force of A acting at Q, and for the same reason the forces of B and C there, will be $\frac{vbbB}{SQ}$, and $\left(\frac{-c \times -vcC}{SQ}\right) \frac{vccC}{SQ}$, now its plain the sum of all these forces must be = m, i. e. $\frac{v \times : Aaa + Bbb + Ccc :}{SQ} = m.$

If we suppose all the bodies placed at Q, their joint force, there will be $v \times SQ \times : A + B + C$, which taken $= m$, we get $v \times SQ \times : A + B + C : = \frac{v \times : Aaa + Bbb + Ccc :}{SQ}$, whence, $SQ = \sqrt{:} \frac{Aaa + Bbb + Ccc}{A + B + C}$: shewing how far Q, the center of gyration is from S, the point of suspension, at which point Q, the same force can stop, or produce, the motion of the whole system, as can stop or produce the motion of a single body ($= A + B + C$) placed at Q.

290. If the system be at rest its force will be $A + B + C$, pressing with that weight on its common center of gravity which let be at the point G in the system, whose velocity being $v \times SG$, this force will then be $v \times SG \times : A + B + C$: put this $= m$, and take the point O instead of Q, then $v \times SG \times : A + B + C : = \frac{v \times : Aaa + Bbb + Ccc :}{SO}$ whence, $SO = \frac{Aaa + Bbb + Ccc}{SG \times : A + B + C :}$, the distance between S the point of suspension and O the center of percussion, or point in the system where the stroke is greatest possible, and appears to be a third proportional to SG the center of gravity, and SQ the center of gyration, that is, $SG : SQ :: SQ : SO$, since the force divided by the weight, gives the velocity, if we divide $\frac{v \times : Aaa + Bbb + Ccc :}{SO}$ by $A + B + C$, and put the quote $= v \times SG$, the velocity of the center of gravity, we'll have $SG = \frac{Aaa + Bbb + Ccc}{SO \times : A + B + C :}$, whence, $SO = \frac{Aaa + Bbb + Ccc}{SG \times : A + B + C :}$, the distance between S, and O the center of oscillation, and appears to be every way the same with the center of percussion last found.

It is the same thing whether the bodies be all in one right line CSAB, or out of it in the same plane CCABB, as is plain by what goes before, suppose them all joined by the line CSAB, and put $p = GA$, $q = GB$, $r = GC$, $g = GS$, and then a, b, c, being as before we'll have,

$$Aaa = \square : g - p : \times A = ggA - 2gpA + ppA$$
$$Bbb = \square : g + g : \times B = gg B + 2gqB + qqB$$
$$Ccc = \square : r - g : \times C = gg C - 2grC + rrC.$$

In these 3 equations, if G be the common center of gravity, B will be in equilib. with C and A, or $q B = p A + r C$, so the three terms $+ 2gpA$, $+ 2gqB$ and $- 2grC$, destroy each other, and the sum of the rest will be, $Aaa + Bbb + Ccc = gg \times : A + B + C : + ppA + qqB + rrC$, divide each side by $g \times$, $A + B + C$: and it will be $\frac{Aaa + Bbb + Ccc}{: A + B + C : \times g} = \frac{App + Bqq + Crr}{: A + B + C : \times g} + g =$ (as before) S O, whence, $\frac{App + Bqq + Crr}{: A + B + C : \times g} = SO - g = OG$, now its plain per fig. that if $OA = p$, $OB = q$, $OC = r$, OS must be $= g$, for $GS - GA = OS - OA = SA = a$, &c. so if g in the last equation stand for S O, we'll get $SO = \frac{App + Bqq + Crr}{: A + B + C : \times OG}$, whence it is evident, that if the plane of the motion be the same, and either O, or S, be made the point of suspension, the other of them will be the center of oscillation, put $SO = S$, OG, or $SG = g$, then $S - g$ being $= \frac{App + Bqq + Crr}{: A + B + C : \times g}$, we'll have $Sg - gg = \frac{App + Bqq + Crr}{A + B + C}$ and if $Sg - gg$ be made a maximum, we'll find $2g = S$, and that the oscillations are made in the least time possible when $OG = SG = \sqrt{: \frac{App + Bqq + Crr}{A + B + G} :}$ where $p = GA$, $q = GB$, $r = GC$.

221. Since the velocity of the system when the point G moves in a right line $\perp$ S O, is to its velocity when it revolves about the point S, as S G to S O (the momentum in each case being the same) it follows that O G the difference of velocities is as the force pressing on S in direction $\perp$ to S O, and consequently the other part S G is spent in accelerating the motion of the system, that is, as $SO : OG ::$ force of gravity in direction $\perp$ S O : force acting at S in the like direction, this also holds true when the axis of suspension has a motion or when the system or body has a progressive motion as well as an angular one, as a cord wrapped about any body, or system of bodies, and one end of the cord made fast, then the body let go it will descend rolling, stretching the cord with a force as O G, and its descent to that of falling freely as S G to S O, i. e. as $SO : SG ::$ falling freely to rolling, and $:: OG :$ weight the cord sustains.

292. Because (by art. 272) $SG \times : A + B + C : = Aa + Bb - Cc$, our equation for S O will become $SO = \frac{Aaa + Bbb + Ccc}{: A + B + C : \times SG}$

$= \frac{Aaa + Bbb + Ccc}{Aa + Bb + Cc}$. Theo. 194. Where if $A = o$, and $Bb = Cc$, it will be $SO = \frac{Bbb + Ccc}{Bb + Cc} = \frac{Bbb + Ccc}{o} =$ infinite, viz. the pendulum will not vibrate but reſt in equilibrio upon S as a center.

Alſo, in this caſe $SG = \frac{Bb - Cc}{C + B} = \frac{o}{B + C}$, that is, the center of gravity will be at S, and ſo produce an equilibrium.

By comparing the laſt theorem with theo. 188, and art. 273, we'll have $SO = \frac{n + 2}{n + 3}e$, inſtead of $AG = \frac{n + 1}{n + 2}e$, ſo $SO = \frac{5}{7}e$, in the common parabola $= \frac{3}{4}e$ in the plane triangle, $= \frac{4}{5}e$ nearly, in the cone or pyramid, $= \frac{5}{4}$ the radius in the plane of a circle $= \frac{2}{3}e$ in a right line, rectangle, ſmall priſm, or cylinder, $= d + \frac{2rr}{5d}$ in a ſphere, where $r =$ its radius, $d =$ diſtance between its center and point of ſuſpenſion, e being the axis of the body which is ſuſpended at the vertex.

PROBLEM CXCVIII.

To determine the ſtreſs and ſtrength of beams &c.

293. Streſs, any force acting againſt a beam to preſs or break it, ſtrength, is the force or power with which the beam reſiſts the ſtreſs.

294. The beams &c. here meant are ſuppoſed to be homogeneous, and placed parallel to the horizon, whether they be fixt at one end as into a wall &c. or ſupported at both ends as upon props &c. except it be ſaid to the contrary. Theſe beams may be of wood, ſtone, iron, &c.

295. Let BCDQ (fig. 159) be a beam with one end BC fixt into a wall &c. and a weight w ſuſpended at DQ the other end of it, then (by art. 280) $AQ \times w$, is as the ſtreſs at any point A, that is, the ſtreſs on this account is as the length, but if e denote the weight of the beam AD, $\frac{1}{2} AQ \times e$, will be as the ſtreſs at the ſame point A, (for the weight of every body) acts in its center of gravity (and this center is in the middle, becauſe the beam is of equal breadth, and of = depth throughout.

296. In ſuch beams, the length AQ, will be as the weight e, and ſo AQ may alſo be put $= e$, and then if $w = e$, we'll have ee for $AQ \times w$, and $\frac{1}{2}ee$, for $\frac{1}{2} AQ \times e$, hence the ſtreſs on A when the weight is equally diffuſed over all the beam, is to the ſtreſs there when

the weight is ſuſpended at the end D Q, as $\frac{1}{2}$ e e to e e, or as 2 to 1, and in general, if g be the diſtance between B C, the fixt end, and the beam's center of gravity, or g = n e, then n e g + e w, is the whole ſtreſs of the point C, on both accounts, e being = C Q, the axis of the beam, let it be of what form it will, and q its weight, alſo w = the weight ſuſpended at D Q.

297 It is manifeſt that the force of w upon the point B, is the ſame if there be another beam = to the beam C D, with one end fixt, and the other end touching D Q, but if it be join'd to D Q, that ſo both beams bear the weight w, then the ſtreſs at B will be but half of what it was before, hence, the ſtreſs on a beam fixt at one end is equal the ſtreſs on one of a double length when ſupported at both ends; the ſame is true if the weight be equally diffuſed over all the beam, as is the caſe when it bears only its own weight, and for the ſame reaſons, the ſtreſs on a ſquare plate &c. ſupported by two ſides, is = the ſtreſs on one of a double ſide or length, when ſupported by all its four ſides.

298. If A a a B (fig. 159) be a beam ſupported at each end, by two props a, a, and bearing a weight at any point P, proportionable to its length, then B P and A P will be as the weights on the ſegments B P and A P, ſo (art. 295) weight A P × diſtance B P, or weight B P × diſtance A P, viz. A P × B P = ſtreſs on the point P, but if the weight be not ſuſpended, but equally diffuſed over all the beam then (art. 297) the ſtreſs at any point P will be as $\frac{AP \times BP}{2}$.

299. If e = B D the length, and b = the breadth of the beam B Q (fig. 159) and if the weight and length continue the ſame, while b varies, its plain the ſtreſs at B, will be inverſely as b, but (art. 295) it is directly as e, ſo the ſtreſs, is as $\frac{e}{b}$, and if there be another beam of the ſame depth, length = E and breadth = B, and if it be as B : E :: b : e, then E b = e B, or $\frac{e}{b} = \frac{C}{B}$. Hence, the ſtreſs on any two ſimilar ſections of the ſame thickneſs, and matter, is equal, if they are alike ſupported.

300. Since the ſtreſs is as the length when the beam is placed horizontally, therefore, if the beam a B (fig. 156) ſtands a ſlope, let fall the ⊥ a P upon the horizontal line A B, then the ſtreſs on the beam B a in this poſition, muſt be as B P, but by trigonometry, as radius (1) : B a :: ſine ∠ B a P (c) viz. co-ſine ∠ of inclination a B P : B a × c, i. e. the ſtreſs on any beam &c. horizontally placed, multiplied by the

co-ſine of any inclination, give the ſtreſs on it when it hath that inclination, and ſo much for ſtreſs.

301. To find the lateral ſtrength, in any part of any beam B C Q D (fig. 160) as at G. Let G n A, be the ſection of the beam there, ſuppoſing it to be cut right a croſs, G A being perpendicular to the horizon, or the beam parallel thereunto, put G A = d, the depth or abſciſſa of the ſection b = A n, its ordinate or breadth.

Note. That ſide of the beam perpendicular to the horizon is called d, the depth, and that ſide of it parallel thereunto is called b, the breadth, ſuppoſe the depth G A to be divided into an infinite number of equal parts, by ſmall threads (n A) paſſing thro' it parallel to the horizon, ſuppoſe a y any one of theſe threads or fibres, as an ordinate to the ſmall variable diſtance G a = e, and e^n = y a, let the ſtrength or reſiſtance of a fibre at A be = 1, then, as d : 1 : : e : $\frac{e}{d}$, ſtrength of a fibre at a; for when the beam is broken, the parts F F (fig. 161) which before were together, are ſeperated at the diſtance E F, and the parts at D not parted at all, now if this diſtance E F, or ſtrength of a fibre at that place be put = 1, then its plain per figure, as D E (d) : E F (1) : : D e (e) : e f, the ſtrength of a fibre at f, but the number of fibres between A and a, are = e, and the equation of the ſection is $e^n = y$, the product of theſe 3, is $\frac{e}{d} \times e^n = \frac{e^{n+2}}{d}$ which by theo. 78, is $\frac{e^{n+3}}{d, n+3,} = \frac{d\,b\,b}{n+3}$, (when e = d and y = b) = T, the total ſtrength of the ſection, G A n, whence, in the common parabola, rectangle, triangle, n = $\frac{1}{2}$, n = 0, n = 1, then T = $\frac{2}{7}$ b d d T = $\frac{1}{3}$ b d d, T = $\frac{1}{4}$ b d d, if the ſection be an ellipſis, or circle, a = A G, that diameter of it perpendicular to the horizon, c = the other diameter parallel thereunto, $\frac{c}{a}$ = D, e = A a, the depth of any ſegment, y = y a, its ſemi-chord, then y = D $\sqrt{}$: a e — e e : ſo $\frac{e}{d} \times e \times D \sqrt{}$: a e — e e : is as the quantity generating the ſtrength of the ſection, this compared with theo. 78, or the general ſeries in art. 215, we'll have T = 0,2451 D a a a, in the ellipſis, and T = 0,2451 a a a in the circle, &c.

Note. Its plain the ſeparating parts, are at the upperſide of the beam when fixt at one end.

302. Let g and S be the distances of the centers of gravity and oscillation of the section G A n from G, a = its area, d = A G, then (art. 292) $g = \frac{n+1}{n+2}d$, and $S = \frac{n+2}{n+3}d$, also, (theo. 78) $a = \frac{db}{n+1}$, the product of these 3 equation is $gSa = \frac{bddd}{n+3}$, whence, $\frac{gSa}{d} = \frac{bdd}{n+3} = T$, as in the last article, now if we take $Ga = \frac{gS}{d}$, then $Ga \times a = \frac{gSa}{d} = T$, the total strength of the section G A n, in which the point a is that where all the fibres being collected, do act with their whole strength, this point a in a rectangle viz. $GA = \frac{1}{3}d$, for $g = \frac{1}{2}d$, and $S = \frac{2}{3}d$, so $gS \div d = \frac{1}{3}d$, in a circle whose diameter = A G, $g = \frac{1}{2}d$, $S = \frac{5}{8}d$, so $Ga = gS \div d = \frac{5}{16} = \frac{1}{3}d$, fere, in the periphery of a circle (the beam being a hollow cylinder) $g = \frac{1}{2}d$, $S = \frac{3}{4}d$, so $Ga = \frac{3}{8}d = \frac{1}{3}d$ nearly, as in the two former, in a square suspended by one of its angles, diagonal = d, $g = \frac{1}{2}d$, $S = \frac{5}{9}d$ nearly, so $Ga = \frac{5}{18}d$.

303. Hence if a power v, be applied to G, as the end of a lever G a, upon the prop a, then $Ga \times v$, is as the force to pull the beam asunder lengthwise, which let be $= GD\,(e) \times w$, the force to break it laterally, i. e. (because $Ga = gS \div d$) $gS \div d = ew$, whence $v = \frac{ewd}{Sg}$. But if z = the weight of the beam, G = the distance between its fixt end and its center of gravity, then $Gz + ew = gSv \div d$, or $v = \frac{dGz + edw}{gS}$, and if y = the weight of the beam below where it is pulled asunder, or that part of its weight which any ways assists in pulling it, then $dGz + edw = : v - y : \times gS$.

Note. If the beam be fixt at one end, g and S must be measured from the under side C Q, but if supported at both ends they must be measured from the upper side thereof B D, and then e taken for half the length, and w half the weight &c. (art. 291) also, if the prop or fulcrum, dent into the beam, it will diminish its strength by so much as d is shortened, by that denting.

304. If B G A C (fig. 160) be a cylinder twined, or twisted about its axis T e T, let e P = e R = d, periphery of the section P R R R = b, e p = any small variable radius e, then if the power of a particle at P to twine about the section be 1 as e P : 1 :: e p : $\frac{ep}{eP} = \frac{e}{d}$, the

power of a particle at p, the number of ſuch particles between e and p are $=$ e, and as d : b : : e : $\frac{eb}{d}$ the periphery of, or particles in p r p r, whence $\frac{e}{d} \times e \times \frac{eb}{d} = \frac{eeeb}{dd}$ (by theo. 78) $= \frac{eeeb}{4dd} =$ (when e $=$ d) $\frac{1}{4}$ d d b, the total twiſting ſtrength of the whole ſection PRPR, and (art. 301) is equal the lateral ſtrength of a triangle $\perp =$ d, and baſe || horizon $=$ b.

Note. This article does not agree ſo well with wood as with metal, for the texture of wood is not the ſame in length and breadth as it is in metal, where the power of ſpliting and breaking is the ſame. Theſe are the principles of ſtrength.

305. If a beam &c. is to be equally ſtrong throughout, or the ſtrength in any part of it proportional to the ſtreſs on that part. Firſt, if it be fixt at one end, and a weight ſuſpended at e diſtance from that end, the ſtreſs there (art. 295) is as w, and (art. 301) the ſtrength as b d d $\div \overline{n+3}$, therefore e $\propto$ b d d (w and n $+$ 3 being conſtant) and if the upper, or under ſides of the beam be a rectangle, or b every where the ſame, then e $\propto$ d d, the equation of the common parabola, if b $=$ d, then e $\propto$ d dd, that of the cubic parabola, but if the weight preſs uniformly over all the beam then (art. 296) e e $\propto$ b d d, and if b be every where the ſame, then e e $\propto$ d d, or e $\propto$ d, the equation of the plane triangle if b $=$ d, then e e $\propto$ d d d, the equation of the ſemicubic parabola.

306 If A B (fig. 159) be a beam ſupported at each end, and bearing a weight on any variable point, or uniformly on all parts of it then (art. 298) the ſtreſs at P is A P $\times$ B P, ſo A P $\times$ B P $\propto$ b d d $\propto$ (if b $=$ d) d d d $\propto$ (if b be conſtant) d d, the equation of the ellipſis, but if a weight be fixt at any point E, then B P $\propto$ b d d, $\propto$ (when b $=$ d) d d d, then A Q E and B Q E are two cubic parabola's, but if b is conſtant, then B P $\propto$ d d, then A Q E and B Q E are two common parabola's.

307. Let there be two beams q and Q of the ſame matter, lengths e and E, each fixt at one end, depths there d and D, breadths b and B, weights of the beams w and z, weights borne at their unfixt ends v and y, the diſtance from their fixt ends to there centers of gravity g and G, then their total ſtreſſes (art. 296) will be g w $+$ e v and G z $+$ E y, and ſtrengths (art. 301) b d d $\div$: n $+$ 3 : and B D D $\div$: m $+$ 3 : now if the ſtrength muſt be proportional to the ſtreſs, then (putting p $=$ 1 $\div \overline{n+3}$ and P $=$ 1 $\div \overline{m+3}$ as p b d d : g w $+$ e v : : P B D D : G z $+$ E y, but becauſe

302. Let g and S be the diſtances of the centers of gravity and oſcillation of the ſection G A n from G, a = its area, d = A G, then (art. 292) $g = \frac{n+1}{n+2}d$, and $S = \frac{n+2}{n+3}d$, alſo, (theo. 78) $a = \frac{db}{n+1}$, the product of theſe 3 equation is $gSa = \frac{bddd}{n+3}$, whence, $\frac{gSa}{d} = \frac{bdd}{n+3} = T$, as in the laſt article, now if we take $Ga = \frac{gS}{d}$, then $Ga \times a = \frac{gSa}{d} = T$, the total ſtrength of the ſection G A n, in which the point a is that where all the fibres being collected, do act with their whole ſtrength, this point a in a rectangle viz. $GA = \frac{1}{3}d$, for $g = \frac{1}{2}d$, and $S = \frac{2}{3}d$, ſo $gS \div d = \frac{1}{3}d$, in a circle whoſe diameter = A G, $g = \frac{1}{2}d$, $S = \frac{5}{8}d$, ſo $Ga = gS \div d = \frac{5}{16} = \frac{1}{3}d$, fere, in the periphery of a circle (the beam being a hollow cylinder) $g = \frac{1}{2}d$, $S = \frac{3}{4}d$, ſo $Ga = \frac{3}{8}d = \frac{1}{3}d$ nearly, as in the two former, in a ſquare ſuſpended by one of its angles, diagonal = d, $g = \frac{1}{2}d$, $S = \frac{5}{9}d$ nearly, ſo $Ga = \frac{5}{18}d$.

303. Hence if a power v, be applied to G, as the end of a lever G a, upon the prop a, then $Ga \times v$, is as the force to pull the beam aſunder lengthwiſe, which let be $= GD\,(e) \times w$, the force to break it laterally, i. e. (becauſe $Ga = gS \div d$) $gS \div d = ew$, whence $v = \frac{ewd}{Sg}$. But if z = the weight of the beam, G = the diſtance between its fixt end and its center of gravity, then $Gz + ew = gSv \div d$, or $v = \frac{dGz + edw}{gS}$, and if y = the weight of the beam below where it is pulled aſunder, or that part of its weight which any ways aſſiſts in pulling it, then $dGz + edw = : v - y : \times gS$.

Note. If the beam be fixt at one end, g and S muſt be meaſured from the under ſide C Q, but if ſupported at both ends they muſt be meaſured from the upper ſide thereof B D, and then e taken for half the length, and w half the weight &c. (art. 291) alſo, if the prop or fulcrum, dent into the beam, it will diminiſh its ſtrength by ſo much as d is ſhortened, by that denting.

304. If B G A C (fig. 160) be a cylinder twined, or twiſted about its axis T e T, let e P = e R = d, periphery of the ſection P R R R = b, e p = any ſmall variable radius e, then if the power of a particle at P to twine about the ſection be 1 as $eP : 1 :: ep : \frac{ep}{eP} = \frac{e}{d}$, the

power of a particle at p, the number of ſuch particles between e and p are $=$ e, and as d : b : : e : $\frac{eb}{d}$ the periphery of, or particles in p r p r, whence $\frac{e}{d} \times e \times \frac{eb}{d} = \frac{eeeb}{dd}$ (by theo. 78) $= \frac{eeeb}{4dd} =$ (when e $=$ d) $\frac{1}{4}$ d d b, the total twiſting ſtrength of the whole ſection PRPR, and (art. 301) is equal the lateral ſtrength of a triangle $\perp =$ d, and baſe $\|$ horizon $=$ b.

Note. This article does not agree ſo well with wood as with metal, for the texture of wood is not the ſame in length and breadth as it is in metal, where the power of ſpliting and breaking is the ſame. Theſe are the principles of ſtrength.

305. If a beam &c. is to be equally ſtrong throughout, or the ſtrength in any part of it proportional to the ſtreſs on that part. Firſt, if it be fixt at one end, and a weight ſuſpended at e diſtance from that end, the ſtreſs there (art. 295) is as w, and (art. 301) the ſtrength as $bdd \div \overline{n+3}$, therefore e $\propto$ b d d (w and n $+$ 3 being conſtant) and if the upper, or under ſides of the beam be a rectangle, or b every where the ſame, then e $\propto$ d d, the equation of the common parabola, if b $=$ d, then e $\propto$ d dd, that of the cubic parabola, but if the weight preſs uniformly over all the beam then (art. 296) e e $\propto$ b d d, and if b be every where the ſame, then e e $\propto$ d d, or e $\propto$ d, the equation of the plane triangle if b $=$ d, then e e $\propto$ d d d, the equation of the ſemi-cubic parabola.

306 If A B (fig. 159) be a beam ſupported at each end, and bearing a weight on any variable point, or uniformly on all parts of it then (art. 298) the ſtreſs at P is A P $\times$ B P, ſo A P $\times$ B P $\propto$ b d d $\propto$ (if b $=$ d) d d d $\propto$ (if b be conſtant) d d, the equation of the ellipſis, but if a weight be fixt at any point E, then B P $\propto$ b d d, $\propto$ (when b $=$ d) d d d, then A Q E and B Q E are two cubic parabola's, but if b is conſtant, then B P $\propto$ d d, then A Q E and B Q E are two common parabola's.

307. Let there be two beams q and Q of the ſame matter, lengths e and E, each fixt at one end, depths there d and D, breadths b and B, weights of the beams w and z, weights borne at their unfixt ends v and y, the diſtance from their fixt ends to there centers of gravity g and G, then their total ſtreſſes (art. 296) will be g w $+$ e v and G z $+$ E y, and ſtrengths (art. 301) b d d $\div$: n $+$ 3 : and B D D $\div$: m $+$ 3 : now if the ſtrength muſt be proportional to the ſtreſs, then (putting p $= 1 \div \overline{n+3}$ and P $= 1 \div \overline{m+3}$ as p b d d : g w $+$ e v : : P B D D : G z $+$ E y, but becauſe

B b

ſolidity is as weight, as c b d e : w : : C B D E : $\frac{wCBDE}{cbde}$ = z, which put in the laſt proportion for z &c. we get p C B D d G E w + p c b d d E e y = P c B D × : D e g w + D e e v. Theo. 195.

308. If the ſections are alike, viz. m = n, then p = P, c = C, S A a and A = their areas then a = c p b d and A = p C B D = P c B D, then the laſt equation will be A d G E w + a d E e y = A D e g w + A D e e v, if y = o, then A d G E w = A D e g w + A D e e v, if the beam q and Q be priſms, or cylinders, then G = ½ E, and g = ½ e, then A d E E w = A D e e w + 2 A e e v D, and if they be ſimilar ſolids, then d E = D e, and ſo E w = e w + 2 e v, alſo if the beams are ſimilar, cylinders or ſquare priſms, then, we may write E and e for 2 G and 2 g, D D and d d for A and a, d E for D e, ſo the laſt theorem will become E E E w + 2 e e e y = E E e × : w + 2 v :, whence y = $\frac{-EEEw + EEe \times : w + 2v :}{2eee}$, in which if e be made variable and the equation a maximum, we'll (art. 221) find E = $\frac{2e \times : w + 2v :}{3w}$.

309. It is prov'd by experiments, that the bending of a beam is as the weights laid on it, alſo by weights hung to the ends of elaſtic ſtrings as wires, hairs &c their lengths by ſtretching, is found proportional to the weights hung at them, except when they are going to break and then this increaſe is ſomewhat more, by taking the weights off none of the bodies are formed to regain their former figures, except well tempered ſprings, tho they by often bending grow weaker, hence there are no natural bodies perfectly elaſtic. Theſe things are alſo proved in art. 263.

310. Let e = A B (fig. 161) the length, d = D E, the depth, and b = the breadth of the ſtreight beam A B, ſupported at both ends, if weights be laid on it till it break the parts E F, which are together at the beginning of the bending, will gradually ſeperate till the beam breaks, at which time they will be all parted, as in the figure, and while they are thus parting the bending or curvature is increaſing, but the greater D E is (all elſe being the ſame), the fewer parts will be ſeperated, ſo the curvature is as $\frac{1}{d}$, and the longer the beam, the leſs weight it will bear, therefore, b d d being as the ſtrength, the weight it will bear is as b d d ÷ e, which put for the greateſt weight, and $\frac{1}{d}$ for the curvature when breaking, w any other weight and C the cor-

responding curvature, then it will be as $\frac{bdd}{e} : w : : \frac{1}{d} : \frac{ew}{bddd} = C$.

311. When C D is very small, A D B is very near a circular arch, whose radius let be r, then $\frac{1}{4}ee$ ($\square$ A D) $= \square$ C D $+ \square$ C A $= 2r \times$ CD $- \square$ CD $+ \square$ CD $= 2r \times$ CD, whence $\frac{CD}{ee} = \frac{1}{8r}$ or $\propto \frac{1}{r}$

but the less r, the greater the curvature, therefore $\frac{CD}{ee} \frac{1}{r} \propto$ (by the last art.) C $\propto \frac{ew}{bdd}$, whence CD $\propto \frac{eeew}{bddd}$, and it was the same in respect of both curvature C, and deflection C D, from the first position of the beam whether it be streight or bended. Also in the outmost strength of beams, or their breaking position b d d $\propto$ e w (art. 305) then C $\propto \frac{1}{d}$, and the deflection C D $\propto$ e e $\div$ d d.

If the length of a beam be e, fixt at one end, and at the other a weight proportional to e, the stress by art. 296, is as e $\times$ e, or e e, but if it be supported at both ends, and the weight in the middle, then the stress by art. 297, is as $\frac{1}{2}$ e $\times$ $\frac{1}{2}$ e, or $\frac{1}{4}$ e e, hence, the stress when fixt at one end, is to the stress when supported at both ends, as 4 to 1, and therefore, if C D $\propto$ e e e e $\div$ b d d d (e being as w) when the beam is supported at both ends, then 16 e e e e $\div$ b d d d $\propto$ C D, when it is fixt at one end, (taking 2 e for e) if a beam be supported by two props within the ends, as at E at F (fig 204) let a equal the whole length D H D, e $=$ E D, and z $=$ F D, then F H E $=$ a $-$ z $-$ e, and therefore G H will be as the biquadrate, of a $-$ z $-$ e, and CD, as 16 e e e e, also d D as 16 z z z z, the sum of these 3 will be as all the deflections of the beam, which call S, and if we take z $=$ e, then $\overline{a - 2e}|^4 + 32eeee = S$, its plain S will be greatest when e $=$ 0, and if the equation be ordered as a minimum (art. 221) e variable and a $=$ 1, we'll have $-8 + 48e - 96ee + 192eee = a$, whence transposing -8, and dividing by 48, we get $e - 2ee + 4eee = \frac{1}{6}$, which solved gives e $= \frac{1}{5}$ nearly, when the bending is the least, if the beam is supported at each end, and other two props to be set each at the distance e from the end, then each deflection for the length e will be as e e e e, so their sum will be as 2 e e e e, and therefore, $\overline{a - 2e}|^4 + 2eeee = S$, so instead of the above equation we'll have $e - 2ee + 1{,}5eee = \frac{1}{6}$, where e $= \frac{1}{3}$, and thus may any number of props be set so as that the beam may have the least bending.

Laſtly. Suppoſe the beam or bar A C B (fig. 161) to bend thro' the ſpace C a, before it begin to break, let $c =$ the weight laid ſoftly upon a, which breaks it when ſo bent, $w =$ a weight falling freely upon C, from the diſtance or height d, juſt to break it or bend it thro' the diſtance C a, put $b = Ca$, $e = Cn$, any ſmall variable diſtance, now (art. 309) the beam bent into the poſition A a B, exerts a force $\propto$ diſtance $Ca = b$, therefore, as $b : c :: e : \frac{ec}{b} =$ force at n, ſo $\frac{ec}{b} - w =$ force at n acted on w, but (theo. 152) $vv = \frac{fS}{q} =$ (in this caſe) $\frac{fe}{w} =$ (becauſe $f = \frac{ec}{b} - w$) $\frac{eec - ebw}{bw}$, $=$ (ſee theo. 78, becauſe e is variable) $\frac{eec - 2ebw}{2bw}$, but (theo. 166) $vv \propto d$, whence, $d = \frac{eec - 2ebw}{2bw}$, and when $e = b$, then $2bwd = bbc - 2bbw$, ſo $w = \frac{bc}{2d + 2b} = \frac{bc}{2d}$, nearly, becauſe b in moſt ſuch caſes is very ſmall.

1. Hence, when $d = 0$, $w = \frac{bc}{2b} = \frac{1}{2}c$, that is, half the weight will break the bar, when bent to its breaking poſition, that will break it when unbent.

2. The weight that by falling a given height will break any beam is nearly as the ſpace (b) thro' which it bends before it breaks, hence brittle bodies break ſooner by percuſſion than others of equal ſtrength.

Note. Here is no notice taken of the bar's weight.

3. If gravity have no concern, or the weight be thrown horizontally againſt the beam, then $\frac{ec}{b} - w$, will only be $\frac{ec}{b}$, ſo proceeding as before, we'll find $2dw = bc$, whence $w = \frac{bc}{2d}$.

PROBLEM CXCIX.

Of Hydroſtatics, hydraulics, and pneumatics.

312. Hydroſtatics, is a ſcience that treats of the properties of fluids.

313. Hydraulics, is the art of raiſing, carrying &c. of water as by pumps, &c.

314. Pneumatics, is a ſcience that treats of the air's properties.

315. The motion of a fluid is accounted for as that of a heavy body, but the motion or preſſure in a fluid is equally diffuſed all around in all manner of directions, and can only be at reſt when its ſurface is parallel to the horizon.

316. If a homogeneous body be immerſed in a fluid of the ſame denſity with itſelf, it will remain at reſt in any place, and in any poſition, but a body of greater denſity than the fluid will ſink to the bottom, and a body of leſſer denſity will riſe to the top and ſwim, hence the body of greater denſity loſes ſo much weight as that of an equal quantity of the fluid, and ſo tends downwards only with the difference of theſe weights, and this is the relative gravity of the body in the fluid, but if the body is ſpecifically lighter than the fluid it ſeems to loſe more weight than it has, and hence the body will tend upwards with the difference of theſe weights, and this is the relative levity of the body in the fluid, and the weight of this body is equal to the weight of a quantity of the fluid as big as the immerſed part of the body.

317. If a plane ſurface be perpendicular to the ſtream of any fluid, the ſtream ſtrikes the plane with the ſquare of its velocity; for its plain, that with n times the velocity the force of each particle will be n, and n times the number of particles will ſtrike the plane in the ſame time, conſequently the plane is ſtruck by n n, and becauſe the whole plane is thus ſtruck, it follows that n n a is as the impreſſion on the plane (a being = the area of the plane.

318. Hence (and by theo., 158) as □ D B : □ D C (fig. 143) : : the abſolute velocity of the fluid to that part of it which impreſſes the plane, and ſo is ſquare radius : ſquare incident angle C B D, when the plane A B is oblique to D B the direction of the ſtream, ſo that if a = area plane A B, r = radius, S = ſine ∠ C B D, we'll have as r r a : S S a : : the impreſſion in a ⊥ direction to that in an oblique one.

319. If a plane in motion be ſtruck perpendicularly by a fluid, it is plain, that each particle ſtrikes that plane with a velocity equal to the difference between the velocities of the fluid and plane, whence, the impreſſion of the fluid on the plane will be as the ſquare of the difference between their velocities, ſo if v = the velocity of the fluid, and v − e = that of the plane, their difference is e, ſo : v — e : × e e = v e e − e e e, the impreſſion, and in caſe of a maximum $e = \frac{2}{3}$ v.

320. If there be two planes whoſe areas are A and a, velocities of the fluids V and v denſities of the fluids D and d, ſines of incidence S and s, then (by art. 318) the impreſſions on the planes are as A V V : a v v, on account of the velocities, which velocities (per laſt art.) are

as S S : s s, and the densities being as D to d, the products of these given as A D V V S S : a d v v s s, the ratio of the impressions.

321. If a fluid falls from any height h it acquires a velocity, with which it moves uniformly, and so (by theo. 167) describes a double space in that time, in which it fell to gain that velocity. Let S = the height in feet fallen by a heavy body in one second of time, v = the velocity of the fluid, or space it describes in 1 second, a = the area of the plane, or base of a column of the fluid, h = the height of this column, fit to acquire the velocity v, then (by theo. 167) 2 S = velocity generated by gravity in falling thro' S, therefore (by theo. 166) as $4SS : S :: vv : \frac{vv}{4S} = h$, the height fallen thro' to gain the velocity v feet per second, so $\frac{vv}{2S} = 2h$, and $\frac{vva}{2S} = 2ha =$ a column of twice the height h, and (by theo. 155) $2S \times 2ha = 4Sha = vva$ the motion generated by the weight of the column in 1 second, viz. the body $\times$ velocity 2 S, this v v a is the same with n n a in art. 317, which shews that the $\perp$ force of any fluid against a plane is equal to the weight of a column of that fluid the base of the column being = to the area of the plane, and its height = twice the height descended by a falling body to acquire the velocity of the fluid.

322. If d = density of the fluid, then $dah = \frac{vvda}{2S} = F$, the force of the fluid against the plane, or $dah = \frac{1}{2}\frac{vvda}{S} = F$, the same $\perp$ force, if the fluid strike the plane obliquely, let z = sine of incidence then $dahzz = \frac{vvdazz}{2S} = F$ (see art. 320) if the plane be also in motion, the relative velocity of the fluid against the plane, must be taken instead of the absolute velocity.

323. If instead of the fluid striking the plane, we suppose the plane to move in the fluid, or a cylinder to move in it in direction of its axis h, the area of its base = a, its velocity or the space it describes in one second, = v feet, and S = as before, then its evident, what was before the impression on the plane, will now be the resistance of the plane or cylinder, that is (by art. 321) $\frac{avv}{2S} = 2ha$ the resistance, were the particles of the fluid all driven directly forward, but since (by art. 315) they move round in all directions, it may be proved (see art. 325)

that this resistance is double too much, and therefore $\frac{vva}{4S} = ha = R$, the true resistance, that is, its resistance is = weight of the cylinder $\frac{vva}{4S}$ of the fluid.

324. The denser the fluid, the greater is the resistance, for the more particles it contains, the more it must resist, and the denser the body, the less the resistance for the more particles it contains the more power it has to overcome the resistance, so if D = the density of the fluid and d = that of the body, we'll have $\frac{vvaD}{4Sd} = R, = \frac{haD}{d}$, and if $v = 2\sqrt{\frac{hSd}{D}}$, or $vv = \frac{4hSd}{D}$, we'll have $R = ha$, in which case, the resistance is = the weight of an equal cylinder of the fluid.

325. If a globe (fig. 162) move uniformly forward in direction C A, draw G B D ∥ C A, draw the tangent D H, and let fall G H ⊥ D H, let G D be the force of a particle of the fluid against the base B, in direction G D, then G H will be the force acting against D, in direction D C, and this force is to the force in direction G D, as D C to D B, whence the force against B is to that against D in direction GD, in a ratio compound of G D to D H, and D C to D B, that is as □ D C to □ D B, so the force of all the particles of the fluid against the base, is to their force against the convex surface as the sum of all the □ D C s is to the sum of all the □ D B's, which sums are as 2 to 1, that is, the resistance of a sphere's surface, is but half the resistance of the base, or of a cylinder of the same diameter.

326. Whence $\frac{vv}{4S}$ (see art. 323) becomes $\frac{vv}{8S}$, that is, if A = the diameter of a globe, its resistance is = the weight of a cylinder of the fluid of the same diameter A, and its length $\frac{vv}{8S}$, and if $vv = \frac{16}{3}$ SA, or $v = 4\sqrt{\frac{SA}{3}}$, then $\frac{vv}{8S} = \frac{16SA}{24S} = \frac{2}{3}A$, viz. its resistance is = weight of an equal globe of the fluid.

327. Let D = density of the fluid and d = density of the globe, then, since a globe whose axis is A, is = a cylinder, whose height is $\frac{2}{3}$ A, the weight of the globe will be = weight of a cylinder of the fluid whose length is $\frac{2Ad}{3D}$ and (by art. 316) the weight of the globe is

the fluid is = weight of a cylinder of the fluid, whose length is $\frac{2}{3}$ A $\times : \frac{d - D}{D}$: but (by art. 321) the resistance of the globe moving with the velocity acquired by falling in vacuo thro' the height $\frac{4}{3}$ A $\times : \frac{d - D}{D}$: is = weight of a cylinder of the fluid whose length is $\frac{2}{3}$ A $\times : \frac{d - D}{D}$, therefore, the weight of the globe in the fluid is = resistance, and consequently cannot accelerate the globe; whence, the greatest velocity a globe can obtain by descending in a fluid, is that which it would acquire by falling in vacuo, thro' a space S, that is to $\frac{4}{3}$ A, as d — D is to D.

328. If $v = 4\sqrt{\frac{d - D}{3D} S A}$, the resistance is = weight of the globe in the fluid.

329. If a cylinder move in a fluid inclosed in a vessel, instead of the absolute velocity, the relative velocity in the fluid must be taken in order to find the resistance, and the narrower the vessel the greater will be the resistance, for then the more particles of the fluid are driven directly before the moving cylinder, and if the vessel be so narrow as none of these particles can diverge in all directions, but be all driven before the cylinder, then the resistance is the greatest possible, and is equal to the resistance of a plane (art. 321) press'd by the fluid, whereas if the particles of the fluid have liberty to deverge in all directions, the force of a cylinder of water against a plane, is double the resistance an equal cylinder meets with, when moving in water with the same velocity, as is plain from art. 321, and 323.

330. If A B (fig. 163) be a cylinder ⊥ to the horizon kept constantly full of water, with a hole at the side or bottom B, constantly running out, let S = space descended by gravity in one second of time t = time in seconds of the waters running out, h = A B the depth of the vessel to the center of the hole, a = area of the hole, now (by theo. 167) if the cylinder of water h a, fall thro' half its height $\frac{1}{2}$ h, by its own weight, it will by that fall acquire such a motion as to pass thro' h uniformly in the same time, and the quantity of pressure at any given depth upon a given surface, being always the same, therefore the water in the hole B, is pressed with the cylinder of water h a, whence, the pressure at B, will generate the same motion in the spouting water, as was generated by the weight of the cylinder h a,

So in the time of falling thro' $\frac{1}{2}$h, a cylinder of water will ſpout out, whoſe length or the ſpace paſſed uniformly over is $=$ h, and in the ſame time repeated another $=$ cylinder will flow out, and in a third part of time, a third &c. therefore, the length of the whole cylinder run out will be proportional to the time, and ſo the velocity of the water at B is uniform and therefore in the time of falling thro' half h, a quantity of the fluid runs out $=$ the cylinder h a, with a uniform velocity at B, $=$ that acquired by a heavy body falling thro' $\frac{1}{2}$ h, and becauſe the velocities of falling bodies are as the ſquare roots of the heights, therefore, the velocities of the fluid ſpouting out at different depths, will be as the ſquare roots of the depths, and theſe velocities will be the ſame in any direction whether ſpouting upwards, downwards, ſideways &c. and if it ſpout upwards it will aſcend nearly to the upper ſurface of the fluid. From what is ſaid in this article, we have $ta\sqrt{2hS} =$ quantity of water in feet, that runs out in the time t, (a, h, and S, being each in feet) $= 6{,}128\,ta\sqrt{2hS}$ ale gallons. Theo. 196.

331. But if there be a rectangular hole in the ſide of a veſſel conſtantly kept full of water then becauſe the velocities are as the ſquare roots of the heights (let e $=$ any height of this hole or ſlit, and y $=$ its breadth) which ratio's form the equation of the common parabola, whoſe area is $= \frac{2}{3}$ e y $= \frac{2}{3}$ a in art. 330, whence the quantity of water diſcharged thro' the ſlit e B a e (fig. 163) is $= \frac{2}{3}$ the quantity diſcharged out of an equal hole plac'd at the whole depth e a, or at the baſe B a in the ſame time, ſo taking h $=$ e a, we'll from the laſt theo. have $\frac{2}{3} \times 6{,}128\,ta\sqrt{2hS}$, for the ale gallons run out in the time t,

332. If the veſſel is not kept full, viz. no water taken in at top, then h a will conſtantly decreaſe, and conſequently the velocity at B, whence, in this caſe the velocity at B, is but half of what it is when the veſſel grows no emptier.

333. A fluid ſpouting thro' a hole, endeavours at a ſmall diſtance from the hole to contract itſelf into a kind of ſpire, by which its velocity is ſomewhat increaſed, and the thinner the hole is, the greater is this increaſe, and if the hole be in a thin plate of metal it will acquire a velocity near $=$ that acquired by a heavy body falling thro' the whole height h; alſo, from the reſiſtance of the air, all bodies projected upwards fall ſhort of theſe projected in vacuo, by ſpaces which are as the ſquares of the heights, as in ſmall heights of fluids ſpouting, is proved by experiments. There is great difference in ſpouting veſſels on account of their forms, bigneſs of the hole, &c. to help which, let A $=$ area at A A the water's ſurface, S, a, t, h as in art. 330, then as $2A^2$

$-a^2 : A^2 :: h : \frac{hA^2}{2A^2-a^2} = H$, the height of a falling body to acquire the velocity of the water at the hole B, and if the area of the hole be but ſmall in reſpect of A, then $H = \frac{hA^2}{2A^2-0} = \frac{1}{2}h$, as before in art. 330, but if $A = a$, or the veſſel have no bottom, then $H = \frac{hA^2}{2A^2-A^2} = h$, (ſee art. 441, and 442) whence it appears that the velocity of the ſpouting fluid can never be leſs than that acquired by falling thro' h, the veſſel being conſtantly kept full, but if no water runs in whilſt it ſpouts out, then it muſt be as $3A^2 - a^2 : A^2 :: h : \frac{A^2h}{3A^2-a^2} = H$, ſo in both caſes it may be $\frac{A^2h}{nA^2-a^2} = H$, the height fallen thro' to gain the velocity at B, ſo (by theo. 167) as $\sqrt{S} : 2S :: \sqrt{:} \frac{A^2h}{nA^2-a^2}$: to $\frac{2S}{\sqrt{S}} \sqrt{:} \frac{A^2h}{nA^2-a^2} : = 2\sqrt{:} \frac{A^2hS}{nA^2-a^2} : =$ the velocity, or ſpace uniformly paſſed thro' by the water at the hole B, in one ſecond which multiplied by t and a the area of the hole, &c. gives $2ta \sqrt{:} \frac{A^2hS}{nA^2-a^2}$: feet for the water run out in one ſecond. Theo. 197.

334. Since water will run thro' a hole in the ſide of any veſſel filled therewith, it is evident that the ſides as well as the bottom is every where preſſed with the fluid, and that with a force proportional to the weight of the fluid above the place preſſed; let therefore, $a =$ area of the cylinder's baſe (fig. 163), and ſuppoſe its height A B, divided into an infinite number of equal parts, beginning at top A, 0, 1, 2, 3 &c. n, then the ſides at each of the parts will be preſſed with the weights 0, a, 2 a, 3 a, &c. n a, ſo $0 + a + 2a + 3a +$ &c. : $+ na = \frac{1}{2}nna$, the whole preſſure againſt the ſides $= \frac{1}{2}ha$, becauſe n n a, is as the whole weight, but the bottom a, bears the whole weight h a, of the fluid, ſo the preſſure againſt the bottom is to that againſt the ſides as $ha : \frac{1}{2}ha$, or as 2 to 1.

335. If A B C D f (fig. 164) be any veſſel containing a fluid, and B L, E D, G F C, and H F O K, be ⊥s to the horizon, G B, F L, C D, ||s thereunto, then if the fluid be poured in at the top A B, till the veſſel is filled to n L, it will riſe in the other part to F a, the ſame level, whence it follows, that if A and a be the baſes of two veſſels of any kind or any ſhape, H and h their heights perpendicular the horizon, the preſſure

will be as h a, this confirmed by experiments, for if a veſſel f Q K, be made ſo as to open like a pair of bellows, and a tube F G H, of any diameter G H, fixed to it by pouring water in this tube, it will raiſe the upper baſe F a, loaded with a weight f, nearly = that of a column of water of the ſame baſe F a, and h = H K, that of the veſſel and tube.

336. The force or preſſure is every where directed ⊥ly againſt the inner ſurface of the veſſel (by theo. 157) ſo at f, it is directed upwards, at L ſideways, and at K downwards, and as the ſame heights are equal, viz. the preſſure at L and F is equal, becauſe B L = G F, alſo, the preſſures at D, O, C, Q, are equal becauſe E D = H O = G C, &c. and at K the preſſure is as H K.

337. Hence (and by art. 334) the preſſure or ſtreſs on any pipe or tube full of water is as the diameter of the pipe and the ⊥ height of the water above that place, ſo, as the internal preſſure on any length of the pipe, is to the ſtreſs it ſuffers as to ſplitting, ſo is 2×3.1416 to 1.

338. Let A and B, be the bulks, or magnitudes of two bodies, A and d, their denſities G and g their weights or gravities, now the denſer bodies are, the more matter they contain, and the greater they are the more matter they alſo contain, and the weights being as the quantities of matter, it will be, as A D : G : : B d : g and if A = B, then as D : G : : d : g, or if D = d, then as A : G : : B : g, but if G = g, then as A D : 1 : : B d : 1, or as A : d : : B : D.

339. Since the ſpecific gravity of bodies, are as the weights of equal bulks, they are therefore as their denſities, ſo whatſoever is ſaid in reſpect to the denſities of bodies, is true in regard to their ſpecific gravities.

340. That is, if the weights be equal, the magnitudes are inverſely as the ſpecific gravities, viz. as A : d : : B : D.

341. Let a = the ſpecific gravity of a body A, B = its weight in water c = the ſpecific gravity of water, then (by art. 316) the weight loſt by the body in the fluid is = A — B = the weight of an = bulk of water, ſo (by the laſt art.) as $A - B : A :: c : \frac{cA}{A-B} = a$, that is, the ſpecific gravities of bodies will be as their weights (A) in the air directly, and as (A — B) in the ſame fluid inverſely, becauſe c is conſtant.

342. If the ſame body is weighed in ſeveral fluids, and be ſpecifically heavier, then from $a = \frac{cA}{A-B}$, we have $c = \frac{a \times : A - B :}{A}$, or c ∝

A — B, hence the ſpecific gravities of theſe fluids are as the weight of the body loſt in the fluids.

343. Let a and b = the ſpecific gravity of a mixture of two bodies A and B, c = the weight of the mixture, d = its ſpecific gravity, then if z = the weight of the body A, c — z will be = that of B, and (by art. 341) $\frac{z}{a}$ and $\frac{c-z}{b}$ will be the weights loſt in the fluid, and $\frac{c}{d}$ that of the mixture, then if $\frac{z}{a}+\frac{c-z}{d}$ be taken $=\frac{c}{d}$, we'll have $z=\frac{ac}{d}\times\frac{b-d}{b-a}$, or $d=\frac{abc}{bz-az+ac}$, &c. for any of the letters, the reſt being given.

345. If we take m = z and n = c — z, then $\frac{z}{a}+\frac{c-z}{b}=\frac{c}{d}$, will be $\frac{m}{a}+\frac{n}{b}=\frac{c}{d}$, whence, $a=\frac{bdm}{bc-dn}$, or as $\frac{c}{a}-\frac{n}{b}$: m :: 1 : a.

346. But if one of the bodies (A) be ſpecifically lighter than the fluid, it will be as m : $\frac{c}{d}-\frac{n}{b}$ :: 1 : a, and thus may a table of ſpecific gravities be calculated, and an irregular ſolid &c. meaſured thereby, thus, let w = the weight of a cubic inch of water, or any other fluid that the ſolid is to be immerſed in, d = the ſpecific gravity of the ſolid, then as 1 : w d :: ſolid content in inches : weight in ounces, &c.

347. A fluid is a body, whoſe parts yield to any force impreſs'd and are eaſily moved amongſt themſelves, there are two kinds of fluids, as bodies, viz. elaſtic and non-elaſtic, a fluid is elaſtic when it can be reduced into a leſſer ſpace by compreſſion, ſuch as air, which differs from other fluids in theſe particulars, (1) it may be preſſed into a leſs ſpace, and ſo can no other fluid, (2) it cannot be congealed or any how fixed, and all other fluids can (3) its denſity every where decreaſes from the earth's ſurface upwards; but other fluids are of an uniform denſity throughout, &c.

Non-elaſtic fluids are ſuch as cannot be reduced to a leſs bulk, ſuch as water, &c.

348. If the particles of air have all the ſame elaſtic force, the compreſſing force muſt be as the number of particles preſs'd, but if there be two equal bulks of air and the denſity of the one be = n times the

density of the other, the latter contains n times more particles than the former. Hence, if all the particles of air have the same elastic force, the force of compression, (by art. 338) is as the density.

349. The elasticity of the air is the foundation of the air pump; for when the piston is forc'd down to the bottom of the barrel and rais'd up again the air in the receiver will expand itself and part of it will enter into the barrel, so that the air in the receiver and that in the barrel will have the same density, which will be to the first density as the capacity of the receiver, is to the capacity of the barrel and receiver together, and by repeating the motion of the piston a second time, the density of the air will again be lessened in the same ratio, and so on; by which means the air in the receiver may be reduc'd to the least density, but can never be entirely exhausted, for the air which is exhausted is only pushed out by the spring of that which is left behind, if therefore every particle were supposed to be exhausted, the last particle would be pushed out without a cause, contrary to art. 233. Hence, if $m =$ the capacity of the receiver, $n =$ that of the barrel, $d =$ the density of the air in the receiver before the pump begin to work, then

As, $n+m : n :: d : \frac{dn}{n+m} =$	the density of the air in the receiver after the	1	stroke of the piston or turn. Theo. 198
$n+m : n :: \frac{dn}{n+m} : \frac{dnn}{\overline{n+m}\vert^{2}} =$		2	
$n+m : n :: \frac{dnn}{\overline{n+m}\vert^{2}} : \frac{dnnn}{\overline{n+m}\vert^{3}} =$		3	
and universally $\frac{dn^{s}}{\overline{n+m}\vert^{s}} =$		4	

350. Hence, since the spring or elasticity of the air is the force it exerts against the force of compression, it follows that the air does the same thing by its spring, as a non-elastic fluid does by its weight.

351. Since the less space a body of air is confined in, the greater must be the force of compression that confines it, it follows that the compressing forces are inversely as the spaces which contain the same quantity of air.

351. Let there be two unequal cylinders A and B, whose diameters are d and r, heights = a and e, filled with two different fluids of densities, n, m, and suppose the sum of these columns in equilib. with another column C, whose diameter is r, height = b, filled with a fluid of m, density, then (by art. 338) $paddn$, $perrm$, and $pbrrm$, are the weights of the columns, so $paddn + perrm = pbrrm$, or

$a\,d\,d\,n + e\,r\,r\,m = b\,r\,r\,m$, and if $r = 1$, then $a\,d\,d\,n + e\,m = b\,m$, or $d\,d = \frac{:b - e:m}{n\,a}$, or if $d = r$, then $e = \frac{b\,m - n\,a}{m}$ &c. for any of these quantities.

352. But if one of these columns (A) be air, and their diameters all the same and the fluids B and C of the same density, suppose a power at B (fig. 165) to sustain the column B D, of the fluid B, or C, but if the column of air E F, be added the same power B can only sustain the column B C of the said fluid B, then its evident the column C D, of the fluid B (being the difference between B D and B C) is = the elastic force of the air in E C (see art. 350) and because cylinders of equal bases are as their altitudes, it will be as the weight of the fluid in D B, is to that in D C, $: : DB : DC = \frac{DC}{DB}$, and (by art. 351) as the force of the air when confined in E C : its force when confined in E F $: : EF : CE = \frac{EC}{EF}$, so, $\frac{EF}{EC}$ is as the force in E F, therefore $\frac{EF}{CE} = \frac{DC}{DB}$, whence, as $BD : CD : : EC : EF$, or as $BD\,(b) : BD - BC\,(b - e) : : BE - BC\,(h - e) : EF\,(c)$ ergo, $b\,c = b\,h - e\,b - e\,h + e\,e$, which by compleating the square &c. we'll find $e = \mp \sqrt{:bc + \overline{\frac{h - b}{2}}\Big|^2} : + \frac{b + h}{2}$ &c. if you'll have other of these quantities.

353. If A I K C, be a compound barometer (fig. 166) close stopp'd at A, and open at C, empty from A to D, filled with mercury from D to B, and with water from B to E, the diameter of the lesser tube F K C, to that of the greater tube F I A, as 1 to d, the density of mercury to that of water as m : 1, take the points H, G, in the same horizontal line with B, then because the air presses equally in all directions, the fluid in G K B, will be in equilib. with itself, as will also the fluid in H I B, whence all the compound between G and H, is in equilibrio with itself; consequently, the column of mercury D H, is in equilibrio with the column of water G E and a column of air of the same base conjointly, and so will vary with the sum of the variations of each of these, let v = the variation of the air's weight, which is measured by the space the mercury moves in the common barometer in a given time, e = the space which the water moves thro' at E, in the same time, then (because cylinderic spaces are inversely as the

ſquares of their diameters) it will be as $dd : e :: 1^2 : \frac{e}{dd}$ = the ſpace moved thro' or variation at B, therefore G E, the difference of the legs E K and K B, will vary in its weight by $e + \frac{e}{dd} = \frac{dde+e}{dd}$. Alſo, ſince the ſpace moved thro' by the mercury at B, is = that moved thro' at $D = \frac{e}{dd}$, the difference D H will vary its weight by $\frac{2e}{dd}$, but this variation of weight is = both the former, and ſince $\frac{dde+e}{dd}$ is an altitude of water, it will be as $m : 1 :: \frac{dde+e}{dd} : \frac{dde+e}{mdd}$, equal the height of the mercury of the ſame weight, conſequently $\frac{2e}{dd} = v + \frac{dde+e}{mdd}$, whence, $e = \frac{vddm}{2m-dd-1}$, or as $e : v :: ddm : 2m-dd-1$.

354. The body of a water pump (fig. 167) n m H C, is called the pumps barrel, A G the ciſtern into which the water comes and runs out at a ſpout A, the part E F C H, which goes down from the barrel into the water at B, is called the pipe, or ſucking pipe, the piſton I D moves in the barrel by means of a lever or handle P G, &c. when the piſton deſcends to H C, the air contained between it and the valve v, (in the pipe) being reduced into a leſſer ſpace will be condenſed, and by its elaſtic force preſſes down the valve v, and ſo forces open the valve D, (of the bucket in the piſton) and ſo riſes above the piſton, then by raiſing the piſton, the weight of the atmoſphere preſſes down the valve D, ſo that no air paſſes thro' the bucket, by which the air that remained between v and D, will be rarified, and as that in the pipe is denſer than that above v, it acquires an elaſtic force fit to raiſe the valve v, and ſo enters into the barrel till the air in both is of the ſame denſity, then the atmoſphere preſſing on the ſurface of the water E F, in the well, cauſes it to riſe into the pipe B v, and in moving the piſton in this manner the water will riſe into the barrel, and from thence into the ciſtern, provided the length of the pipe E H, between H and the ſurface of the water in the well (ſee queſtion 178) do not exceed 35 feet, but becauſe part of the air which enters into the barrel is forc'd down again into the ſucking pipe, by the ſhutting of the valve, the air cannot be (ſee art 349) intirely taken out, and ſo this height is ſettled at 34 feet which anſwers experiments. Let a = D H the diſtance

moved by a ſtroke of the piſton, b = E H the height of the pipe, c = 34 feet, e = the height to which the water riſes in the pipe (at the firſt ſtrokes of the piſton) and ſuppoſe the diameter of the barrel equal that of the pipe, otherwiſe they muſt be reduced to the ſame diameter by dividing the capacity of the barrel from H C to the end of the ſtroke, by the ſection of the pipe, thus if the diameter of the pipe be d, and that of the barrel D, and the length of a ſtroke l, then l D D = the capacity, ſo l D D ÷ d d, ſo, if the piſton moves l inches in the barrel, it anſwers to l D D ÷ d d inches in the pipe, the piſton in every ſtroke ſhould deſcend cloſe to the valve v, at the pipe's head, otherwiſe the riſing of this valve will be partly ſtop'd by the waters weight above it, which ſo much ſtops the working of the pump. Then, when the piſton is raiſed the height a, the air in the pipe would be rariſied into the ſpace a + b, did the water not riſe in the pipe, but the water riſing to the height e, this air is reduced into the ſpace a + b — e, ſo (by art. 351) its denſity before the ſtroke, will be to that after the ſtroke inverſely as a + b — e : b, but becauſe the air in the pipe was in equilib. with the atmoſphere before the ſtroke, it will be of the ſame denſity, and ſo its elaſtic force will be as c, whence as a + b — e : b :: c : $\frac{bc}{a+b-e}$ = the denſity or elaſtic force of the air after the ſtroke, which together with the weight of the water e, are in equilib. with the atmoſphere immediately after the ſtroke, viz. $e + \frac{bc}{a+b-e} = c$, ſo, ea + eb — ee = ca — ce, or : a + b + c : × e : — ee = ac, or putting 2 z = a + b + c, then 2 z e — e e = a c, which by compleating the ſquare &c. gives e = z — √ : z z + a c :.

355. (By theo. 168) we have √c for the velocity with which it enters the pipe at E, and (by theo. 169) √c — √b, for the velocity with which it enters into the barrel at H, whence the piſton moving uniformly in the barrel. The water in the pipe follows it with a variable velocity, the greateſt velocity being to the leaſt as √c : √c — √b.

356. But it's evident, a pump diſcharges moſt water with the greateſt eaſe, when the water in the pump riſes cloſe after the bottom of the piſton for then there is no vacancy between the water and bottom of the piſton, and ſo no part of the ſtroke is loſt, nor any of the water hindered, as is the caſe when the water moves faſter than the piſton, now becauſe the piſton, and conſequently the water in the barrel moves uniformly it is evident that the uniform velocity of the water at the bottom of the piſton, will be the velocity of the piſton, when the

pump is in its best perfection. Let S = the space descended by a heavy body in the first second of time, then (by theo. 167) as $\sqrt{S} : 2S ::$ $\sqrt{c} - \sqrt{b} : \frac{2S\sqrt{c} - 2S\sqrt{b}}{\sqrt{S}} = 2\sqrt{Sc} - 2\sqrt{Sb} = v$, feet per second, the uniform velocity of the piston.

357. Suppose the highest elevation of the piston to be D v, and let E v + D v = b put v = the velocity of the piston, D = the diameter of the barrel d = that of the pipe, V = the least velocity of water that rises in the pipe, which (by the last art.) is found $V = 2\sqrt{Sc} - 2\sqrt{Sb}$, now if (DD) the bore of the barrel be greater than (d d) that of the pipe, its manifest that the water will run out of the pipe into the barrel with a greater velocity, in proportion as the bore of the barrel is greater, so it will be as DD : V : : d d : v, whence D D v $= ddV$, or $V = \frac{DDv}{dd} = 2\sqrt{Sc} - 2\sqrt{Sb}$. Theo. 199.

Any one of these quantities may be found if all the rest are known, by which a water pump may be fitted to the best advantage, but here observe that the water as it rises in the pipe meets with some resistance from the sides of the pipe so the velocity as before found may be a small matter too much, and that the water may meetwith the least resistance, let the diameter of the bucket's cavity be = that of the pipe, and the insides of the barrel and pipe truly cylinderical; the discharge of water is the same, whether the valve v be placed at the bottom of the barrel, or bottom of the pipe at E or near E, as is easily gathered from the foregoing demonstrations, when the valve v is placed at H, it will be easier come at when it wants repairing, which it will oftner do than if it were placed near E where it is always kept wet.

358. Since the rising of water in a pump depends intirely on the air being taken out above it, the parts thereof should be so tight as to let in no air, and then the water (see art. 354) will rise to the height of 34 feet below the lowest descent of the piston, but by reason of the water's weight &c. (in this sort of pump called the common, or sucking pump) it cannot be carried much above the piston, so in heights above 34 feet there is use made of two other kinds of pumps working in frames and forcing the water above them, &c. called forcing, and lifting pumps.

PROBLEM CC.

To find the path of a projectile in a non-resisting medium.

359. Suppose a body projected from the point A (fig. 168) in direction A C with a velocity v, fit to carry it over a given distance d, in the time t. Let b = the distance fallen by a heavy body from a state of rest in that time, S = sine ∠ C A B, which A m E B the path of the projectile, makes with the horizon A B, c = its co-sine, r = radius, then (by theo. 48) $\frac{re}{c}$ = A G and $\frac{eS}{c}$ = G H (e being = A H) now because the ball, or projectile in motion, is by the force of gravity continually compelled to leave its right line or direction A C, it must therefore describe some curve, as A m E B, where A C is a tangent to it at the point A, but as gravity acts always || G H, it does not alter the velocity of the body in that direction, so if we suppose E H to move parallel to its self along with the ball, its plain the motion of the ball in this direction may be looked upon as uniform, and then as d : t : : $\frac{re}{c} : \frac{ret}{dc}$ = time of the balls describing A m, and (by theo. 166) as $tt : b :: \overline{\frac{ret}{dc}}\Big|^2 : \frac{rreeb}{ddcc}$ = G m the distance fallen thro' in that time so, G H — G m = m H = e s ÷ c : $-\frac{rreeb}{ddcc} = \frac{ddesc - rreeb}{ddcc}$, and when m H is a maximum we'll have $ddSc - 2rreb = 0$, whence $e = \frac{scdd}{2rrb}$ = A D and then H m becomes = E D = $\frac{ssdd}{4brr}$, again if e be taken = A B, then if $m = 0 = \frac{ddesc - rreeb}{ddcc}$, whence e $= \frac{Scdd}{rrb}$ = A B, which is double to the value of A D last found, and so proves the ball to be highest when half its flight is performed, when $e = \frac{Scdd}{2rrb}$ = A D, then $\frac{eS}{c}$ = G H, becomes $\frac{SSdd}{2rrb}$ = F D, which is double to E D = $\frac{SSdd}{4rrb}$, whence the path A m E B, is a common parabola, for in that curve, the sub-tangent F D is double E D. Theo. 200.

360. From this theorem is deduced the principles of gunnery, where note,

361. The angle B A C, which the gun's axis makes with the plane of the horizon A B, is called the elevation.

362. E D the greatest height to which the ball rises in its flight, is called the height or altitude of the projection.

363. If the ball were thrown directly upwards with the same velocity, the height A P, to which it would rise, is called the impetus.

364. The distance A B, between the gun A, and B, where the ball first strikes the horizon, is called amplitude, random, or horizontal range, which is greatest when the elevation is 45°, because the sine and co-sine of 45° are equal viz. $c = S$, and then $AB = \frac{SSdd}{rrb}$ for $S \times S$, is greater than any $S \times c$.

365. Because $\frac{ssdd}{4rrb} = ED$, and $\frac{scdd}{rrb} = AB$, therefore as, $ED : AB :: \frac{ssdd}{4rrb} : \frac{scdd}{rrb} :: \frac{1}{4}s : c$, hence as $c : s :: AB : 4ED$, so $4ED = AB$, when $s = c$.

366. When the ball is projected directly upwards then $c = 0$, and $r = S$, and then $\frac{ssdd}{4rrb} = ED$, becomes $\frac{dd}{4b} = AP$, but if $S =$ sine of 45° then $rr - ss = ss$, or $ss = \frac{1}{2}rr$, then $ED = \frac{rrdd}{8rrd} = \frac{dd}{8b}$, whence, as $AP : ED :: \frac{dd}{4b} : \frac{dd}{8b} :: 2 : 1$. But per last art. $4ED = AB$, so as, $AP : AB :: 1 : 2$, i. e. the greatest amplitude is always double to the impetus.

367. Its manifest, while the ball performs its whole flight, a heavy body from rest would fall from G to B, so let $n =$ tangent $\angle CAB$ $a = AB$, $t =$ time of the balls flight, $r =$ radius, then (by theo. 47) as $r : n :: a : \frac{na}{r} = BC$, and (by theo. 166) as $b : \square$ 1 second $:: \frac{na}{r} : \frac{na}{rb} = tt$, but if $n =$ tangent 45°, then $n = r$, and so $\frac{a}{b} = tt$, in second, and a in feet.

368. Because the motion of the ball, or the line G H, along the line A C and A B is uniform, therefore as a (AB) $: \sqrt{\frac{na}{rb}}$ (t) $:: e$ (AH) $: e\sqrt{\frac{n}{arb}} =$ time describing A m.

369. Let a = amplitude, s = ſine, c = co-ſine of the elevation, and y = ſine of its double. Alſo, let A, C, S and Y, denote the ſame things in ſome other projection of an equal impetus, then from $\frac{scdd}{rrb}$ = A B, it will be, as A : a $\frac{CSdd}{rrb} : \frac{csdd}{rrb}$ :: SC : sc, but (by theo. 40) SC = $\frac{1}{2}$ rY, and sc = $\frac{1}{2}$ ry, whence, as A : a :: $\frac{1}{2}$ rY : $\frac{1}{2}$ ry :: Y : y, and if S = ſine 45° then Y = r, and as r : A :: y : a.

370. When S = ſine 45° then A = greateſt amplitude = twice impetus, ſuppoſe = 2 m, (ſee art. 364 and 366) therefore as r : 2m :: y : $\frac{2my}{r}$ = a.

371. Let v = the velocity with which the ball ſets out, which being thrown directly upwards would riſe the height A P = m, ſo (by theo. 169) the ball muſt fall thro' A P, to acquire ſuch a velocity, which in direction A C being uniform, it will (by theo. 167) be as, b : 4 bb :: m : 4 bm = v v, ſo m = $\frac{vv}{4b}$.

372. Let h = E D, the height of the projection, then (by art. 365) as c : s :: a : 4 h, ergo, h = $\frac{sa}{4c} = \frac{s}{c}\frac{1}{4}$ a, from theſe equations we get as follow, viz.

373. $tt = \frac{na}{rb} = \frac{2myn}{rrb} = \frac{4scmn}{rrb} = \frac{4scmrs}{rrbc} = \frac{4ssm}{rb}$ (ſee art. 176) $= \frac{ssvv}{rbb}$.

374. $a = \frac{2my}{r} = \frac{vvv}{2rb} = \frac{scvv}{rb} = \frac{4scm}{r}, = \frac{4ch}{s}$.

375. $h = \frac{s}{4c}a = \frac{ssvv}{4rb} = \frac{ssm}{r}, = \frac{1}{4}ttb$.

376. (By theo. 47) as (A D) $\frac{1}{2}$ a : (D F) 2 h, :: r : $n = \frac{4rh}{a}$ (= tangent ∠ C A B) and (by art. 167) $tt = \frac{na}{rb}$, ſo $tt = \frac{4rha}{rba} = \frac{4h}{b}$, and becauſe $\frac{4}{b}$ is conſtant, therefore $tt \propto h$, or $t \propto \sqrt{h}$, be the elevations and ranges what they will.

377. From art. 373 and 376, we have $tt = \frac{4ssm}{rb} = \frac{4h}{b}$, so $ssm = rh$, also by art. 273 $tt = \frac{ssvv}{rbb}$, or $rbbtt = ssvv$, so per last equation, $rbbtt = \frac{vvrh}{m}$, whence $v = bt\sqrt{\frac{m}{h}}$, or (by art. 375) $v = \frac{2\sqrt{rbh}}{S}$, &c.

378. If S = the sine of elevation, c = its co-sine, a = amplitude, i = impetus, h = height of the projection, t = time of flight, and n = tangent elevation of a gun g, and S, C, A, I, T, H, N, the like things of another gun G, r = radius, then (by art. 374) $\frac{4sch}{SS} = \frac{4rsc}{r}$, or $rh = iss$, as also, $rH = ISS$, now if $h = H$, then $iss = ISS$, so as $\sqrt{i} : \sqrt{I} :: S : s$.

379. If $i = I$, then $\frac{rH}{SS} = \frac{rh}{ss}$, or $SSh = ssH$, hence, as $\sqrt{h} : \sqrt{H} :: s : S$.

380. If $s = S$, then $\frac{rH}{I} = \frac{rh}{i}$, or $Hi = hI$, hence, as $I : i :: H : h$.

381. If $a = A$, then $ics = ICS$, hence, as $I : i :: sc : SC$.

382. Also, when $a = A$, then $Sch = sCH$, so as, $H : h :: Sc : sC$, but (by art. 176) $rs = cn$, so $\frac{c}{s} = \frac{r}{n}$, whence $\frac{4rh}{n} = \frac{4rH}{N}$, or $Nh = nH$, therefore, as $H : h :: N : n :: Sc : sC :: I : i$, (by art. 381 and 382) $:: TT : tt$ (by art. 376).

383. If e = the horizontal distance of any object and q, the time of flight thereunto, of the gun g, and E, Q, be the like things of the gun G, then (by art 368) as $A : T :: E : \frac{TE}{A} = Q$, and also $\frac{te}{a} = q$, now if $q = Q$, then $TEa = teA$, and if $H = h$, or $T = t$, then $Ae = aE$, and then as $A : E :: a : e$.

384. If v and V, be the velocities of the balls from the gun g and G, t and T, the times of flight over a and A, then (by theo. 153), $TV \propto A$, and $tv \propto a$, hence, as $TV : A :: tv : a :: E : e$, and if $T = t$, then as $V : v :: A : a :: E : e$ (or if $V = v$) as $T : t :: A : a :: E : e$.

385. (By art. 373) $\frac{ssvv}{rbb} = tt$, and $\frac{SSVV}{rbb} = TT$, and if $t = T$, then $ssvv = SSVV$, or $sv = SV$, hence, as $V : v :: s : S$, or if $s = S$, then $\frac{T}{V} = \frac{t}{v}$, or $Tv = tV$, so as $V : T :: v : t$, and when neither are equal, it is as $V : v :: \frac{T}{S} : \frac{t}{s}$, whence velocity, time and sine of elevation here, are the same as velocity, time, and space in uniform motion.

386 But the velocity of a projectile in any point of the path, is as the secant of the $\angle$ of its direction above the horizon, for A H, the horizontal velocity is the same at all points of the curve, and the velocity A G at A in the path A m E B, is the secant of the $\angle$ of elevation G A H, hence, the velocities at any two points in the curve A m E B, equally distant from E its middle, are equal.

387. If the body or ball be projected on an inclined plane A m, let v = the velocity of the ball in A, or the space it moves thro' in time 1, b = the space fallen thro' by a heavy body from rest in that time, s = $\angle$ C A m, c = sine $\angle$ G A P, or A G m, z = sine $\angle$ P A m, or A m H = sine $\angle$ A m G, and a = A m, then (by theo. 48) as $c : a :: z : \frac{az}{c} = AG$, and as $c : a :: s : \frac{sa}{c} = Gm$, but (by theo. 152) as $v : 1 :: (AG)\ \frac{az}{c} : \frac{az}{cv} = t$, time of describing A G, also (by theo. 166) as $b : \square\, 1 :: (Gm)\ \frac{sa}{c} : tt = \frac{sa}{cb} = \square$ time, falling from rest thro' G m, and that $t = t$ is manifest, because A G and G m are both described in the same time, therefore $tt = tt$, viz. $\frac{aazz}{ccvv} = \frac{sa}{cb}$, or $\frac{azz}{cvv} = \frac{s}{b}$, whence, $a = Am = \frac{scvv}{zzb}$, or $a \propto \frac{scvv}{zz}$, (because b is constant.)

388. If A d = m d, then e is the vertex of the path A e E m, in respect of the plane A m, and then (by art. 366) $de = \frac{1}{4}Gm = \frac{sa}{4c}$ (because $a = \frac{scvv}{zzb}$) $\frac{SSvv}{4zzb}$, which call h, viz. $de = h$, the height of the oblique projection.

389. Let t = time of flight, from A to m, then (by art. 387) $t = \frac{az}{cv} = \frac{Sv}{zb}$, (because $a = \frac{scvv}{zzb}$).

390. Hence, as $h : t :: \frac{ssvv}{4zzb} : \frac{sv}{zb} :: \frac{ssvv}{zz} : \frac{sv}{z}$.

391. Because $a = \frac{scvv}{zzb}$, therefore, z and b, continuing the same, a will be as S c, which will be greatest when S = c, or $\angle m A G = \angle G A P$.

392. Let m = the impetus, then (by art. 371) $m = \frac{vv}{4b}$, let e = A H n = tangent $\angle H A m$, U = m H, or L m, radius = 1, then (by theo. 47) $n e = u$, and $y a = u$ (y = sine $\angle H A m$ and z = its co-sine A m H) or $z a = e$.

393. Whence $e = z a = \frac{scvv}{zb}$, or $e \propto \frac{scvv}{z}, \propto \frac{scm}{z}$, or $= \frac{4scm}{z}$.

394. These things are true whether m be above, or below, the horizon A B, and from these equations, we get these following, viz.

395. $a = \frac{scvv}{bzz} = \frac{4scm}{zz} = \frac{e}{z} = \frac{u}{y}$.

396. $h = \frac{ssvv}{4bzz} = \frac{ssm}{zz} = \frac{sa}{4c} = \frac{es}{4cz} = \frac{us}{4yc}$.

397. $t = \frac{sv}{bz} = \frac{2s}{z}\sqrt{\frac{m}{b}}$.

398. These things may also be proved from the problem itself, thus, Let radius = r, and c, s, d, b, h, a, e, be the same as in art 359 and 369, then because, $\frac{es}{c} = \frac{rreeb}{ddcc} = H m = h$, (and by trigonometry) $rraa + cc = dd$, and $\frac{sa}{c} = b$, (taking b = B C, d = A C, a = A B) we'll have $\frac{es}{c} - \frac{ees}{ac} = h, = \frac{s}{ca} \times : ae - ee :$ but (by art. 176) $\frac{s}{c} = \frac{t}{r}$ (t = tangent $\angle C A B$) so $h = \frac{t}{ra} \times : ae - ee :$ from whence we get $A H = c = \frac{1}{2}a + \sqrt{: \frac{1}{4}aa - \frac{rah}{t} :}$.

399. If e and h, with a the greatest amplitude be given to find the elevation, then (per art. 369) $\frac{sca}{2rr}$ = present amplitude, whence h = $\frac{es}{c} - \frac{ees}{ca}$ becomes $h = \frac{es}{c} - \frac{rree}{2acc}, = \frac{te}{r} - \frac{:rr+tt:\times ee}{2arr}$ (because by art. 176, $rs = A$, and $\frac{rr}{cc} = \frac{rr+tt}{rr}$) $= 2arrh = 2aret - rree - ttee$, which solved gives $t = \pm \frac{r\sqrt{:aa-2ah-ee:}+ra}{e}$ (but when $e = AL$) $= \pm \frac{r\sqrt{:aa+2ah-ee:}+ar}{e}$ (but if $m = \frac{1}{2}$ the impetus be required, we'll have $m = \frac{1}{2}a = \frac{:tt+1:\times ee}{4et+4h}$, radius r being taken = 1, and $4et - 4h$, when $e = AH$, but $4et + 4h$, when $te = AL$, if the ∠ of elevation (t) be required, so as m the impetus may be the least possible, then by making t variable in $m = \frac{:rr+tt:\times ee}{4ret \mp 4rrh}$, we'll have $2tte \pm 2trh - rre - ett = 0$, = a minimum, whence $t = \mp \frac{rh}{e} + \sqrt{:\frac{rrhh}{ee}+rr:}$ but (by theo. 47) $rh = en$ (n = tangent ∠mAH), so $t = \mp n + \sqrt{:nn+rr:}$, but (by art. 176) $nn + rr = ss$ (s = secant ∠mAH) so $t = \mp n + s$, whence ∠mAG = ∠GAP (as by art. 391) so the required least impetus will be $= \frac{Hm+mA}{2} = \frac{se \mp rh}{2r} = \frac{1}{2}a$, or as r : t∠GAB :: e : a, Also, because $\frac{en}{r} = h$, we have (by art. 399) $\frac{1}{2}a = \frac{err+tte}{4rt \mp 4rn}$ (n = tangent m AH), the object's elevation or depression.

400. From the last art. we'll have $e = \frac{2ar\times:t\mp n}{rr+tt} = \frac{rd}{s}$ (s = secant ∠m AH, the inclination of the plane and $d = Am$) whence $d = \frac{2sa\times:t\mp n:}{rr+tt} = \frac{2sa}{cc}\times:t\mp n:$ (c = secant ∠ m AH, the elevation.

401. The greatest random upon a horizontal plane being (by art. 364) when the elevation is 45° but by reason of the air's resistance, this elevation must be somewhat less than 45°, and the ranges above 45° will scarce go so far, or those so much below 45°. Also, the lighter matter bullets are made, the greater will be the resistance,

for take two equal quantities of any matter ſuppoſe lead, and of the one make a globe, but of the other a ſphere of a double ſurface, then its plain the reſiſtance of the ſphere will be double to that of the globe, (every thing elſe being the ſame) for a double area or ſurface muſt ſtrike a double number of particles of the fluid. Hence, with the ſame charge of powder a ball will go much farther than a charge of ſmall ſhot of equal weight. But in the practice of gunnery the reſiſtance of the air is not regarded, becauſe when it is, the work is very tedious, and indeed need not to be obſerved much, becauſe the greateſt random of every piece is had by trial made with a proper ball and charge of powder, by which any other random &c. is found ; in this its plain this reſiſtance is in ſome meaſure conſidered, the theorems for this purpoſe where the reſiſtance of the air is conſidered are as follow.

Let $t =$ tangent $\angle$ of elevation, $s =$ its ſecant, $m =$ height, $e =$ the height fallen through to acquire the greateſt velocity in the medium, $c =$ co-ſine of the elevation that gives the greateſt random, $z =$ ſine of any elevation to an amplitude a, and $A =$ ſine of that elevation in vacuo ; then, if the reſiſtance of the medium be as the ſquare of the ball's velocity, we'll have (fig. 168).

$$1.\ h = ED = \frac{ttm}{ss} - \frac{4tttmm}{3esss} + \frac{2t^5mm}{3es^5} + \&c.$$

$$2.\ a = AB = \frac{4tm}{ss} - \frac{32ttmm}{3es^4} + \frac{16t^4mm}{3es^5} + \&c.$$

$$3.\ AD = \frac{2tm}{ss} - \frac{4ttmm}{es^3} + \frac{4t^4mm}{3es^4} + \&c.$$

$$4.\ cc = \frac{96e - 235m}{192e - 512m},\ \text{nearly.}$$

$$5.\ z = A + \frac{aa}{6em} + \frac{a^4}{192em^3} + \&c.\ \text{nearly.}$$

For the demonſtration of theſe, ſee art. 432 and 436.

Note. When the reſiſtance is $= 0$, then e is infinite, and then all the terms divided by e are $= 0$, then theſe theorems hold in a non-reſiſting medium as before.

But if the body be projected directly upwards with a given velocity g, then q being put $=$ the uniform velocity acquired by falling from reſt in the fluid in the firſt ſecond of time it will be 6, $\sqrt{2qe} =$ the greateſt velocity acquired by falling in the fluid 7, $m = e \times 2{,}302585$ times log. of $\frac{+}{-}\ \frac{gg \frac{+}{-} 2qe}{2qe + vv}$, ſee queſt. 233, and thoſe following it.

PROBLEM CCI.

Being as a SUPPLEMENT to the foregoing problems, ſhewing the nature of fluxions, and fluents, with ſeveral uſeful articles in mechanics, thereon depending.

402. Fluxion, according to Sir I. NEWTON, is the ſame with velocity, is by all allowed to be but a ſimple idea.

403. Whence, the fluxion of any quantity is but an idea of ſomething inconceivably ſmall, by whoſe continual motion the quantities or fluents are generated.

404. The very ſmall parts of the fluent which are generated in very ſmall parts of time, are called moments or increments, or if the fluent decreaſes, the parts continually deſtroyed are called decrements.

405. Moments being the effects of fluxions, fluxions are as moments generated in the ſame time, for the effects of like cauſes are proportional.

406. The velocity, variation, or quickneſs of increaſe (or decreaſe) of any fluxion is called the ſecond fluxion, and the ſame thing of the ſecond fluxion, is called the third fluxion, &c. for as the parts of a fluent may be generated, faſter or ſlower, ſo theſe parts will cauſe a variation in the fluxion &c.

407. If z be a flowing or variable quantity its fluxion is written thus $\dot{z}$, the fluxion of that fluxion, or ſecond fluxion of z thus $\ddot{z}$, the fluxion of that fluxion, or third fluxion of z thus $\dddot{z}$, &c. but becauſe theſe points above the letters are often wrong printed, I ſhall uſe different letters for fluxions.

408. In the following articles, e, y and R, are taken for the abſciſſa ordinate (or ſemi-ordinate) and curve reſpectively, and their reſpective fluxions are denoted by u, v, and z,

409. A conſtant quantity has neither increaſe nor decreaſe, and ſo can have no fluxion, ſo b u, is the fluxion of e b, as alſo of e b $\mp$ b $\pm$ f, if b and f, be both conſtant, or ſtanding quantities, and on the contrary, e b is the only fluent that will be found to u b, unleſs by the nature of the queſtion, as ſometimes it happens, there muſt be ſomething put to this fluent, and then it is called the correct fluent, or fluent corrected.

410. Let A R B (fig. 210) be a curve, draw F n parallel to, and very near E B; alſo, draw C B parallel to Q A, and D n, and tangent to the curve, in the point let e = A E, y = E B, R = arch A B, then if

the moment of y be $r n = v$, that of e will be $B r = E F = u$, and that of $R = n B = z$ that is v, u, and z (becauſe the moments are as the fluxions) are the fluxions of the ordinate, abſciſſa and curve reſpectively.

411. If by the parallel motion of y the area or ſpace E A B be deſcribed, then $u y$, viz. the very ſmall rectangle F E B r, will be as the fluxion of the ſaid ſpace, which if the equation of the curve be $c e^n = y$, will be $u c e^n = u y$, but (prob. 185) the area, viz. the fluent of this fluxion $u y$ is $= \frac{c e^{n+1}}{n+1}$, whence, the fluxion of any fluent $c e^n$ is $= n u c e^{n-1}$ and on the contrary, the fluent of the fluxion $n c e^{n-1} u$, is $= c e^n$, that is, the fluent of any expreſſion conſiſting of conſtant quantities, with only one variable quantity and its powers and fluxion, is found by the rule in prob. 185, and the reverſe of that rule will find the fluxion of ſuch a fluent.

412. From the laſt general fluxion may be had the fluxions of ſurds or of fractions, thus the fluxion of $\sqrt{:2ae - ee:}$ is $= \frac{au - eu}{\sqrt{:2ae - ee:}}$, for here $n = \frac{1}{2}$, ſo $n - 1 = -\frac{1}{2}$, and the fluxion of $2ae - ee$ is $= 2au - 2eu$, and ne^{n-1} is $= \frac{1}{2} \times \overline{2ae - ee}\big|^{-\frac{1}{2}}$, therefore, $n c e^{n-1} u$, is $= :2au - 2eu: \times \frac{1}{2} \times \overline{2ae - ee}\big|^{-\frac{1}{2}} =$ as before. Alſo, $\frac{a}{e} = a \times e^{-1}$, we have $n = -1$, ſo $n - 1 = -2$, therefore, $a n e^{n-1} u = -1 \times a e^{-2} u = -\frac{au}{ee}$, for the fluxion of $\frac{a}{e}$, &c. for any ſuch like.

413. If the fluxion of $e y$, the product of two variable quantities be required, let each be added to its moment, and then multiplied together i. e. $:e + u: \times :y + v: = ey + ev + uy + uv$, wherein uv, being very ſmall in reſpect of the other terms may be rejected, and then it will be $ey + ev + yu$, from which taking $e y$ the given product there leaves $ev + yu$, for the moment of $e y$, which is as the fluxion required, in like manner the fluxion of the product of $R e y$, three variable quantities will be found $= Rev + Ruy + zey$, &c. for any other number of variable quantities. Hence, if $e = R = y$, we'll have $2eu$ for the fluxion of ee, alſo $3eeu$ for the fluxion of eee, and in general $n e^{n-1} u$ for the fluxion of e^n the ſame as in art. 411, alſo, if $R = e + \frac{a}{y}$, we'll by what goes before, have $z = u -$

$\frac{av}{yy}$, again if $R = \frac{ae}{y}$, or $R = aey^{-1}$, then $z = \frac{au}{y} - \frac{ave}{yy} = \frac{auy - ave}{yy}$, if $R = \sqrt{:aa - ee:}$ then $z = \frac{-eu}{\sqrt{:aa - ee:}}$, if $R = \sqrt{:2ae - ee:}$ then $z = \frac{au - eu}{\sqrt{:2ae - ee:}}$, &c. &c.

414. In exponential expressions, or those whose indexes are variable, as y^e put R = hyperbolic log. of y, then y^e becomes eR, whose fluxion is $= ez + uR = \frac{ev}{y} + uR$, because (art. 416) $z = \frac{v}{y}$.

415. By these articles any expression may be put into fluxions, and tho' the reverse methods give the fluent, yet if the fluxion is altered as it often happens, it must have a different fluent, which when all other methods fail may be had by infinite series, as directed in prob. 185, therefore to find the fluent of any fluxion, try to find such an expression, as being put into fluxions may be exactly = the fluxion given, which if it be, you have hit on the fluent required, otherwise you must use infinite series.

416. It is proved in the construction of $\frac{1}{1+e}$, being put into a series &c. gives the hyperbolic log. of $1+e$, whence $\frac{u}{1+e}$, is the fluxion of the hyperbolic log. of $1+e$, but $\frac{u}{1+e}$ is = fluxion of $\overline{1+e}, \div : 1+e :$ i. e. the fluxion of any expression, divided by that expression is the hyperbolic log. of that expression, whence the fluent of such a fluxion will be the hyperbolic log. of the said expression, thus the fluent of $\frac{u}{e}$ is the hyperbolic log. of e, or $= 2,3058$ tabular log. e, also the fluent of $u = \frac{au}{\sqrt{:yy - aa}}$, is $e = 2,3058\, a \times$ log. $y + \sqrt{\overline{yy - aa}}$; again, the fluent of $\frac{au}{\sqrt{:2ae + ee:}}$ is $= 2,3058\, a \times$ log. $a + e + \sqrt{:2ae + ee:}$ fluent of $\frac{u}{\sqrt{:ee \pm aa:}} = 2,3058$ log. $e + \sqrt{\overline{ee \pm aa}}$, &c. for any such like.

417. Sometimes one fluent may be found by another fluent given, called the comparison of fluents. Thus, given A = fluent of $\frac{\dot{u}}{\sqrt{:aa+ee:}}$ to find B = fluent of $\frac{ee\dot{u}}{\sqrt{:aa+ee:}}$, here $\frac{ee\dot{u}}{\sqrt{:aa+ee:}} \times e = \frac{eee\dot{u}}{\sqrt{:aaee+eeee:}}$, then from a notion of fluxions it appears that the last expression wants $\frac{1}{2}aae\dot{u}$ in the numerator to make it a complete fluxion, viz. $\frac{\frac{1}{2}aae\dot{u}+eee\dot{u}}{\sqrt{:aaee+eeee:}}$, whose fluent will be $\frac{1}{2}\sqrt{\overline{a^2e^2+e^4}}$ $=\frac{1}{2}e\sqrt{:aa+ee:}$ from which take the fluent of $\frac{\frac{1}{2}aae\dot{u}}{\sqrt{:a^2e^2+e^4:}}$ the part added, which fluent as that of $\frac{e\dot{u}}{\sqrt{:a^2e^2+e^4:}} = \frac{\dot{u}}{\sqrt{:aa+ee:}}$ is given $=$ A, will be $\frac{1}{2}aa$A, whence $\frac{1}{2}e\sqrt{:aa+ee:}-\frac{1}{2}aa\text{A}$ $=$ B, the fluent sought; again, given A $=$ fluent of $\frac{\dot{u}}{\sqrt{:aa-ee:}}$, to to find B $=$ fluent of $\frac{ee\dot{u}}{\sqrt{:aa-ee:}}$. Here as before, the fluxion of $\frac{1}{2}\sqrt{:a^2e^2-eeee:}=\frac{1}{2}e\sqrt{:aa-ee:}$ is $=\frac{-\frac{1}{2}aa\dot{u}e-eee\dot{u}}{\sqrt{:a^2e^2-e^4:}}$ therefore, from $-\frac{1}{2}e\sqrt{:aa-ee:}$ take $\frac{1}{2}\text{A}a^2$ the fluent of $\frac{-\frac{1}{2}aae\dot{u}}{\sqrt{:a^2e^2-e^4:}}=\frac{-\frac{1}{2}aa\dot{u}}{\sqrt{:aa-ee}}$, and there leaves $-\frac{1}{2}\sqrt{:aa-ee:}$ $+\frac{1}{2}\text{A}aa=$ B, the fluent sought.

Note. A $= 2{,}3058$ log. $e+\sqrt{:a^2+e^2}$, in the first of these examples, viz. $=$ fluent of $\frac{\dot{u}}{\sqrt{:aa+ee:}}$ by the last art. and A $=$ arch of a circle, radius unity and sine $\frac{e}{a}$ $=$ fluent of $\frac{\dot{u}}{\sqrt{:aa-ee:}}$, for per similar Δs (fig. 210) as EB ($\sqrt{:aa-ee:}$) : QB (1) :: rB ($\dot{u}$) : nB z, there are other methods more general, for the comparison of fluents but they are very prolix, and so are here neglected.

418. In fluxional equations, where only one of the two variable quantities (e and y) enters, you may substitute for the ratio of the two fluxions ($\dot{u}$ and $\dot{v}$) and so find the required quantity; thus, if $a\dot{u}\dot{v}\dot{v}\dot{v}$ $=y\times\overline{\dot{u}\dot{u}+\dot{v}\dot{v}}|^2$, let $\dot{u}=n\dot{v}$ and then $an\dot{v}^4=y\times\overline{nn\dot{v}\dot{v}+\dot{v}\dot{v}}|^2$,

whence $y = an \div \overline{nn+1}|^2$, which in fluxions (taking $m =$ fluxion of n) is $v =: am - 3annm :\div \overline{nn+1}|^3$, whence $u = nv =: amn - 3annnm :\div \overline{nn+1}|^3$, now let $s =$ fluxion of w, and put $ww = nn+1$, then $ww - 1 = nn$, in fluxions $ws = nm$, which subſtituted in the value of u, gives $u = \frac{aws - 3aws \times : w^2 - 1 :}{w^6} = 4sw^{-5} - 3sw^{-3}$, and therefore e (the fluent of u) $= \frac{4aw^{-4}}{-4} - \frac{3aw^{-2}}{-2} = \frac{3a}{2ww} - \frac{a}{w^4} = \frac{3ann+a}{2 \times \overline{nn+1}|^2}$, as required. Again, if $ayuvvv = \overline{uu+vv}|^2$, put $nv = u$, then $aynv^4 = \overline{nnvv+vv}|^2$, so $y = \frac{\overline{nn+1}|^2}{an}$, which in fluxions is $v =: 3nn + 2 - \frac{1}{nn} : \times \frac{m}{a}$, conſequently $nv = u =: 3n^3 + 2n - \frac{1}{n} : \times \frac{m}{a}$, therefore $e =: \frac{1}{a} \times : \frac{3}{4}nnnn + nn - 2{,}3025 \log. n$, as required.

419. Fluxional equations, may alſo be ſolved by an aſſumed ſeries, as directed at the latter end of algebra, only here you muſt have equations for both the required quantity and its fluxion or fluxions, thus in $\frac{u}{1+e} = v$, or $u - v - ev = 0$, to find e in the terms of y, aſſume $e = Ay + Byy + Cy^3 + Dy^4 + Ey^5 +$ &c. and then the fluxion of e, viz. $u = Av + 2Byv + 3Cy^2v + 4Dy^3v + 5Ey^4v +$ &c. which values of e and u, ſubſtituted in $u - v - ev = 0$, gives

$$\left.\begin{matrix} Av + 2Byv + 3Cyyv + 4Dyyyv + \&c. \\ -v - Ayv - Byyv - Cyyyv - \&c. \end{matrix}\right\} = 0.$$

Whence $A - 1 = 0$, or $A = 1$, $2B - A = 0$, or $B = \frac{A}{2} = \frac{1}{2}$, $3C - B = 0$, or $C = \frac{1}{3}B = \frac{1}{2,3}$, $4D - C = 0$, or $D = \frac{1}{4}C = \frac{1}{2,3,4}$ conſequently $e = Ay + Byy + Cyyy +$ &c. $= y + \frac{yy}{2} + \frac{yyy}{2,3} + \frac{yyyy}{2,3,4} + \frac{y^5}{2,3,4,5} +$ &c.

Again, to find y in terms of e, in $ceeu + yu = av$, or $av - yu - yuceeu = 0$. Here assuming $y = Ae + Bee + Ceee + De^4 + \&c.$ as before we'll get

$$\left.\begin{array}{l} aAu + 2aBeu + 3aCeeu + 4aDeeeu + 5aEe^4u + \&c. \text{ for } a\dot{v} \\ 0 - Aeu - Beeu - Ceeeu - De^4u - \&c. \text{ for } y\dot{u} \\ 0 \quad\quad 0 - ceeu \end{array}\right\} = 0.$$

Whence $A = 0$, $2aB = A = 0$, $3aC = B + c$, or $C = \frac{c}{3a}$, $4a D = C = \frac{c}{3a}$, or $D = \frac{c}{3,4aa}$, $5E = D = \frac{c}{3,4aa}$ or $E = \frac{c}{3,4,5aaa}$, and consequently $y = Ae + Bee + Ceee + \&c. = \frac{ce^3}{3a} + \frac{ce^4}{3,4aa} + \frac{ce^5}{3,4,5a^3} + \frac{ce^6}{3,4,5,6a^4} + \&c.$

Again, to find e in terms of z in $a\dot{u}u + eevv - aavv = 0$, ($v$ being put for the fluxion of z), first, for the indexes of the assumed series, work as directed, at the latter end of algebra. Thus, write z^n for e, and $nz^{n-1}v$ for u, and then the indexes of z in the equation, will be $2n - 2$, $2n$, and 0, the two least of which ($2n - 2$ and 0) being put equal each other, we get $n = 1$, whence the indexes ($2n - 2$, $2n$ and 0) are 0, 2, 0, and the differences 0, 2, to which by continually adding 2, these turn out 0, 2, 4, 6, 8 &c. each of which added to 1 the value of n, gives 1, 3, 5, 7, 9, &c. for the indices of the assumed series, therefore, put $e = Az + Bz^3 + Cz^5 + Dz^7 + \&c.$ and to facilitate the operation, let $v = 1$, then $u = A + 3Bz^2 + 5Cz^4 + 7Dz^6 + \&c.$ which two values of e and u, squared and substituted in the given equation for ee and uu, it becomes

$$\left.\begin{array}{l} aaAA + 6aaABzz + 10aaACz^4 + 14aaADz^6 + \&c. \\ \quad\quad\quad\quad + 9aaBBz^4 + 30aaBCz^6 + \&c. \\ \quad\quad AAzz + 2ABz^4 + 2ACz^6 + \&c. \\ aa \quad\quad\quad\quad\quad + BBz^6 + \&c. \end{array}\right\} = 0.$$

Whence $AAaa = aa$, so $A = 1$, $6aaB = -AA$, so $B = -\frac{AA}{6aa} = \frac{-1}{6aa} = -\frac{1}{2,3aa}$, $10aaAC + 9aaBB + 2AB = 0$, so $C = \frac{1}{2,3,4,5a^4}$, $14aaAD + 30aaBC + 2AC + BB = 0$, so $D = -\frac{1}{14.36aa^4}$, consequently $e = z - \frac{zzz}{2,3aa} + \frac{z^5}{2,3,4,5a^4} -$

$\frac{z^7}{2,3,4,5,6,7a^6}$ required y, in $aaez - 2aauv + aeuu + eeey = 0$ (z being the second fluxion of y, and u and v, the fluxions of e and y as before) then putting e^n for y, the indexes will be $n-1$, $n-1$, 1 and $n+1$, where making $n-1=1$, we get $n=2$, whence, the differences being 0, 2, we must put $y = Ae^2 + Be^4 + Ce^6 + De^8 +$ &c. from which making $u=1$, we get

$$v = 2Ae + 4Be^3 + 6Ce^5 + 8De^7 + \&c.$$
$$z = 2A + 12Be^2 + 30Ce^4 + 56De^6 + \&c.$$

These values of v and z substituted in the given equation, &c. as before, we'll find $y = \frac{ee}{2a} - \frac{e^4}{4a^3} + \frac{e^6}{6a^5} - \frac{e^8}{8a^7}$, &c. this series, is the fluent of $\frac{2eu}{aa+ee} = 2{,}3058\frac{1}{2}a \times \log. \frac{aa+ee}{aa} = y$, also

Required y and e, from the two equations $yz = ru$, and $rv = rz - ez$.

Let $e = AR + BR^2 + CR^3 + DR^4 + ER^5 +$ &c. ($R =$ fluent of z) and $y = aR + bR^2 + cR^3 + dR^4 + eR^5 +$ &c. then by substitution and transposition, our two equations will become.

$$\left.\begin{array}{l} aRz + bRRz + cR^3z + dR^4z + fR^5z + \&c. \\ -rAz - 2rBRz - 3rCR^2z - 4rDR^3z - 5rER^4z, \end{array}\right\} = 0.$$

$$\text{and} \left\{\begin{array}{l} raz + 2rbRz + 3rcR^2z + 4rdR^3z + 5rfR^4z, \\ -rz + ARz + BR^2z + CR^3z + DR^4z + \&c. \end{array}\right\} = 0.$$

Whence, $A = 0$, $a = 2rB$, $b = 3rC$, $c = 4rD$, $d = 5rE$, by first equation and $a = 1$, $b = -\frac{A}{2r}$, $c = \frac{B}{3r}$, $d = \frac{C}{4r}$ &c. by second equation, therefore, $2rB = 1$, $3rC = \frac{-A}{2r}$, $4rD = \frac{-B}{3r}$, $5rE = -\frac{C}{4r}$, &c. consequently, $B = \frac{1}{2r}$, $C = 0$, $D = -\frac{B}{3,4rr}, = -\frac{1}{2,3,4r^3}$, $E = 0$, $F = -\frac{D}{5,6rr} = -\frac{1}{2,3,4,5,6r^5}$ &c. Also $b = 3rC = 0$, $c = 4rD = -\frac{1}{2,3rr}$ &c. &c. Whence $y = aR + bR^2 +$ &c. $= R - \frac{R^3}{2,3rr} + \frac{R^5}{2,3,4,5r^4} - \frac{R^7}{2,3,4,5,6,7r^6}$ &c. and $e = AR + BR^2 +$ &c. $=$

$\frac{RR}{2r} - \frac{R^4}{2,3,4r^3} + \frac{R^6}{2,3,4,5,6r^5} -$ &c.

Note. Here y = ſine, and e = verſed ſine of any circular arch R (fig. 210) = A B, radius Q B = Q A = r, for per ſimilar Δs as r : (BE) y : : (Bn) z : (Br) u, ſo y z = r u, alſo, as r : (Q E) r − e : : (Bn) z : (nr) v, whence r v = r z − z e, the ſecond given equation.

420. To ſolve problems or queſtions in fluxions, you muſt always by the nature of the problem get the moment of the thing wanted, whoſe fluent is the anſwer required, theſe moments are ſo ſmall, as in all caſes, to be looked upon as generated uniformly, or made up of ſtreight lines; moments increaſing or increments are poſitive, but if decreaſing, called decrements, are noted with the ſign —, what is ſaid about moments holds in fluxions art.

421. When any quantity or expreſſion, is required to be the greateſt, or the leaſt poſſible, its evident at that time its variable parts can neither increaſe, nor decreaſe, whence any expreſſion is a maximum, or a minimum, when its fluxion is made = 0, in ſuch caſes where there is but one variable quantity, the fluxion of that quantity being in every term, divides off, ſo in ſuch queſtions I leave it out, whereby it ſuits either this article, or the prob. for that purpoſe, in theſe articles (ſee art. 410) I take y, e, and R, for variable quantities, and v, u, and z, for their fluxions, when any thing elſe happens, it is mentioned. Theſe few foregoing articles, if well conſidered, will teach the principles of fluxions; what follow, are ſome very curious articles by way of illuſtration.

422. It has been proved that the reſiſtance of a body moving in a perfect fluid is as the ſquare of the ſine of the incident angle, if therefore A R B (fig. 210) be ſuppoſed to move in ſuch a fluid in direction C B parallel to its axis Q A, then the curve in the point B, (viz. the line n B D, being a tangent to the curve in that point) makes with the ſaid direction C B, the incident angle r B n (art. 410) therefore, the ſines of the ∠s being as their oppoſite ſides, it will be as z z : v v (viz. □ B n : □ r n) : : force of a particle of the fluid againſt the baſe B Q : force of a particle againſt the ſide at B, i. e. as 1 : $\frac{vv}{zz}$, now the quantity of the fluid which ſtrikes againſt the curve in B to reſiſt its motion, i. e. the reſiſtance of B n, is = r n (v) becauſe its motion is ⊥ n F, whence, the laſt ratio 1 to $\frac{vv}{zz}$, becomes v to $\frac{vvv}{zz}$, or becauſe z z =

$uu + vv)$ v to $\frac{vvv}{uu + vv}$, the moment of resistance of a plane, out of which by the equation of the curve exterminate u u, and the fluent of what leaves, is the answer, thus if the figure be a circle, radius QB $=$ QR $=$ r, then per similar $\triangle$s QBE and rnB, as QE : EB : : rn : rB, whence $uu = \frac{yyvv}{rr - yy}$, so v to $\frac{vvv}{uu + vv}$, becomes v to $\frac{vvv \times : rr - yy :}{yyvv + vv \times : rr - yy :}$, that is, v to $\frac{rrv - yyv}{rr}$, whose fluent is y to $\frac{rry}{rr} - \frac{yyy}{3rr}$, or 3 r r to 3 r r $-$ y y, and when y $=$ r it is 3 to 2, so is resistance against base, to resistance against circle or cylinderic surface

Again, If $e = cy^n$, then (art. 410) $u = ncy^{n-1}v$, so $vv = ccnny^{2n-2}vv$, which substituted in v to $\frac{vvv}{uu + vv}$, for uu, it becomes v to $\frac{v}{ccnny^{2n-2}}$, if the figure be a plane triangle moving in direction of e its perpendicular, base $=$ y, then $cc = \frac{ee}{yy}$ and $n = 1$, then the last general ratio is v to $\frac{v}{cc + 1}$, or (art. 410) y to $\frac{y}{cc + 1}$, that is, $ee + yy$ to yy, so is resistance of the base to resistance of the side, if the triangle move in direction parallel to y its base, then $cc = \frac{yy}{ee}$ so instead of $ee + yy$ to yy, we'll have $yy + ee$ to ee : : resistance base : resistance side.

Note. What is here meant by a plane figure, is a prismatic solid of any given depth, whose base is that (plane figure) floating on and parallel to the surface of the fluid, and its depth, or thickness perpendicular to the said surface.

423. The quantity of the fluid striking against z (B n) the surface of the small solid, formed by A D B, turning about A Q, will be as y v, so our ratio 1 to $\frac{vv}{zz}$, becomes y v to $\frac{yvvv}{zz} = \frac{yvvv}{uu + vv}$, then as in the last article with the nature of the generating curve, expunge u u, the fluent of what leaves will be the resistance. Thus, if the quadrant A B R, be turned about A Q, it will form a hemi-sphere, and as before

$\dot{t}\dot{u} = \frac{y y \dot{v} \dot{v}}{rr - yy}$, so the last ratio becomes $\dot{y}\dot{v}$ to $\frac{rr y \dot{v} - y y y \dot{v}}{rr}$ therefore (art. 410) $\frac{1}{2} yy$ to $\frac{1}{2} yy - \frac{yyyy}{4rr}$, and when $r = y$, it will be 2 to 1, : : resistance base to resistance solid. Again, if (as before) $c y^{n} = e$, our general ratio, will become $y\dot{v}$ to $\frac{y\dot{v}}{ccnny^{2n-2}+1}$ and if the solid be a cone, formed as before R B A, being supposed a streight line, A E = e, radius E B of the base = r, then n = 1, and $cc = \frac{ee}{rr}$, when the cone moves in direction E A of its axis, so the last ratio becomes $y\dot{v}$ to $\frac{y\dot{v}}{cc+1}$, whence (art. 410) $\frac{1}{2} yy$ to $\frac{yy}{2cc+2}$ or 1 to $\frac{1}{cc+1}$, or $ee + rr$ to rr, or ss to rr (s being = side of the cone) : : resistance base to resistance side, or convex surface of the cone, &c for any solid.

424. Let (fig. 211) A E = e E B = y, A Q = c, $B n = \dot{z}$ $r n = \dot{v}$, E F = u, arch C B = R, and suppose A Q ⊥, and C Q ∥ horizon, and that a heavy body is falling freely in the curve R B A, from R, at rest; it has been proved that the velocities of falling bodies (in vacuo) are as the square roots of the heights fallen thro', so the velocity at any point B is as $\sqrt{CB} = \sqrt{QE} = \sqrt{:c-e:}$, and the heights fallen thro' are as the times and velocities conjointly, whence $\dot{z} = \dot{t}\sqrt{:c-e:}$ so $\dot{t} = \frac{\dot{z}}{\sqrt{:c-e:}}$ = moment of the time of falling in any curve, by whose equation you may exterminate $\dot{z}$, the fluent of what leaves will be = T, the time of descent, thus if $ae = RR$ (a = any constant quantity) then (art. 410) $\sqrt{ae} = R$, so $\frac{\dot{u}\sqrt{a}}{2\sqrt{e}} = \dot{z}$, whence $\dot{t} = \frac{\dot{u}\sqrt{a}}{2\sqrt{:ce-ee:}}$ = the fluxion (art. 219) of a circular arch, versed sine = e and radius = $\frac{1}{2}$ c, and therefore these two moments or fluxions $\dot{t}$ and $\dot{z}$, are always as $\sqrt{a}$ to c, whence T must be always to that arch as $\sqrt{a}$ to c, but when e = c the said arch becomes a semicircle whose diameter is p c, and then as c : p c : : $\sqrt{a}$: p a = T, which being no ways affected with c, will still be the same, let the distance descended be what it will, that is all bodies descending, or all bobs of pendulums vibrating from any different points R, B, in such

a curve (called a cycloid) are performed in the same time. Again, if R B A (fig. 211) be a circular arch, $\frac{1}{2}$ a $=$ r $=$ O A its radius, then per similar $\triangle$s, or the property of the circle, $z = \frac{ur}{\sqrt{:ac - ee:}}$, whence $t = \frac{ur}{\sqrt{:c - e:} \times \sqrt{:ae - ee:}}$, which by an infinite series (prob. 185) will give the value of T, or by the comparison of fluents (art. 417) when c $=$ e, $T = 3{,}1416 \sqrt{\frac{a}{4}} \times : 1 + \frac{c}{2,2a} + \frac{3,3cc}{2,2,4,4aa} + \frac{3,3,5,5cc}{2,2,4,4,6,6,aaa} +$ &c. $=$ time of describing the arch R A.

425. If P A R (fig. 211) be a vessel filled with water, standing upon A, its vertex, axis A Q $\perp$ horizon, a hole at A to let the water run out, its plain the surface of the descending water describes the solidity of the vessel's cavity, therefore, let c $=$ A Q, e $=$ any height A E, y $=$ B D, &c. as before, then if $pe^n = yy$, $upe^n = uyyy$, is $=$ moment of solidity viz. as the fluxion of it, and because the velocity of water, or any other heavy body is ($\sqrt{e}$) the square root of the height fallen thro', there $\frac{pe^n u}{\sqrt{e}} = pe^{n - \frac{1}{2}}u = t$ (see the last art.) $=$ moment of time of evacuation, whose fluent (art. 410) is $\frac{pe^{n+\frac{1}{2}}}{n+\frac{1}{2}} = T$, so in a cylinder, or prism standing upright, n $=$ 0, p $=$ 1, whence $2\sqrt{e} = T$, a cone standing upon its vertex, n $=$ 2, so $\frac{2}{5}\sqrt{eeeee} = T$, a common parabolic conoid, standing on its vertex, n $=$ 1, so $\frac{2}{3}\sqrt{eee} = T$, if R A P be the segment of a sphere standing upon A its vertex, then $\frac{pu \times :ae - ee:}{\sqrt{e}} = t$ in fluents, $T = p \times : \frac{2ae^{\frac{3}{2}}}{3} - \frac{2e^{\frac{5}{2}}}{3} : = (\text{w}^n\ c = e)\ p \times : \frac{10ac^{\frac{3}{2}} - 6c^{\frac{5}{2}}}{15}$: if the segment stand on its base R P, then e must be $=$ Q E, put r radius of the sphere, then $pu \times : \frac{rr - ee}{\sqrt{e}} : = pu \times : rre^{-\frac{1}{2}} - e^{\frac{3}{2}} = t$, so in fluents $p \times : 2rr\sqrt{e} - \frac{2}{5}e^{\frac{5}{2}} : = T = (\text{if } c = e \text{ and } a = 2r)\ p \times : \frac{5aa\sqrt{c} - 4c^{\frac{5}{2}}}{10}$:

Let Q $=$ time in which this vessel would be emptied with the first or greatest velocity, for instance, suppose it to hold 69 gallons, and to run out 1 gallon in the first second of time, then at that rate it would

be emptied in 69 seconds $=Q$), then its solidity be $= pc \times : \frac{1}{4}aa - \frac{1}{3}cc$: it will be as $\frac{pc}{\sqrt{c}} \times : \frac{1}{4}aa - \frac{1}{3}cc$: is to Q, so is $p \times : \frac{5aa\sqrt{c} - 4c^{\frac{5}{2}}}{10}$ to T, the true time of emptying in seconds, and so on for any vessel.

426. If R B D P (fig. 211) be the frustum of a cone standing on the lesser base B D, let $e = EQ$, its height, $c = AE$, the height of the remaining part of the cone, then $p \times : \overline{c+e}|^2 = p \times : cc + 2ce + ee : = yy$ ($\square$ QP) so per last art. $\frac{pu}{\sqrt{e}} \times : cc + 2ce + ee : = t$, so (art. 410) $p \times : \sqrt{e} \times : 2cc + \frac{4}{3}ce + \frac{2}{5}ee : = T$, if the frustum stand on the greater base R P, put $e = QE$ its height, $c = QA$ the height of the whole cone, then $p \times \overline{c-e}|^2 = p \times : cc - 2ce + ee : = yy$ ($\square$ ED) so $\frac{pu}{\sqrt{e}} \times : cc - 2ce + ee : = t$, influents $p\sqrt{e} \times : 2cc - \frac{4}{3}ce + \frac{2}{5}ee = T$, if these two times are equal, the quantity run out in the first case, will be to the quantity run out in the second case, as $2cc + \frac{4}{3}ce + \frac{2}{5}ee$ to $2cc - \frac{4}{3}ce + \frac{2}{5}ee$, where, if $c = 0$, in the first, and $c = e$, in the second, then $e = AQ$, or the frustum becomes a cone, and then the last ratio, will become, as $\frac{2}{5} : \frac{16}{15}$, or as $3 : 8 ::$ quantity run out when the cone stands upon its vertex, to that when it stands upon its base, at an equal orifice, and in the same time, &c. for a frustum of any other solid.

427. Suppose the curve R A P (fig 211) to be a heavy flexible line hung over two pins R, P, parallel to the horizon, draw C r and Q A, each $\perp$ R P, also B D and r F each $\parallel$ R P, and B n a tangent to the curve in B, the part B A is sustained by 3 forces, first its gravity in direction B r, which is as the weight (suppose n) below the point B of suspension. Secondly, it is drawn at A, in direction $\parallel$ R P, or B D for if the curve were cut asunder at A, it would hang as two $\perp$s, on the two pins. Let $a =$ this force, which having nothing to do with the chains weight, is constant, or still the same. Thirdly, the force in direction B n, by which it is sustained; let $AE = e$, $EB = y$, $Bn = z$, $rn = v$, $Br = u$, then as $v : a$ (force at a $\parallel$ BD) $:: u : \frac{au}{v} =$ (force at B $\parallel$ QA) n, by which the ratio of weights hung to a flexible line, so as to make it form any proposed curve may be found, as follows.

428. If it must be a semicircle, radius $AQ = QR = QP = a$, $QE = e$, $EB = y$, then $yy = aa - ee$, in fluxions $yv = -eu$, whence $n = \frac{au}{v}$, becomes $n = \frac{-ay}{e}$, which in fluxions (putting m = fluxion of n, and a as before constant) is $m = \frac{ayu - aev}{ee}$, now if w = the force pressing the curve in the point B upon the small space z, then $m = wz = \frac{ayu - aev}{ee}$, but similar Δs (fig. 210) as $EB : QB :: rB : nB$, whence, $\frac{au}{y} = z$, so $wz = \frac{wau}{y} = \frac{ayu - aev}{ee}$ $=$ (because $yu = -eu$, or $v = -\frac{eu}{y}$) $\frac{auvy - auee}{yee}$ $=$ (because $yy + ee = aa$) $\frac{aau}{yee}$, whence $w = \frac{aa}{ee}$. Again, if $ae^r = y$, be the equation of the curve required, then in fluxions $rae^{r-1}u = v$, and squaring each side, $uu = \frac{vv}{rraae^{2r-2}}$, which put in $\sqrt{:}\, uu + vv :$ $= z$ instead of uu, we get $z = \sqrt{:}\, \frac{vv}{rraae^{2r-2}} + vv :$ in which putting yy for its equal aae^{2r}, we get $z = \frac{v}{ry}\sqrt{:}\, ee + rryy :$; or substituting $rraae^{2r-2}uu = \frac{uurryy}{ee}$ for vv we get $z = \frac{u}{e}$ $\sqrt{:}\, ee + rryy$ then from $rae^{r-1}u = v$, and $n = \frac{au}{u}$, we get $n =$ $\frac{1}{re^{r-1}}$, in fluxions $m = \frac{:rr - r: \times e^{r-2}u}{rre^{2r-2}} = \frac{rru - ru}{rre^r} = wz =$ $\frac{wu}{e}\sqrt{:}\, ee + rryy :$ so $w = \frac{re - e}{re^r \sqrt{:}\, ee + rryy :}$, &c. for any curve.

429. If the ratio of the weights be given, and the nature of the curve required. Let p = fluxion of n and q = that of v, also m = fluxion of n as before, then n being $= \frac{au}{v}$, its fluxion $m = \frac{avq - anp}{vv}$, and in $uu + vv - zz = 0$ taking z constant, we'll have (art. 410) $uq +$

$\dot{y}p = 0$, or $-\frac{\dot{v}p}{u} = \dot{q}$, which two equations cleared of q, we get m $= \frac{-\dot{a}v\dot{v}p}{uvv} - \frac{a\dot{u}p}{vv} = \frac{ap \times : u\dot{u} + v\dot{v}}{uvv} = \frac{ap\dot{z}z}{uvv}$. Hence if the curve R A P (fig. 211) be preſſed in each point by a force which is as any power r, of its diſtance from R P, we ſhall have $m = \dot{z}e^{r} = \frac{ap\dot{z}z}{uvv}$, whoſe fluent (z being as before, conſtant, and writing a^{r+1} for a, to make the equation of like terms) is $\frac{e^{r+1}}{r+1} = \frac{\dot{z}a^{r+1}}{v}$, the fluent of which will be the equation of the curve required, and may be found when r is known. Thus,

430. If the equation of the curve forming the ſtrongeſt arch poſſible be ſought, here it is plain all the particles of the arch muſt be of the ſame weight, each weight tending towards the horizon perpendicularly whence in this caſe $r = 0$, ſo our equation becomes $e\dot{v} = a\dot{z}$, which ſquared is $ee\dot{v}\dot{v} = aa\dot{z}\dot{z} = aa\dot{u}\dot{u} + aa\dot{v}\dot{v}$, ſo $\dot{v}\dot{v} \times : ee - aa = aa\dot{u}\dot{u}$, and $\dot{v}\sqrt{: ee - aa :} = a\dot{u}$, but in this caſe, (art. 428) $\dot{R} = \dot{n} = \frac{a\dot{u}}{\dot{v}}$, ſo $\sqrt{: ee - aa :} = R$, for the length of any arch A B (fig. 211), and when $AB = R = 0$, then $\sqrt{: ee - aa :} = 0$, or $a = e$, therefore, if the abſciſſa e, begin at the vertex A, we muſt take $a + e$ inſtead of e, and then $R = \sqrt{: ee - aa :}$ will become $R = \sqrt{: 2ae + ee :}$ the equation of the catenary ($AE = e$, $EB = y$, $AB = R$ (fig 211) $a =$ any conſtant quantity,) this equation in fluxions is $\frac{a\dot{u} + e\dot{u}}{\sqrt{: 2ae + ee :}} = \dot{z}$ ſquared is $\frac{: aa + 2ae + ee : \times \dot{u}\dot{u}}{2ae + ee} = \dot{z}\dot{z} = \dot{u}\dot{u} + \dot{v}\dot{v}$, take $\dot{u}\dot{u}$ from each ſide then $\frac{aa\dot{u}\dot{u}}{2ae + ee} = \dot{v}\dot{v}$, ſo $\frac{a\dot{u}}{\sqrt{: 2ae + ee :}} = \dot{v}$, then (art. 416) $2{,}3025\ a \times \log.\ a + e + \sqrt{: 2ae - ee :}$ but when $e = 0$, $y = 2{,}3025\ a \log.\ a$, therefore, the fluent corrected (art. 409) is $2{,}3025\ a \times \log.\ a + e + \sqrt{: 2ae + ee :} - 2{,}3025\ a \log.\ a = 2{,}3025\ a \times \log.\ \frac{a + e + \sqrt{: 2a + ee :}}{a} = y$,

in the same manner, you'll find $y = 2{,}3025\, a \times \log. \frac{R + \sqrt{: aa + RR:}}{a}$.

by any of which 3 equation the catenary may be constructed. Lastly, if the point B be so taken as $\angle nBE = \angle nBr = 45°$, then $rn = rB$, $(v = u)$ and then $R = \frac{au}{v} = a$.

Note. its plain that any flexible line (of little, or no weight) being pressed by the wind or any other fluid uniformly in direction perpendicular to R P, will put itself into this curve.

431. If the pressure be perpendicular to the curve, then draw O B perpendicular to B n, viz. to the curve in any point B, now if O B be the force in direction O B, O E, will be the power of that force in direction Q A, or C B, but by the similar Δs r n B and E O B, as O B $: rn :: m : \frac{vm}{z}$ = power in direction B C and (art. 427) as $z : v :: \frac{vm}{z} : \frac{vvm}{zz}$ = (because as before $m = \frac{apzz}{uvv} : \frac{ap}{u}$, the fluxion the pressure against the curve perpendicularly, in any point B, now the quantity of the curve pressed being $= z$, if as before this pressing force be as any power r, of the distance from R P, we'll have $ze^{r} = \frac{ap}{u}$, or $= \frac{pa^{r+1}}{u}$, so $zue^{r} = pa^{r+1}$, where z as before being constant, we'll have the fluent $\frac{ze^{r+1}}{r+1} = va^{r+1}$, where the index r being known, the fluent, or required equation of the curve will be found, thus, if it be required to find the nature of a curve that a flexible line will form when filled with a fluid of any kind. Here by the nature of fluids, the pressure of any point B, is as the height C B, whence $r = 1$, so our last equation becomes $zee = vaa$, which squared is $vva^4 = zze^4 = e^4 \times : uu + vv :$, whence $eeu = v\sqrt{: a^4 - e^4 :}$ whose fluent is the equation required, but if the fluid be air, the altitudes C B, Q A, &c. may be taken as equal (because in such small heights of the atmosphere the pressure differs but little) and then $r = 0$ and so $ez = av$, the equation of the circle (art. 421).

432. To find the velocity of a body descending, or ascending in a right line R C Q, (fig. 210) in a perfect fluid, where the resistance is as the square of the velocity. Let c = velocity (uniformly generated in the time r) with which the body begin its motion at R or Q, y = that in the point C, T = time of describing R C or Q C, e = R C, or

Q C, put u, v, t, for the moments of e, y, T, and suppose the first velocity c would be destroyed in the time n, by moving uniformly over a distance d, then as n : d : : r : c (for the spaces c, d, described with the same velocity, are as the times r, n, of description), whence, $\frac{c}{d} = \frac{r}{n}$, again, as n : c : : r : $\frac{cr}{n}$, or $\frac{cc}{d}$ (because $\frac{c}{d} = \frac{r}{n}$) that part of motion which would be uniformly destroyed in the time r; now the resistance at R, being to that at C as c c to y y, its plain the velocity destroyed in the time r, by a force = resistance at C, will be $\frac{c^*c}{d} \times \frac{yy}{cc} = \frac{yy}{d}$ = measure of resistance. Let b = velocity, generated in the medium, or fluid, by the force of gravity in the time r, then b added to, or taken from $\frac{yy}{d}$, according as the body ascends, or descends, gives $\frac{yy}{d} \pm b = f$, the whole force affecting the motion at C, but by the laws of motion, $\frac{u}{y} = r$, and $fr = \pm v$, so $f = \pm\frac{yv}{u} =$ (in this case, because y decreases while e increases) $-\frac{yu}{y}$, and therefore $\frac{yy}{d} + b = f = -\frac{yv}{u}$, whence $u = \frac{-dyv}{yy \pm bd}$, and $t = \frac{ru}{y} = \frac{-rdv}{yy \mp bd}$ the fluent of the first is (art. 416) $e = -2,3025 \times \frac{1}{2} d \times \log.\ yy \pm bd$: but when e = o, then c = y, and so the correct fluent is $e = -2,3025 \times \frac{1}{2} d \times \log.\ yy \pm bd : + 2,3025 \times \frac{1}{2} d \times \log. : cc \pm bd: = -2,3025 \times \frac{1}{2} d \times \log. : \frac{yy + bd}{cc \pm bd} : = 2,3025 \frac{d}{2} \times \log. : \frac{cc \div bd}{yy \pm bd}:$ if $bd = aa$, then the fluent of $t = \frac{-rdv}{yy - bd}$, is $\frac{ra}{b} \times D$, where D = difference of the two circular arches, $\frac{c}{a}$ and $\frac{y}{a}$, (radius = 1) so the time of the whole ascent is $T = \frac{ra}{b} \times \frac{c}{a}$.

Note, $\frac{c}{a}$ and $\frac{y}{a}$ are the tangents of the arches to be used, but when $t = \frac{-rdv}{yy + bd}$, or the body descends, then T, the time of descent

is $= \frac{ra}{2b} \times$ difference of the hyperbolic log. of $\frac{a+y}{a-y}$ and $\frac{a+c}{a-c}$, so, if $c = 0$, or the body fall from rest, $T = \frac{ra}{2b} \times 2{,}3025$ log. $\frac{a+y}{a-y}$

Note. $b = 32\frac{1}{2} \times \frac{B-M}{B}$, where B to M, is as the specific gravity of the body to that of the fluid, and $32\frac{1}{2}$ being the velocity acquired by gravity in the first second of time in vacuo.

433. If the body move directly forward, then $b = 0$, whence $\dot{u} = \frac{-dy^{-1}\dot{v}}{1}$, so $c = -2{,}3025 \times d \times$ log. y, which corrected (by taking $e = 0$ and $y = c$) gives $e = 2{,}3025\, d \times$ log. $\frac{c}{y}$, also $\dot{t} = -rdy^{-2}\dot{v}$, so $T = + \frac{rd}{y} =$ (when corrected) $\frac{rd}{y} - 1 = \frac{nc}{y} - 1$, (because $nc = rd$).

434. Since any expression is got a maximum, or a minimum, when its moment is made $= 0$, if therefore, the fluxion of any moment be made $= 0$, such a moment (allowing the expression) will be had a maximum or a minimum, if therefore, things be as in art. 410, 422, 423, 424, then $\dot{u}$, $\dot{u}y$, $\dot{u}yy$, $y\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:}$, $\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:}$, $\frac{y\dot{v}y\dot{v}}{\dot{u}\dot{u}+\dot{v}\dot{v}}$, $\frac{\dot{z}}{\sqrt{e}}$, are as the moments of the abscissa, area, solidity, surface, length of the curve, resistance, descent, respectively, so if $\dot{u} + b\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:} + c\dot{u}y + d\dot{u}yy + fy\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:} + \frac{hy\dot{v}\dot{v}\dot{v}}{\dot{u}\dot{u}+\dot{v}\dot{v}} + g\sqrt{:\frac{\dot{u}\dot{u}+\dot{v}\dot{v}}{e}:} = m$, a maximum, or a minimum, ($\dot{u}$ alone variable) we'll have $1 + \frac{\dot{u}b}{\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:}} + cy + dyy + \frac{fy\dot{u}}{\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:}}$ $\frac{-2h\dot{u}y\dot{v}\dot{v}\dot{v}}{\overline{\dot{u}\dot{u}+\dot{v}\dot{v}}|^2} + \frac{g\dot{u}}{\sqrt{:\dot{u}\dot{u}e+\dot{v}\dot{v}e:}}$ $0 =$, wherein the co-efficients a, b, c, d, &c. may be any constant quantities, positive or negative, as the nature of the quest. requires, any two or more of these terms made $= 0$, the equation of the curve &c. may be found, when the terms so taken are a maximum, or minimum, thus, if the length of the curve be a minimum or the area a maximum, then $cy + \frac{\dot{u}b}{\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:}} = 0$, so, $y\sqrt{:\dot{u}\dot{u}+\dot{v}\dot{v}:} = -\frac{b}{c}\dot{u} =$ (putting $r = -\frac{b}{c}$) $r\dot{u}$, squaring each side, $yy \times :\dot{u}\dot{u}+\dot{v}\dot{v}: = rr\dot{u}\dot{u}$, so $:rr - yy: \times \dot{u}\dot{u} = yy\dot{v}\dot{v}$, the equation of the circle, (art. 422).

435. If the equation of the generating curve be required, when the surface is a minimum or the solidity a maximum then $d\,yy + \frac{f\,y\,u}{\sqrt{:uu+vv:}} = 0$, whence $-uf = dy\sqrt{:uu+vv:}$, or $y\sqrt{:uu+vv:} = -\frac{f}{d}u =$ (putting $r = -\frac{f}{d}$) $r\,u$, and squaring each side, $yy \times :uu + vv: = rruu$, the same as before, viz. the solid is a globe, if the length and greatest diameter of a solid be given, to find the equation of the generating curve, when the convex surface is a minimum, or the solidity a maximum.

Note. The ordinate is always supposed given, because when the equation of the curve is determined, the abscissa may be found to it, so here we have $1 + \frac{f\,y\,u}{\sqrt{:uu+vv:}} = 0$, or putting $a = \frac{-1}{f}$ squaring each side &c. we get $uu \times :yy - aa: = aavv$, whence, $u = \frac{av}{\sqrt{:yy-aa:}}$, whose fluent, (art. 415) is e equal $2{,}3025\ a \times$ log. $\frac{y + \sqrt{yy-aa}}{a}$ (after correction) the equation of the catenary (art. 430) wherein y must always be greater than a, otherwise it will not hold. If the curve of swiftest descent be required, the abscissa is given, we'll have $1 + \frac{g\,u}{z\sqrt{e}} = 0$, or $gu = -z\sqrt{e}$, so $gguu = zze$, or (taking $\frac{1}{4}a = gg$) $\frac{uua}{4} = zze$, whence $z = \frac{u\sqrt{a}}{2\sqrt{e}}$, the same as in art. 424. the equation of the cycloid ($e = QE$, fig. 211) if the points R, A (fig. 211) or the abscissa A Q, ordinate R Q, and length R B A, of the curve be given, then $1 + \frac{gu}{\sqrt{:uue+vve:}} + \frac{ub}{\sqrt{:uu+vv:}} = 0$, or (because $zz = uu + vv$) we'll have $z\sqrt{e} + gu + ub\sqrt{e} = 0$, so $z = -\frac{gu}{\sqrt{e}} - ub =$ (writing $a = -g$, and $b = -b$) $\frac{au}{\sqrt{e}} + uu$, whose fluent shews in this case, the nature of the curve of swiftest descent. Further if the velocity at any point B, be as any power n of the distance C B (e), the last equation will be $z = \frac{au}{e^n} + ub = ae^{-n}u + ub$, which in fluents is $R = \frac{ae^{1-n}}{1-n} + eb$, $=$ ($b = 0$ and $n = \frac{1}{2}$) $2a\sqrt{e}$, or $\sqrt{e}\,a$, the equation of the cycloid as before.

If the length and greatest diameter of a solid be given, required the equation of the generating curve, when the solid is that of least resistance, here $1 + \frac{-2hyuvvv}{zzzz} = 0$, whence, (putting $a = 2h$)

$a\,y\,u\,v\,v\,v = z^4$; again, if instead of the greatest diameter, or length, the solidity be given, then $d\,y\,y + \frac{-2\,h\,y\,u\,v\,v\,v}{z\,z\,z\,z} = 0$, whence $y\,z^4 = a\,u\,v^3$. Also, if the length, greatest diameter, and solidity be given, then $1 + d\,y\,y + \frac{-2\,h\,y\,u\,v\,v\,v}{z\,z\,z\,z} = 0$, if the length and surface be given, then $\frac{f\,y\,u}{z} + \frac{-2\,h\,y\,u\,v\,v\,v}{z\,z\,z\,z} = 0$, whence $f\,z^3 = 2\,h\,v\,v\,v$ and putting $a = \frac{2h}{f}$, it will be $a\,v\,v\,v = z\,z\,z$, which being no way affected with any of the variable quantities (e, y, R) will still be the same, and so the figure will be a conoid or the frustum of one, and it will be the same when the length, or ordinate and area of the generating plane is given, for then $c\,y + \frac{-2\,h\,y\,u\,v\,v\,v}{z\,z\,z\,z} = 0$, so $a\,u\,v\,v\,v = z\,z\,z\,z$.

436. If a body, instead of ascending directly upwards (art. 432) &c. be projected obliquely, it will describe a curve as A B C (fig. 209) whose equation may be thus found, draw B Q, perpendicular to the horizon, or parallel to A H the axis of the curve, and P B ⊥ A H, let A P = e, P B = y, arch A B = R, then their fluxions, B M = N b = u, M N = B b = v, B N = z. Also, let r = second fluxion of e, and s = third fluxion thereof, now if Q = velocity of the body at B in direction P B, the decrease of velocity in the said direction is caused by the resistance alone (art. 395) and must therefore be = − q, the fluxion of Q, and therefore, as $v : z :: -q : -\frac{qz}{v}$ = decrease of motion in direction B N and as $v : u :: -q : -\frac{qu}{v}$ = that in direction B M, also so is Q to $\frac{Qu}{v}$ = velocity in direction B M, whose fluxion $\frac{Qr + qu}{v}$ must be as the whole alteration of motion in that direction, both gravity and resistance considered, from which taking $\frac{qu}{v}$, the part owing to the resistance, there leaves $\frac{Qr}{v}$ for the effect of gravity, whence as $\frac{Qr}{v} : -\frac{qz}{v} :: 1 : -\frac{qz}{Qr}$ :: force of gravity : force of resistance, in direction B N, but in falling bodies, the velocities are as the times, therefore the velocity generated by gravity in the time ($\frac{v}{Q}$, space ÷ velocity) of describing B b, with velocity Q, will be as

$\frac{v}{Q}$, for another expression of the effect of gravity, so $\frac{v}{Q}=\frac{Qr}{v}$, or $QQ=\frac{vv}{r}$, but to clear this equation of Q, let $2Qq=-\frac{vvs}{rr}$, the fluxion thereof be divided by $QQ=\frac{vv}{r}$, the fluent, and then $\frac{Q}{q}=\frac{-s}{r}$, whence $-\frac{qz}{Qr}$ becomes $\frac{z\dot{s}}{2rr}$, for the true force of resistance the weight, of the body, or gravity $=1$.

437. If the resistance be as the product of the square of the velocity and (d) the density of the medium, then $\frac{Qz}{v}$ being $=$ absolute velocity at B, we'll have $\frac{QQzz}{vv}=$ resistance at that point, and therefore $\frac{QQzz}{vv}d\times\frac{v}{Q}=$ velocity destroyed thereby in the time $(\frac{v}{Q})$ of describing B N, which put $=-\frac{qz}{v}$, the effect of the same resistance found before, and you'll have $d=\frac{-q}{Qz}=\frac{s}{2zr}$, by writing $\frac{-s}{2r}$, for its equal $\frac{q}{Q}$, whence, $s=2rzd$, or $ss=4rrzzdd=4dd\times:uu+vv:\times rr$, now (art. 419) we'll find $e=\frac{ayy}{2}+\frac{2ady^3}{9}+\frac{addy^4}{6}+\frac{4ad^3y^5+a^3dy^5}{60}+\frac{5a^3d^3y^6+4ad^4y^6}{120}$ &c. when the body is in the descending part of the curve, and if the signs are made negative in every term, where the odd powers of d are found the series will then be for the ascending part of the curve. If there be no resistance, or $d=0$, then $e=\frac{1}{2}ayy$, so $\frac{2e}{a}=yy$, the equation of the common parabola, as in art. 359, perameter $=\frac{2}{a}$, but if it be as $p:1::$ the density of the body : density of the medium, and if the body be a globe, the diameter $=D$, then the above density $d=\frac{3}{2pD}$ (art. 327) and $\frac{p}{p-1}\times\frac{2}{a}=h$, the perameter of the perabola, that would be described in vacuo, by a body having the same velocity at the vertex A, whence $a=\frac{2p-2}{ph}=\frac{2}{h}$, nearly, whence e is found, when y is any given ordinate.

438. In the two laſt, as alſo in ſome foregoing articles (as is common in the ſolution of ſuch problems) I have made ſome one of the firſt fluctions, of ſome one variable quantity (e, y, z,) conſtant which both avoids trouble, and ſerves as a ſtandard, to refer the variable fluctions of the other quantities too.

439. Fluctions of all orders, are contemporaneous, or generated with their reſpective velocities in one and the ſame infinitely ſmall part of time (art. 402) alſo (art. 409) any variable quantity can have but one fluxion, of the ſame order but any fluxion may have many fluents out of which one muſt be taken, that ſuits the conditions of the problem.

440. Since, $\frac{zs}{2rr}$ is to 1, as the reſiſtance is to the gravity (art. 437) we may from the given equation of any curve, find the ratio of the reſiſtance to the gravity, when the body deſcribes that curve; thus, if the curve be a common parabola, and $ae = yy$, in fluxions $au = 2yv$, in ſecond fluxions $ar = 2vv$, in third fluxions $sa = 0$, whence $\frac{zs}{2rr} = 0$, and therefore to deſcribe ſuch a curve, the body muſt meet with no reſiſtance, but move in vacuo. Again, in a cubic parabola, where $ae = yyy$, we have $au = 3yyv$, and $ar = 6yvv$, alſo, $sa = 6vvv$, which ſubſtituted in $\frac{zs}{2rr}$, for r and s, gives $\frac{az}{12yv}$ for the reſiſtance when the gravity is 1, again if R B A (fig. 210) be a quadrant of a circle, you'll find $\frac{zs}{2rr} = \frac{3u}{2z} =$ (per circle) $\frac{3\,CB}{2\,QB}$, that is, as 3 C B : 2 Q B :: reſiſtance : gravity, and v the velocity, will be every where as $\sqrt{BE}$.

441. In an upright veſſel A B A B (fig. 163) kept conſtantly full of water &c. to find the quantity thereof diſcharged in any given time thro' an orifice at the bottom B. Let $V =$ the deſcent of the water's center of gravity, acquired by the fall of its ſurface from A to B, $e =$ A B, which will alſo be as the deſcent of the ſaid ſurface A A at B, R equal the quantity of water in the veſſel, v, u and z, the reſpective fluxions of V, e and R. Alſo, put $A =$ area of the water's ſurface, and $a =$ area of the orifice at B, then V R, will be e momentum of the water in A B A B, whoſe fluxion $F = Vz + Rv$, ſuppoſe at the beginning of the evacuation the column of water A B A B, becomes g B m n B g. Now the time of running out muſt be inverſely as the orifice (for the greater the one, the leſs the other) and the deſcents of heavy bodies being as the ſquares of the times of deſcent, it will be as A A : e :: a a : $\frac{aae}{AA} =$ the deſcent of the ſurface A A to q g, and

therefore, $e - \frac{aae}{AA} =$ the difference of these two descents, whose fluxion $\dot{u} - \frac{aa\dot{u}}{AA}$, multiplied by A, gives $A \times : \dot{u} = \frac{aa\dot{u}}{AA} : = f$, for the fluxion of the momentum of the water in g B m n B q which (it is plain) must be $= z$, and because $A e =$ the water in A B A B, therefore $A e = R$, which values of z and R, substituted in $F = V z + R \dot{v}$, we get $A V \dot{u} + A e \dot{v} - \frac{aaV\dot{u}}{A} - F$, again, since $A e =$ water in A B A B, its fluxion $A \dot{u}$, multiplied by e, the descent of the surface A, A, gives $A \dot{u} e = F$, the fluxion of the same momentum, and therefore $A V \dot{u} + A e \dot{v} - \frac{aaV\dot{u}}{A} = A \dot{u} e$ or (putting $n = 1 - \frac{aa}{AA}$) $n A V \dot{u} + A e \dot{v} = A e \dot{u}$, or $n V \dot{u} + e \dot{v} = e \dot{u}$, multiply each side by e^{n-1}, and then, $n V e^{n-1} \dot{u} + e^n \dot{v} = e^n \dot{u}$, whose fluent (art. 413) is $V e^n = \frac{e^{n+1}}{n+1}$, so $V = \frac{e}{n+1}, = e \div : 1 - \frac{aa}{AA} :$, whece $(2 - \frac{aa}{AA} = 1 + n$ because $n = 1 - \frac{aa}{AA}$, $V = \frac{AAe}{2AA - aa}$.

442. If the water in the vessel A B A B, descends without being supplied, or its surface A, A, descends by its own gravity, it is plain that the velocity acquired in the descent thro' A q. is accelerated, whilst that in the descent thro' B m is for a small time uniform, and therefore describes a double space (theo. 167) in the same time to what it would describe if accelerated, so that in this case, to find the difference of descents, mentioned in the last art. we must take the descent $\frac{aae}{AA}$, thro A q, from 2 e, twice the descent thro' B m, in order to have $2\dot{u} - \frac{aa\dot{u}}{AA}$, the fluxion of the said difference, which wrought as in the last art. we'll find $V = \frac{AAe}{3AA - aa}$.

443. If a body B (fig. 211) or a pendulum be oscillating in a cycloid R A P, and be resisted by an uniform force, which is to the force of gravity as m to 1, and also by the fluid or medium in which it moves where the resistance is as the square of the body's velocity, to find the time of oscillation &c. let d = as in art. 432, or to the small arch B m, as the whole motion of the body at B to that lost in moving thro' B m, by the resistance of the fluid, put e B = R, B m = z, A e = a, A R or the pendulum's length = b, h E = e = the distance the body must freely fall in vacuo by a force = its specific gravity in the fluid, to acquire the velocity it has at B, then because (art. 424) the descents from

any points R, e, B, &c. in a cycloid to A, the bottom is made in the same time, it will be as b is to 1 the force of gravity so is a — R to $\frac{a-R}{b}$ = that part of gravity which accelerates the motion of the body at B, and because the uniform velocity acquired by falling thro' any distance e is (theorem 167) = 2e, therefore, as d : 1 : : 2e : $\frac{2e}{d}$ the resistance caused by the fluid, which added to m, the other part of resistance, and that taken from $\frac{a-R}{b}$, gives $\frac{a-R}{b} - m - \frac{2e}{d}$, for the whole accelerative force of the body at B, now (theo. 151) $t = \frac{s}{v}$, so $\frac{z}{\sqrt{2e}}$ = time of describing B m, and (theo. 152) $v = ft$, that is : $\frac{a-R}{b} - m - \frac{2e}{d} : \times \frac{z}{\sqrt{e}}$ = velocity generated in that time, which, its plain, must be $= \frac{u}{\sqrt{2e}}$, the fluxion or increase of $\sqrt{2e}$, the velocity at B, hence, we have $\frac{a-R}{b} - m - \frac{2e}{d} = \frac{u}{z}$, or putting $c = a - mb$, it will be $-\frac{c}{b} + \frac{R}{b} + \frac{2e}{d} + \frac{u}{z} = 0$, which (art. 419) solved gives $e = \frac{cR}{v} - : \frac{a-2c}{2bd} : \times : RR - \frac{2RRR}{3d} + \frac{4R^4}{3,4dd} - \frac{8R^5}{3,4,5d^3}$ &c. which series substituted in $\frac{z}{\sqrt{2e}} = t$, instead of e, will give the fluxion of the time, whose fluent is found, $T = 3,1416 \sqrt{b} \times : 1 + \frac{cc}{6dd} - \frac{2ccc}{9ddd}$ &c. $\propto$ the time of one oscillation. When the body comes to I, the height of its ascent, then $e = 0$, and from the above value of e we'll find $R = 2C - \frac{4cc}{3d} + \frac{16ccc}{3dd}$ = arch B A I, whence arch $ID = 2a = 2c + \frac{4cc}{3d} - \frac{16cc}{9dd}$ &c. = (because $c = a - mb$) $2mb + \frac{4cc}{3d} - \frac{16ccc}{9dd}$, &c.

The End of the Second Part.

THE UNIVERSAL

MEASURER

AND

MECHANIC.

PART THIRD.

CONTAINING

1. *Decimal arithmetic made easy, with new constructions; and the extraction of roots.*
2. *Description, construction, and use of Coggleshal's sliding-rule.*
3. *Multiplication of feet and inches, generally called cross multiplication.*
4. *Superficial measure of planes and of solids; with the methods of taking dimensions with, or without inches.*
5. *The several artificers works.*
6. *Mensuration of solids.*
7. *Surveying, plotting, and dividing of lands.*
8. *Gauging in all its parts. With the reduction of measures, description of instruments, new improvements on the sliding-rule, gauging without inches, &c.*
9. *Questions and solutions; exercising mensurations, surveying, gauging and mechanics: as the mechanic powers, pendulums, musical strings, barometers, pumps, mills, engines, gunnery, &c.*

SECTION I. *Notation of Decimals.*

EXPLANATIONS.

1. IF the denominator of any fraction be 1 and cyphers, such a fraction is called a decimal fraction, as $\frac{5}{10}$, $\frac{5}{100}$, &c. are decimal fractions. Now if for the unit 1 in the denominator we write a comma, or full point . and suppose every figure in the numerator to possess the place of a cypher in the denominator, then the above fractions will stand thus ,5 ,05 &c. whence decimal fractions may be expressed without their denominators, and so wrote in one line like whole numbers, being always careful of the comma and cyphers on the left hand of the figures; for the comma is in the place of units, the next to it the place of tens, the next in the place of hundreds, &c. i. e. $\frac{5}{10}$ = ,5 or = 0,5, which is 5 tenths or no whole numbers 5 tenths, and $\frac{35}{100}$ = ,35 = 3 tenths 5 hundred parts, or 35 hundred parts of an unit, and $\frac{35}{1000}$ = ,035 = no tenths, 3 hundreds, 5 thousands, or 35 thousand parts of an unit, &c. Cyphers on the right hand side of decimals are of no value and may be rejected, except for illustrations sake, $\frac{5}{10} = \frac{50}{100} = \frac{5000}{10000}$ i. e. ,5 = ,50 = ,5000, &c. each of these being half of an unit; for if an unit be divided into 10 = parts 5 is half of them, if 100 = parts 50 is also half of them.

2. If an integer or unit be divided into 10, 100, 1000, &c. = parts, the figures in the decimal shew how many of such parts are to be taken; so in ,25 the integer, whether a foot, yard, pound, &c. is divided into 100 = parts, and ,25 denotes 25 of them; also ,2345 denotes ,2345 parts of 10000. Now because 10 × 10 = 100, and 100 × 10 = 1000, it follows that the 10's are 10 times greater than the 100's, the 100's ten times greater than the 1000's &c. if the figures were all =; so that if ever so many figures follow the ,2 it is greater than them all, even if they were so large as to be all 8's or all 9's; i. e. unity is greater than ,99999, $\frac{1}{10}$ is greater than ,099999, &c.

3. Therefore ,2345 may be taken for ,2345897, or ,0028 for ,00287648 &c. for all the figures after 5, viz. 897 do not make $\frac{1}{10000}$, nor all after 8, viz. 7648 &c. do not amount to ,0001. But if two or more decimals are to be wrought with, and you would be more exact, you may take the sum of all the figures after the breaking off figures, and add the number of points to any one of the breaking off figures; thus if ,23456 ,007892, and ,013479602 were the decimals, and you would work with three places in each, viz. ,234, ,007, and ,013, here the sum of two of the following figures in each is 56 + 89 + 47

= 192, and becauſe it is nearer 200 than 100, add 2 to ſome of the breaking off figures 4, 7, 3, ſuppoſe to 3, and then the 3 contracted decimals will be ,234 ,007 and ,015 which in working will anſwer nearly the ſame end as if you work with the three firſt taken ones. For the ſame reaſons 3,1416 may be taken for 3,1415927 &c. But if the figure following the breaking off figure be under 5 this increaſe of 1 may be too much, as 3,78912 you may take 3,79 in a common way, but to be more exact 3,789 may be taken, &c. for others as you'll find made uſe of in the following pages, where 3 or 4 places of decimals are thought ſufficiently exact in common caſes, which method ſpares a great deal of labour.

In addition and multiplication theſe methods may anſwer, but in ſubtraction and diviſion it is plain, if one of the places be increaſed by unity, &c. and not the other, the error muſt conſequently be greater.

Reduction of Decimals.

To reduce a vulgar fraction to a decimal one of the ſame value.

Rule 1. Annex or bring down cyphers to the numerator, and divide it by the denominator till nothing remain, (if poſſible) and the quotient is the decimal required.

Ex. 4. 5. and 6. Reduce $\frac{1}{2}$, $\frac{1}{20}$, and $\frac{1}{200}$, each to decimals.

2)1,00(,5 = $\frac{1}{2}$ 20)1,00(,05 = $\frac{1}{20}$ 200)1,000(,005 = $\frac{1}{200}$
10 100 1000

7. If the numerator and denominator of any fraction be each multiplied by one and the ſame number, the fraction will ſtill retain its firſt value, ſo in $\frac{1}{200}$, if the numerator 1 and denominator 200 be each multiplied by 5, we ſhall have $\frac{1}{200} = \frac{5}{1000} =$,005.

8. Hence to determine the value of any decimal, ſay how often the whole denominator 200 in the numerator 1, it goes no times, ſo make a comma in the quotient place; then ſay how often the whole denominator 200 in the whole number 1 and one cypher, it goes no times again, ſo write 0 after the comma; again, how often 200 in 1 and two cyphers, it yet goes no times, ſo write a ſecond cypher in the quotient; again try with 200 in 1 and three cyphers, and finds it goes juſt 5 times ſo ſetting 5 in the quotient after the ſecond 0 you find the decimal ,005 = $\frac{1}{200}$, the given fraction. Obſerve the ſame method in all examples of this kind, and you'll eaſily find the value of the decimal, which is the chief thing in decimals; for by writing 5 for 0,5, or 0,5 for ,005, the error in the end may be much greater than if you uſed no decimals or parts at all, ſince decimals are only for working ſuch parts as are leſs

than a unit of the integer of the whole part. i. e. having taken the decimal of these parts, you join it to the right hand side of the whole numbers, and so work with that mixt number instead of the given one, as 7.5 for $7\frac{1}{2}$, ,5 being the decimal of $\frac{1}{2}$.

Ex. 9. Reduce $\frac{5}{13}$ to a decimal.

13)5,000(,384615,384615, &c.
5 the remainder.

Here I annex 3 cyphers and bring down 3 more, and then find that 5 remains, which 5 is = to the first dividend, and therefore the figures in the quotient will return again or circulate in the same order as before; so therefore add as many figures to the quotient as you please with continuing the division the more the nearer the truth you'll come, as by ex. 2 and 3. tho' it never will be quite exact. These are called repeating, circulating, or infinite decimals.

Ex. 10, 11, 12. Reduce $\frac{1}{3}$ and $\frac{2}{3}$ and $\frac{11}{3} = 3\frac{2}{3}$ each to decimals.

3)1,00(,3333 + = $\frac{1}{3}$
9
———
1 &c.

3)2,00 $\left\{\begin{array}{l},6666+\\,6667-\end{array}\right\} = \frac{2}{3}$
18
———
2 &c.

3)11,00 $\left\{\begin{array}{l}3,666+\\3,667-\end{array}\right\} = \frac{11}{3}$
9
———
20
18
———
2 &c.

See example 3.

The sign + annexed to any number shews it to be too little, but — denotes it something too much.

Ex. 13, and 14. Reduce 5 inches to the decimal parts of a foot, and also of a yard. Here 12 inches are one foot, and 36 one yard, so we only have $\frac{5}{12}$ and $\frac{5}{36}$ to reduce.

12)5,000(,4167— feet.
8 the remainder

36)5,000(,1389— yards.
32 the remainder

Ex. 15, 16, 17. Reduce $2\frac{1}{2}$ or 2,5 inches to the decimal of a foot; also 17s 6d, and $8\frac{1}{2}$d, each to the decimal of a pound sterling.

24)5,000
———
$2\frac{1}{2}$ in. = ,2083 f.

40)35,00
———
17s 6d = ,875 £.

480)17,000
———
$8\frac{1}{2}$d ,35416 £.

Otherwise.
12)2,5000
———
$2\frac{1}{2}$ in. = ,2083 f. = $\frac{2\frac{1}{2}}{12}$

First. Becauſe $2\frac{1}{2}$ inches is $=$ 5 half inches, and 24 half inches $=$ the integer 1 foot, therefore divide 5 and cyphers by 24 or 2,5 and cyphers by 12. Secondly. 17s 6d is $=$ 35 ſixpences and 40 ſixpences, $=$ the integer 1 pound. Alſo $8\frac{1}{2}$ d $=$ 17 half pence and 480 halfſpence $=$ 1 pound. And thus may coin, weight, time meaſure, &c. be reduced. So if the integer be a foot, the decimal of $1\frac{1}{2}$ inches will be $,125 = \frac{1\frac{1}{2}}{12} = \frac{1}{8}$ of 3 inches $= ,25 = \frac{3}{12} = \frac{1}{4}$, of 9 inches $= ,75 = \frac{9}{12} = \frac{3}{4}$ of $\frac{1}{2}$ an inch $= ,0416$ &c. &c.

Addition and Subtraction of Decimals.

Rule 2. Theſe are performed in all reſpects like whole numbers, regard being had to place units under units, (viz comma's under comma's) tens under tens &c. both in the integers and decimals.

Examples in Addition.

Ex. 18.	Ex. 19.	Ex. 20.	Ex. 21.
,0057	,785623	25,687	7,5
,0002	,02	1032,02	27,07
,06	,5	4,5	123,009
,083	,079	2432,0689	6,0008
,1489	1,384623	3494,2759	163,5798

Examples in Subtraction.

	Ex. 22.	Ex. 23.	Ex. 24.	Ex. 25.
From	,3123	25,02	270	1
Take	,0102	3,98	,028	,98769
	,3021	21,04	269,972	,01231

Multiplication of Decimals.

Rule 2. Having multiplied the factors together as if they were whole numbers, prick off as many decimal places in the product towards the right hand as there are in both factors, viz: in both the multiplier and multiplicand. For if 10 and a 100, or 100 and 1000, &c. be the denominators of two decimals, their products will be $10 \times 100 = 1000$, and $100 \times 1000 = 100000$, i. e. there will be as many cyphers in the product as there are in both factors.

Note. If there be not a ſufficient number of figures in the product to prick off, you muſt ſupply that want by prefixing cyphers.

Ex. 26.	Ex. 27.	Ex. 28.	Ex. 29.
Mul 3,024	32,12	,0347	,000627
by 2 23	24.3	,0236	1,0002
9072	9636	2082	1254
6048	12848	1041	627000
6048	6424	694	
6,74352	780,516	,00081892	,0006271254

30. When the factors together contain more places of decimals than you chuse to have in the product, they may be contracted in the working to any number of places, and the product true to that number as if the factors had been multiplied at large. Thus, place the units place of the multiplier under the number of decimals in the multiplicand that you would have in the product, which done set all the other figures in the multiplier in an inverse order; then begin to multiply each figure in the multiplicand by the figure in the multiplier which stands under it, setting it down under the units place in the multiplier, and the rest as in common multiplication, the sum of all these products is that required. The greatest difficulty is to get the first figure, begin therefore 2, 3, or 4 figures before that which you are to set down, so you'll have the tens to carry to it, which increase must be carefully observed.

Ex. 31. 32. 33. Let it be required to multiply 3,141592 by 52,7438, and to have 4 places of decimals in the product. Also 257,356 by 76,48, and to have no decimals in the product. Also, 0,248264 by 0,725234 and to have 5 places of decimals in the product.

Ex. 31.	Ex. 32.	Ex. 33.
3,141562	257.356	,248264
8347,25	84,67	432527,
1570796	18015	17378
62832	1544	497
21991	103	134
1257	20	5
94		
25	19682	,18004
165,6995		

Since (by Ex. 3) 3 or 4 places of decimals are sufficient, and this way of multiplying answers that end with so much ease, you'll find it often used in the following pages, and to make it yet more plain. See these examples here at large.

Ex. 31.		Ex. 32.		Ex. 33.	
3,141592		257,356		,248264	
52,7438		76,48		,725234	
25	132736	20	59848		993056
94	24776	102	9424		744792
1257	6368	1544	136	4	96528
21991	144	18014	92	124	1320
52832	84	19682	,58688	496	528
570796	0			17378	48
65,6995	001296			,18004	9493776

34. To find a vulgar fraction nearly = to a given decimal one. (See prob. 173) But if the decimal expression be a circulate it may be done thus, for every cypher in the first circulate write a 9, so you have a vulgar fraction = the given decimal, which vulgar fraction by reduction will turn out the same decimal, so in ,189189189 &c. here the circulate is 189 so 189189189 becomes $\frac{189}{999} = \frac{21}{111}$ in lower terms $= \frac{7}{37}$ in the lowest terms, also ,666 &c. $= \frac{6}{9} = \frac{2}{3}$, and ,123123 &c. $= \frac{123}{999} = \frac{41}{333}$, also ,15666 &c. $= ,15\frac{6}{9} = ,15\frac{2}{3}$ and so on, observing to omit the one in the denominator when you write 9's for 0's. A fraction is brought into lower terms by dividing its denominator and numerator by any number that measures them both without a remainder.

35. To find the value of any decimal, is the converse of finding the decimal of such a value, therefore, in ,587 and in ,587333 &c. how many pence and farthings sterling.

Ex. 36.	Ex. 37.	
,587	,587333 &c. =	$,587\frac{1}{3}$
12	12	12
d 7,044	d 7,047996	d 7,044
4	4	add 4 = 000 $\frac{1}{3}$ of 12
qrs 0,176	qrs. 0,191984	d 7,048
		4
		qr. 0,192

Ans. 7d. 0 qrs. 176 = 587 sh.

Answer ,587333 &c = 7d 0 qr. ,191984, or nearer = 7d 0 qr. ,192.

Ex. and 39. In ,5234 of a flat foot how many square inches, and in ,1251212 &c. of a yard in length how many feet and inches?

,5234	,1251212	=	,125 $\frac{4}{11}$
144	3		3
20936	,3753636		,375 $\frac{12}{11}$
20936	12		12
5234	4,5043632		4,500 $\frac{144}{11}$
ſq. in. 75,3696	An. 0 feet but 4,50436 in. nearer 0 f.		4,504$\frac{12}{11}$ in.

In theſe and many other examples of the ſame kind, you muſt prick off as many decimals in every product as you have in the multiplicand, and if any figures remain towards the left hand they are whole numbers of the ſame name with the multiplier. In circulating decimals if you put the vulgar fraction for its equal you'll have its juſt value. (See ex. 37 and 39,) wrought both ways.

Diviſion of Decimals.

Rule 4. Perform diviſion as in whole numbers; then prick off as many decimals in the quotient : as that their number added to that of the diviſor may be equal to thoſe in the dividend : ſo if the decimal places in the diviſor and dividend be made equal by annexing cyphers, the quotient will be whole numbers till you bring down more cyphers and then you get decimals ; ſo after diviſion is ended if any thing remain, you may bring down cyphers and continue the quotient at pleaſure. See ex. 3.

Ex. 40. 24,3)780,516(32,12

Ex. 41. ,0236),00081892(,0347

When the quotient figures are too few the want muſt be ſupplied by prefixing cyphers to the quotient. Diviſion and multiplication are the converſe of one another.

Ex. 42, 43, 44, 45. Divide unity by ,7854, by 282, by 231, and by 2150,42.

,7854)1,000(1,2732+		282)1,000(,003546+
5752	remainders	130
231)1,000(,004329+		2150,42)1,000000(,000,465+
67	remainders	5470

46. Any whole number is multiplied by a vulgar fraction when it is multiplied by its numerator and divided by its denominator. So to multiply any whole number by $\frac{1}{282}$ is but to divide it by 282. But (by ex. 43) $\frac{1}{282}$ = ,003546+, therefore multiplying by ,003546 and dividing by 282 are the ſame, or multiplying by 282 and dividing by ,003546 give the ſame anſwer. And thus the 4 laſt examples being di-

visors are turned to multipliers, or being multipliers are turned to divisors, and so may any other by making 1 the dividend and the given factor the divisor. And because multiplication is easier than division, you may thus turn any constant divisor into a constant multiplier or factor.

47. Since, by the last rule, the number of places in the quotient must be = to the difference between that number in the divisor and dividend, you may therefore know by observing the divisor and dividend what places there must be in the quotient, and so contract your division, which will greatly lessen the work when there are many places in the divisor and dividend.

48. Divide 70,23 by 7,9863.

Contracted.

```
7,9863)70,2300(8,7938
        638904
        ------
         63396
         55904
        ------
          7492
          7187
        ------
           305
           239
        ------
            66
            63
        ------
             3
```

The same at large.

```
7,9863)70,2300(87,938
        638904 |
        ------
         63396 | 0
         55904 | 1
        ------
          7491 | 90
          7187 | 67
        ------
           304 | 230
           239 | 589
            64 | 6414
            63 | 8904
        ------
             0 | 7516
```

For every figure you write in the quotient leave out one in the divisor towards the right hand; having due regard to the increase that would arise from that figure so left out, and then divide on as usual.

49. The square root of any number is such a number, as being multiplied by its self may produce the first taken number, which is called the square of that root; so 2 is the square root of 4, 3 of 9, 5 of 25, and 4 is the square of 2, 9 of 3, 25 of 5, &c. In the same manner the cube of any number is known by multiplying the number twice into its self; so 8 is the cube of 2, for $2 \times 2 \times 2 = 8$, also 64 is the cube of 4, 8000 is the cube of 20, &c. and 2 is the cube root of 8, 4 of 64, 20 of 8000, &c. by which method the following table is calculated,

Cubes	8	27	64	125	216	343	512	729	1000
Squares	4	9	16	25	36	49	64	81	100
Roots	2	3	4	5	6	7	8	9	10

50. When any number as 2578,3692 is given to be extracted, you must first divide it into periods by points made over every second figure for the square root, and over every third figure for the cube root, beginning to point at the units place whether whole numbers or decimals, so the above number pointed for the square root stands thus $2\dot{5}7\dot{8},3\dot{6}9\dot{2}$, and for the cube root thus, $\dot{2}57\dot{8},36\dot{9}2$, this shews what the first figure in the root will be, by comparing the first period with the above table, and also how many figures will be in the root, there being a figure for every full point except one, so the square root will consist of 4 places, and the root of the first period 25 is 5; but the cube root will have only 3 places, and the nearest root in the table to the first period 2 is 1, for the square of 10 being = 100, the square of 100 being = 10000, the cube of 10 being = 1000, the cube of 100 being = 1000000, &c. it follows that two figures cannot have above one figure in their square root, nor three figures more than one figure in the cube root.

51. Hence, also, if you are to take the square root of any decimal it must be made to consist of an even number of places, otherwise you cannot have the true root of the denominator. Also for the cube root of a decimal, it must be made to consist of 3, 6, 9, &c. places, by annexing a cypher or two; for if you were to take the cube root of 0,8 and call it 0,2 (because by the table 2 is the cube root of 8) then 0,2×0,2×,02 is (by rule 3d) = ,008, which (by explanation 49) should be = 0,8, so 0,2 is not the cube root of 0,8, but the cube root of 0,800 will be the cube root of 0,8.

To extract the square root.

Having pointed the given number as before directed into periods of two figures each, take the nearest less square root of the first period towards the left hand (which is soon had by the table) and set it in the quotient, and take its square from the said first period for a dividend, double the quotient for a divisor, then see how often it may be had in that dividend, so as the figure thence arising being annexed to the divisor and multiplied by the said figure, the product may be =, or the nearest less to the dividend that can be; subtract this product from the dividend, and to the remainder bring down the next period for a new dividend, double the whole quotient for a divisor and proceed as before. Thus go on until all the periods are brought down, if nothing remain you have the true root, but if there be a remainder bring down cyphers two at a time, and so carry the root into decimals at pleasure.

Ex. 52. What is the square root of 321489?

```
        . . .
        321489(567 the root.
        25
      ----------
 106)   714
        636
      ----------
1127)    7889
         7889
```

Here the nearest less root to 32 the first period is 5 its square 25 so there remains 7, to which bring down 14 and the dividend is 714, and 5 doubled is 10, say how often 10 in 71 (leaving out the 4 because the figure arising is to be annexed to 10) it goes 6 times, so set 6 after 10 and it will be 106 the first divisior, which multiplied by 6 gives 636 and taken from 714 leaves 78, to which bring the next period 89, so 7889 will be a second dividend, the whole quotient 56 doubled is 112, which seems to go 7 times in 788, so 1127 is the second divisor, which multiplied by 7 gives 7889 = to the dividend; so 567 is the exact square root of 321489, as will appear by multiplying 567 by its self, which is the way to prove the square root, and when there is a remainder it must be added.

Ex. 53, 54, 55. Required the square root of 32,1489, of 321,4890, and of ,0321489.

```
       . . .               . . . .                        . . . .
      32,1489(5,67       321,4890(17,9301               ,03214890(0,1793
      25      root       1              root             1           root
     ------             ------                          ------
106)  714           27)  221                        27)  221
      636                189                             189
     ------             ------                          ------
1127)  7889        349)  3248                      349)  3248
       7889              3141                            3141
                        ------                          ------
                  3583)   13790                   3583)  10790
                          10749                          10749
                         ------
                358601)   410000                 remains    41
                          358601
                remains   051399
```

Ex. 56, 57. What is the ſquare root of $\frac{2}{3}$ and of $1\frac{1}{2}$?

$\frac{2}{3}$ = ,666666(,8164 root
64
161) 266
161
1626) 10566
9756
,16324) 81066
65296
remains 15770

$1\frac{1}{2}$ = 1,50(1,2247 root
1
22) 50
44
242) 600
484
2444) 11600
9776
24487) 182400
171409
remains 10991

Extraction of the cube root.

58. Having pointed the given number into periods, as by ex. 50. take the neareſt leſs root to the firſt period and write it in the quotient, take its firſt cube from the ſaid firſt period, and to the remainder bring down the firſt figure of the next period for a dividend, ſquare the quotient or firſt root and treble it for a diviſor, perform diviſion and ſet this quotient figure after the laſt. Again, cube this whole quotient and take it from the two firſt periods, bring down to the remainder the firſt figure of the third period for a ſecond dividend, and multiply the ſquare of the quotient by 3 for a diviſor, perform diviſion, and place this quotient figure after the laſt; thus go on until all the periods have been ſubtracted from, and if at laſt any thing remain, bring down cyphers and carry the root into decimals as far as you pleaſe. It is to be obſerved, that after you have got 2 or 3 figures in the quotient, you may bring down a whole period at a time, and ſo get three places of figures more by diviſion, as in ex. 60, &c.

Ex. 59. Required the cube root of 1728.

1728(12 root
1
3) 7
6
1

Here the neareſt leſs root of the firſt period 1 is 1 whoſe cube is 1, which taken from the firſt period 1 leaves 0, to which bring down 7 the firſt figure in the next period and you'll have 7 for the dividend,

and 3 times the square of 1 = 3 for the divisor which goes twice in 7, so set 2 after 1 in the quotient and it makes 12 whose cube is 1728, which taken from 1728 the two first periods (in this case all the given number) leaves 0; so 1728 is the exact cube of 12.

Ex. 60. What is the cube root of ,067507824239?

```
                        ,067507824239(,407178 the root required
                        ,064  = the cube of 4
3 times □ of 4 = 48)      35
                        ,067507   two first periods
                        ,064000   = cube of ,40
3 times □ of ,40 = ,4800)  35078
                          33600
                           1478
                        ,067507824   = 3 first periods
3 times □ of ,407 =     ,067419143   cube of ,407
         ,496947)       ,000088612
                            496947
                           3898653
                           3478629
                           4200249   &c.
```

Ex. 61, 62. Required the cube roots of a gallon of ale and one of wine; viz. 282 and of 231.

```
                        282)6,558 = root
                        216  = cube of 6
6×6×3 = 108)             66,0
                         540
                        282,000   two first periods
                        274,625   = cube of 6,5
6,5×6,5×3 = 126,75)       73750
                          63375
                         103750
                         101400
```

231(6,136 = root
216 = cube of 6

$6 \times 6 \times 3 = 108$) 150
108

231,000 = two firſt periods
226,981 = cube of 6,1

$6{,}1 \times 6{,}1 \times 3 = 111{,}63$) 40190
33489
67010
66978

In Ex. 60. becauſe the cube of ,407 is ſo near = the given number I take in 4 of the laſt figures in the root by diviſion, in each of the laſt examples, two are taken in by diviſion, but if this method in any caſe ſeem not to hold true, you may proceed according to the rule, viz. by cubing the quote for every new figure annexed &c. this method of extracting the cube root is from the converging ſeries (prob. 171. art 84.) by which method any root may be taken, but other roots being of little uſe, and are much eaſier done by (prob. 171) are here omitted.

Note. If at any time your cubed quotient be too much, as it would be in ex. 61. if you took 0,6 (viz. the number ariſing by dividing 6,60 by 108) inſtead of 0,5 you muſt take a unit leſs in the laſt figure &c. as is done there.

Take theſe examples for exerciſe.

cube	its root	ſquare	its root
2150,42	12,9081	7	2,6457513
$1\frac{1}{3}$ = 1,333+	1,1006	,000762	0,0275 +
7121,102	19,238	4,00006712	2,000016+
7612,812	19,67+	300	17,3205 +
		192	13,8564 +
		$\frac{4}{9}$	$\frac{2}{3}$

SECTION II. *Deſcription, conſtruction, and uſe of Coggleſhal's, or the common ſliding-rule,*

63. On this rule are two lines, one on each ſide of the ſlider, one of them is exactly the ſame with that on the ſlider, numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, to the middle where they again begin at 1 and ends at 9; theſe are commonly called Gunter's lines being the ſame with the line

of numbers on Gunter's ſcale. The other line on rules ſometimes marked with D at one end, and girt line being ſet at 12 being the gauge point for timber. (Alſo, on ſome of theſe rules are A. G. the gauge point for ale gallons, and W. G. for wine;) this line is of a double radius to the other two, the diſtance here between 5 and 6 being double that between 5 and 6 on the other two lines, &c. in any other like numbers; this line begins commonly about 4 and ends about 40, numbered 4, 5, 6, 7, 8, 9, 10, 20, 30, 40. D.

64. Theſe lines are numbered from the left hand towards the right according to the order of figures; thus, if the firſt 1 (beginning at the left hand) be 1, they read 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, if the firſt 1 denote 10, they read 10, 20, 30, 40, &c. to 900, if the firſt be 100, then they read 100, 200, &c. to 9000, if the firſt 1 be 0,1, they read 0,1. 0,2, 0,3. 0,4. 0,5. 0,6. 0,7. 0,8. 0,9. 1, 2, 3, 4, 5, 6, 7, 8, 9, &c. theſe you are carefully to obſerve, or you cannot know the value of the anſwers.

65. The ſpace between every figure is divided into 10 parts called the large diviſions, and each of theſe into 10 other parts called ſmall diviſions, which are hundred parts the other being tenths; to find any number on theſe lines ſuppoſe 346, for the 300 take the figure 3 and for 40 take 4 off the large diviſions, and for 6 take 6 off the ſmall ones, ſo you'll have the point where 346 ſtands, and at the ſame point will alſo ſtand ,346, or 3,46, or 34,6, or 346000, or ,00346 by taking the figure 3 to be 0,3, or 3, or 30, or 300, or 300000, or ,003, &c. Alſo, if you would find 4567891 on the lines, for 4 take the figure 4 (always take the figure on the ſlider the ſame with the firſt figure in the given number) for 5 take 5 off the large diviſions, for 6 take 6 off the ſmall diviſions, then becauſe there are no more diviſions, you muſt compute for the other figures as near as you can, ſo 7891 being to compute, take ſomewhat more than three fourths of the next ſmall diviſion (becauſe 75 viz. 0,75 is $= \frac{3}{4}$ but 0,78 is a little more) that is the required point where 4567891 muſt be is at $6\frac{3}{4}$ + ſmall diviſions paſt 5, large ones paſt the figure 4, which is alſo the place of 4,567891 ,004567891 or 45,67891, according as you take figures 4 to be 4 or ,004 or 40, &c. But when there are not ſo many ſmall diviſions as are mentioned, which is the caſe in all common ſliding-rules, you muſt take what diviſions there are and compute for the reſt as above directed: ſo if between any two figures 8 and 9 on theſe rules there are but 5 large diviſions, and you would find 875, for 8 take the figure 8, then for 7 you muſt ſuppoſe them ſubdivided ſo 7 will be $3\frac{1}{2}$ of them and 3 will be but $\frac{5}{100}$ parts of one of

these divisions, which is so small that you can only take good measure of $3\frac{1}{2}$ divisions past the figure 8 for 875, or even for 87 and any number of figures to follow it. This method of numbering the rule called notation should be well unde stood, for it is the greatest difficulty you'll meet with in the use of this or any other rule. This method of notation will serve for any rule or scale.

66. The construction of this scale is such that if any 4 numbers be directly proportional, the distance on Gunter's lines between the first and second terms is = to the distance between the third and fourth terms, i. e. the distance between the first and second terms being doubled or added to itself gives the distance between the first and fourth terms which plainly shews this rule to be a logarithmetic scale of the numbers 2, 3, 4, 5, 6, 7, 8, 9, and is thus made. From a nice diagonal scale take the numbers 301, 477, 602, &c. (viz.) the logs. of 2, 3, 4, &c. and set them in a right line one after another, so you have the true distances on the rule between the figures 1, 2, 3, 4. &c. for the large divisions take the logs. of 1,1 1,2 1,3 &c. and 2,1 2,2 2,3 &c. from the same diagonal scale and set them after the lines 1, 2, 3, &c. on the rule, &c. for the small divisions, by which means Gunter's lines are soon constructed, the line D or gauge point line is made the same way only its divisions are double to the respective divisions on the other lines.

Note, For distinction I call that line which begins 1 on the left hand of the rule the former line, and the following line beginning at near the mid rule I call the latter line, both of which are constructed alike.

67. The use of the sliding-rule must be easy from the construction thereof; because in any 4 proportionals, the distance taken with a pair of compasses between the first and second terms, will the same way extend from the third to the 4th term. Now in a sliding-rule the compasses are supplied by the slider; whence this

Rule 5. Set the first term on the slider to the second term on the rule, then against the third term on the slider stands the fourth term on the rule; or, set the first on the rule to the second on the slider, then against the third on the rule stands the fourth on the slider; so that the first and third terms must still be found on one line and the second and fourth terms on the other: or you may use the second term for the third and the third for the second. This rule answers for multiplication and division. For, as 1 is to one of the factors (in multiplication) so is the other factor to the product. Also in division, as the divisor is to the dividend so is 1 to the quotient.

68. By ex. 64, if the firſt 1 be 1 the laſt 9 will be 90, if it be 10 the laſt 9 will be 900, i. e. the laſt 9 is always 10 times the firſt 9 or 90 times the firſt 1. Hence in 4 proportionals when the ſecond and third terms are each more than 80 or 90 times the firſt term: or contrary, ſuch a proportion will fall off the rule and ſlider; in ſuch caſes it is beſt to ſuppoſe the ſmall term to be multiplied by 10 or 100 or 1000, &c. or the great one to be divided by 10 or 100, &c. till they will come on the rule and ſlider, and then multiply your anſwer by the ſame number that the firſt term was multiplied by, or that either the ſecond or third terms were divided by; but if the firſt term be divided, or either of the other two multiplied, then the anſwer or fourth term muſt be divided by the ſame number to give the true anſwer. Now to multiply by 10, 100, &c. is but to annex a cypher or two to a whole number, or in a mixt number to remove the decimal point 1 or 2 places nearer the right hand, and diviſion is juſt the reverſe of this, ſo that this may eaſily be performed in your mind. And here it is to be obſerved that the line D is of a double radius, the numbers on the ſlider will be ſquares to the numbers on it; ſo that if you divide by 10 on the line D it will anſwer to 100 on the ſlider, &c. for by ex. 65, the firſt and ſecond terms being taken, the value of the other figures on the lines are thereby known, and by obſerving the ſaid example it muſt be eaſy to bring any numbers on the lines by removing the decimal point &c.

69. If 1 coſt 7 s and 6 d, what will 3 coſt at that rate? Here as 1 on the ſlider : 7,5 (for 6 d is 0,5 of a ſhilling) on the rule : : 3 on the ſlider : 22,5 s on the rule, the anſwer. But to have the uſe of the ſliding rule, ſhorter, or in fewer words, let A and B denote Gunter's lines on the rule and ſlider, and D the girt, or gauge point, then the laſt proportion will ſtand thus, as 1 on A : 7,5 on B : : 3 on A : 22,5 on B the anſwer.

70. If 1 coſt 7,5 what will 300 coſt at that rate? As 1 on A : 300 on B : : 7,5 on A : 2250 on B the anſwer.

71. If 1 coſt 300 what will 750 coſt? Or, multiply 750 by 300, (By ex. 68) As 1 on A : 750 on B : : 3,00 on A to 2250 on B; or, as 1 on A : 300 on B : : 7,50 on A to 2250 on B; or, as 100 on A : 300 on B : : 750 on A to 2250 on B, which multiplied by 100 (becauſe the ſecond or third term was divided by 100, or the firſt term multiplied by 100) gives 225000 for the anſwer.

72. When the proportion is increaſing (as in the 3 laſt examples) find the two firſt terms on the former lines; but if it is decreaſing, find them on the latter lines, as in examples 73 and 74.

73. If $22\frac{1}{2}$ cost 3, what will 1 cost? That is, divide 22,5 by 3. As 3 on A : 22,5 on B : : 1 on A to 7,5 on B the answer.

74. Divide 2250 by 300. As 300 on A : : 1 on B : : 2250 on A : 7,5 on B the answer.

75. Divide 225000 by 750. By example 68, it will be as 7,50 on A : 225000 on B : : 1 on A : 30000 on B, or as 75,0 on A : 225000 on B : : 10 on A : 30000 on B; or, as 750 on A : 215000 on B : : 100 on A to 30000 on B, which divided by 100 (because the first term was so divided, or the first term divided by 10, and the second or third multiplied by 10, 10 times 10 being = 100, or the second or third term multiplied by 100) gives 300 for the answer.

76. If the square of 3 require 7, what will the square of 60 require? As 3 on D (viz. the figure 30 must be taken because 3 is not on that line, for it begins with 4) : 7 on the slider : : 6 on D (this 6 by the question should be 60, but it is only 0,6 because 30 is taken for 3, so this line D in this case is divided by 100) to 0,28 on the slider, which multiplied by the square of 100, viz. by 10000 (see ex. 68) gives 2800, for the answer.

77. What is the square root of 15129? As 1 on the slider : 1 on D : : 1,5129 (see ex. 68) on the slider : 1,23 on D, which multiplied by 100 (because 15129 on A was divided by the square of 100) gives 123 for the square root of 15129.

78. Between any two given numbers suppose 4 and 9, to find a mean geometrical proportional. If you multiply the two numbers together, the square root of that product will be the mean proportional sought. Or, by the sliding-rule, as the lesser number 4 on A : the said lesser number 4 on D : : the greater number 9 on A : 6 the mean proportional on C; or, as 9 on A : 9 on D : : 4 on A to 6 on D as before.

79. To find the first of two mean proportionals between any two given numbers; you must take the cube root of their product which will be the proportional sought: this cannot be done on this sliding-rule for want of the line E, which see in section 8.

80. On the edge of this sliding-rule is a line divided into 100 equal parts ending at 12 inches which is a foot decimally divided, and serves to find the decimals of inches when the integer is 1 foot; thus, against 6, 9, $1\frac{1}{2}$ &c. on the inches stands ,50, ,75, ,$12\frac{1}{2}$ &c. on the said decimal line, i. e. 0,5 for the decimal of 6 inches, 0,75 for that of 9 inches, 0,$12\frac{1}{2}$ or 0,125 for that of $1\frac{1}{2}$ inches, &c.

81. On this sliding-rule, is a table marked at top D. L. S. D. viz. pence, pounds, shillings, pence; this table shews the value of 50 feet of timber called a load, at all rates per foot from 6 to 24 pence, so if

you would know what 50 feet comes to at 10d, or 10¼d, 10½d, or 10¾d, per foot, here under the first D, and against 10, and the first or second or third line below 10 you'll find 2l 1s 8d the price at 10d a foot, or 2l 2s 8d the price at 10d ¼ a foot, or 2l 3s 9d the price at 10d ½ per foot, &c. See Ex. 91.

82. Lastly, It is plain from what goes before that if the second or third terms fall on the line D the square root of the first term must also be on D. Also, if any of the terms be a product of two numbers, a mean proportional between such two numbers must be found, and used on D, instead of such a product.

SECTION III. *Multiplication of feet and inches.*

There are several ways to work multiplication of feet and inches, but the best is as follows: where the 12th part of a foot, whether lineal, superficial, or solid, is called an inch, the 12th part of an inch is called one part, and so on, as in this table.

12 inches	make one	foot	consequently	f. × f. give feet	divided by 12 gives	
12 parts		inch		f. × in. give inches		feet
12 seconds		part		in. × in. give parts		inches
12 thirds		second		in. × parts give sec.		parts
12 fourths		third		parts × parts, thirds		second

That is, feet into feet give feet, feet into inches give inches, and divided by 12 give feet and inches remain; inches into inches give parts which divided by 12 gives inches and parts remain, &c.

83.

F.	I.	P.
8 :	9	
3 :	6	
26 :	3	
4 :	4 :	6
30 :	7 :	6

84.

F.	I.	P.
24 :	10	
8 :	9.	
198 :	8	
18 :	7 :	6
217 :	3 :	6

85.

F.	I.	P.			
3 :	4 :	6	note	½ an inch = parts	
2 :	8 :	3		¼ an inch = parts	
6 :	9 :	0	5.	1.	
2 :	3 :	0 :	0 :	0	
0 :	0 :	10 :	1 :	6	
9 :	0 :	10 :	1 :	6	the answer

In Ex. 84. I first multiply 10 inches by 8 feet, saying 8 times 10 is 80 inches, which divided by 12 quotes 6 feet and 8 inches remains, set 8 under inches, and carry 6 to the feet, saying 8 times 4 is 32 and 6 is 38, set 8 down and carry 3, saying 8 times 2 is 16 and 3 is 19, which is set down under feet, again for the second row, 9 inches into 10 inches gives 90 parts or 7 inches and 6 parts, set down 6 under parts then multiply the 24 by 9, and to the product 216 inches add the 7 and the sum is 223 inches or 18 feet 7 inches as you see set down in the work, add these two rows together pricking at 12's under inches,

parts, &c. and you'll have 217 feet 3 inches and 6 parts for the Anſ. Ex. 83. is wrought the ſame way. In Fx. 85, begin with 6 ~~inches~~ Parts by two feet and ſo get the firſt row, then begin with 6 parts and 8 inches and ſo get the ſecond row, laſtly, with the 3 parts get the third row, always pricking at 12's except in feet, which prick at 10's.

F.	I.	P.	S.	T.				
8	4	3	5	6				
3	3	7	8	2				
25	0	10	4	0				
2	1	0	10	4	6			
	4	10	6	0	2	6		
		5	6	10	3	8	0	
			1	4	8	6	11	0
27	7	3	5	1	8	8	11	0

But if there are more figures in the firſt than one, it will be eaſier to work by aliquot parts, thus, multiply all the feet, inches, &c. in the multiplicand, by the feet in the multiplier as before, and take ſuch aliquot parts of the multiplicand as ſuit the inches, parts, &c. in the multiplier, which added together gives the anſwer. Here follows a table of aliquot parts. Take ex. 83 and 84 by this method.

1	inch	is	one twelfth	of a foot.
2	inches	is	one ſixth	of a foot.
3	inches	is	one fourth	of a foot.
4	inches	is	one third	of a foot.
5	inches	is	one third and one twelfth	of a foot.
6	inches	is	one half	of a foot.
7	inches	is	one third and one fourth	of a foot.
8	inches	is	one third and one third	of a foot.
9	inches	is	one ſecond and one fourth	of a foot.
10	inches	is	one half and one third	of a foot.
11	inches	is	one third 1 third and 1 fourth	of a foot.

1	9		
3	6		
26	3		
4	4	6	$=\frac{1}{2}$ of 8 ft. 9 in.
30	7	6	product

24	10		
8	9		
198	8		
12	5		$=\frac{1}{2}$ of 24 feet 10 inches.
6	2	6	$=\frac{1}{4}$ of 24 ft. 10 in.
217	3	6	the product

In ex. 83, I firſt multiply 8 feet 9 inches by 3 feet and ſo get the firſt row 26 3, then 6 inches being half a foot I take half of 8 ft. 9 in. which is 4 f. 4 i. 6 p. In ex. 84, multiply 24 f. 10 i. by 8 f. and then 9 i. being $\frac{1}{2}$ and $\frac{1}{4}$ take thoſe parts of 24 f. 10 i. viz. $\frac{1}{2}$ and $\frac{1}{4}$. But if the feet in the multiplier are above 12, (as in ex. 87 they are 17 ft.) it will be eaſier firſt to multiply the feet together not regarding the inches, then take ſuch aliquot parts of the feet in the multiplier as ſuits the inches, parts, &c. in the multiplier as before.

Ex. 87.

75	9		
17	7		
525			
75			
8	6		$= \frac{1}{2}$ of 17 ft.
4	3		$= \frac{1}{4}$ of 17 ft.
25	3		$= \frac{1}{3}$ of 75 ft. 9 in.
18	11	3	$= \frac{1}{4}$ 75 ft. 9 in.
1331	11	3	the product

Ex. 88.

97	8	
8	9	
781	4	= 97 ft. 8 in. × 8 ft.
48	10	$= \frac{1}{2}$ of 97 ft. 8 in.
24	5	$= \frac{1}{4}$ of 97 ft. 8 in.
854	7	product

Ex. 89.

37	7	5			
4	8	6			
150	5	8			= 37 f. 7 i. 5 p × 4 f.
12	6	5	8		$= \frac{1}{3}$ 37 f. 7 i. 5 p.
12	6	5	8		$= \frac{1}{3}$ of ditto
1	6	9	8	6	= ditto × 6 p.
177	1	5	0	6	product

Ex. 90.

40	10	3			
30	6	6			
1200					= 40 f. × 30 f.
25					= 30 ft. × 10 in. = 300 in.
0	7	6			= 30 ft. × 3 p. = 90 p.
20	5	1	6		$= \frac{1}{2}$ of 40 10 3
1	8	5	1	6	= d° × 6 parts
1247	9	0	7	6	product

Thus multiplication of feet and inches, like practice, may be wrought ſeveral ways.

91. There is yet another article belonging to the ſliding rule, viz. when one or more of the terms is a ſum or difference: thus, if 10 give 20, what will the ſum of 30, 40, and 50 give? or, (which is the ſame) what will 120 give? Set 10 on A to 20 on B, then againſt 30, 40, and 50 on A ſtands 60, 80, and 100 reſpectively, which added together gives 240 for the anſwer. Alſo, if 9 gives 20, what will the ſum of the ſquares of 30 and 40 give? Here, becauſe the ſquare of the ſecond or third term is to be uſed it muſt be found on D, ſo by rule 5, the firſt term 9 (viz. 3 its ſquare root by ex. 68,) muſt alſo be on D, i. e. the 30 on D muſt be 3, ſo 40 will be 4, and then as 3 on D : 20 on A : : 3, 4 on D : 20 and 35,5 + on A reſpectively, whoſe ſum is 55,5 + which multiplied by 100 (becauſe D is divided by the ſquare root of 100) gives 5550 + for the anſwer. Alſo, if the difference of theſe ſquares were to be uſed it would be the ſame on the rule, only taking 20 from 35,5 + leaves 15,5 +, which multiplied by 100 gives 1550 + for the anſwer. This is evident, if you conſider, that the ſum of all the parts is equal to the whole.

Section IV. *Superficial Measure.*

92. The area and dimensions are commonly kept in one name; that is, if the area be required

in { inches / feet / yards } the dimensions are taken in { inches / feet and inches / yards feet and in.

then those odd parts being reduced to decimal parts of the integer, the area is had by working as the nature of the figure requires, or without decimals, the area may be had by multiplication of feet and inches, but easiest of all by taking the dimenfions by decimally divided rules, and then the odd parts are ready reduced to hand, and is also truer than to take the dimensions in feet and inches &c. and to reduce the inches to decimal parts of a foot, for by such reduction often something remains, besides the inches are divided into more parts decimally, than among themselves, which is only into 8th's or half-quarters, so that a foot contains 12 times 8 or 96 parts, but decimally divided it contains 100 parts, hence, when the area is required in feet, take the dimensions in feet and for the odds look not at the inches, but at the line on the edge of the sliding-rule decimally divided into a 100 = parts in the length of a foot or 12 inches, so may you have your dimensions in feet and decimal parts of feet true, there are also yards decimally divided for the same end, and for the like reason, Gunter's chain is divided into 100 = links, &c. in most cases the content is required in inches, feet, or yards, and being in any one of these it may be reduced to any other of them, by these three tables.

Long Measure.			Superficial Measure.			Solid Measure.		
I.	F.	Y.	I.	F.	Y.	I.	F.	Y.
12 =	1		144 =	1		1728 =		
36 =	3 =	1	1296 =	9 =	1	46656 =	27 =	1

To reduce measures from one denomination to another, observe this general

Rule 6. Less numbers are brought into greater by multiplication, and greater into less by division.

Ex. 93, 94. In 54,5 yards in length how many feet and inches? And in 54,5 square or superficial yards how many feet and inches.

54,5	54,5
3	9
163,5 feet in length	490,5 square feet
12	144
1962,0 inches in length	70632,0 square inches

Ex. 95, 96. In 0,52 parts of a ſolid foot how many ſolid inches? Alſo, in 1730,63 ſquare inches how many ſquare feet?

1728	144)1730,63)12,014 +, ſquare feet
0,52	47 the remainder.
898,56 ſolid inches	

To meaſure a ſquare.

Rule 7. Multiply any ſide of the ſquare by itſelf, and the product it its area. By theorem 22.

Ex. 97. If each ſide A B = B D of a ſquare (fig. 17) be 7 feet 6 inches what its area.

By feet and inches.	Decimally.
7 6	7,5
7 6	7,5
56 3 the anſwer	56,25 the anſwer

By the ſliding-rule. As 1 on A : 7,5 on B : : 7,5 on A : 56¼ on B.

To meaſure a rectangled parallelogram.

Rule 8. Multiply the length by the breadth for the area. Theorem 22.

By feet and inches.				Decimally.
10 9	=	length	=	10,75
2 0	=	breadth	=	2
21 6	=	anſwer	=	21,50

Sliding-Rule. As 1 on A : 10,75 on B : : 2 on A : 21,5 on B.

To meaſure an oblique angled parallelogram, as a rhombus or rhomboides.

Rule 9. Mnltiply the perpendicular let fall from one of the obtuſe, angles upon the oppoſite ſide, by that ſide, for the area. Theorem 23.

Ex. 99. Given the ſide A B = D C 2 feet 10½ inches, the perpendicular D P = C E 9 inches to find the area. Fig. 21.

By feet and inches.				Decimally.
2 10 6	=	ſide	=	2,875
9	=	perpendicular	=	,75
2 1 10 6	.	anſwer		2,15625

Sliding-rule. As 1 on A : 2,875 on B : : 0,75 on A : 2,15 + on B.

To meaſure any plane triangle.

Rule 10. Multiply the baſe and perpendicular one by half the other or multiply them together and take half the product; by either method you may find the area. Theorem 24.

Ex. 100. In a right angled plane triangle C BP, given the baſe B P = 7 feet 10 inches, and perpendicular C P = 2 feet 3 inches, required the area. Fig. 11, 12, 13.

Feet and inches.			Decimally.
7 10	= baſe =		7,8333
2 3	= perpendicular =		2,25
15 8		2)17,625	
1 11 6			8,8125
17 7 6 ÷ 2 = 8 f. 9 in. 9 p.			

Sliding-rule. As 2 on A : 7,833 on B : : 2,25 on A : 8,81 on B; the ſame as if you take C B and a perpendicular to it from P.

Ex. 101. In an oblique angled plane △ A B C, given the baſe A B = 10 feet 3 inches, the ⊥ C P = 6 feet 10 inches, to find the area. Fig. 11, 12, 13.

Feet and inches.		Decimally.
10 3 = A B		3,416 = ½ C P
3 5 = ½ C P		10,25 = A B
30 9		35,02083
4 3 3		
35 0 3		

Sliding-rule. As 2 on A : 10,25 on B : : 6,83 on A : 35,02 on B; or, as 1 on A : 3.416 on B : : 10,25 on A : 35,02 on B, the anſwer.

102. The plane △ is the moſt uſeful figure in menſurations, &c. for if a figure be ever ſo irregular it may be reduced into △s, by drawing or meaſuring diagonals, and ⊥ s from thoſe diagonals to their oppoſite angles, by which the area of every △ may be found; and then it is evident, the ſum of all the areas of theſe △s will be the area of the irregular polygon, if there be bended ſides, you may take offsets as taught in ſurveying, or by rule 24 and 25.

To meaſure any plane △ by having its 3 ſides given.

Ex. 103. In the oblique △ A B C are given the 3 ſides A B = 56, BC = 41, and C A = 29, required its area.

Rule 11. From half the ſum of the 3 ſides ſubtract each ſide ſeverally, multiply the ſaid half ſum and 3 remainders together according to continual multiplication, and the ſquare root of the laſt product is the area. Theorem 73.

NB. Pa: 25 follows pa: 32.

$29 = A\,C$ $41 = B\,C$ $56 = A\,B$

$2)\ 126 =$ sum sides

$63 = \frac{1}{2}$ sum

63	63	63
56	41	29
7	22	34

63
7 = first difference

441
22 = second deffer.

9702
34 = third differ.

329868(574,34 = area

25

107) 798
1144) 4968
11483) 39200
114864) 475100

In the following examples, you must suppose the dimensions to be taken as directed in ex. 92; so that let them be what they will the area will be of the same name.

Sliding-rule. As 1 on A : 22 on B : : 7 on A : 154 on B; and, as 1 on A : 154 on B : : 34 on A : 5236 on B. Then as 63 on A : 63 on D : : 52,36 on A (for 5236 is off the rule) to 57,4 on D, which (by ex. 78 and 91) multiplied by 10, because 5236 was on A, divided by the square of 10, gives 574 = area.

To measure a trapezia.

Rule 12. Take the sum of the two ⊥s and diagonal on which they fall, half the product is the area; or, the one multiplied by the other gives it. Theorem 25.

Ex. 104. In the trapezia A H G I given the diagonal A G = 28,2 the ⊥ e H = 10,5 and the ⊥ I a = 8, to find the area. Fig. 14.

10,5 = ⊥ e H
8 = ⊥ a I

18,5 = sum ⊥s
14,1 = $\frac{1}{2}$ diagonal

260,85 = area

Sliding-rule. As 2 on A : 28,2 on B : : 18,5 on A : 260,85 on B the answer.

To measure any irregular polygon.

Ex. 105. In the irregular pentagon a b c d e (fig. 15) is given the diagonals a c = 24, and a d = 20,72, the ⊥s b n = 10, d q = $12\frac{1}{2}$, and d u = 10,2, required the area. Here two ⊥s b n and d q fall upon one diagonal a c, so the figure (see ex. 102) is divided into a trapezia a b c d and a △ a d e.

10 = b n
12,5 = d q

22,5 = sum ⊥s
12 = $\frac{1}{2}$ diag. a c

270,0 = trap. a b c d

20,72 = d a
5,1 = $\frac{1}{2}$ d u

105,672 = △ d a e
270 = trapezia a b c d

375,672 = area required.

Sliding-rule. As 1 on A : 22,5 on B : : 12 on A : 270 on B the area of the trapezia ; as 2 on A : 20,72 on B : : 10,2 on A : 105,6 on B, &c.

To meaſure any regular polygon.

Rule 13. Multiply the ⊥ let fall from the center of the polygon to the middle of one of the ſides, and one of the ſides, and the number of ſides into one another, half the laſt product is the area, or the product of any two of theſe three into half the third gives the area. Theorem 27.

Ex. 106. If each ſide C D of a regular pentagon be 500, and the ⊥ Q P 360, what is its area? Fig. 25.

500	= a ſide
5	= number of ſides
2500	
180	= ½ ⊥
450000	= area

Sliding-rule. When three numbers are to be multiplied together, you may turn ſome one of them into a diviſor, or find a mean proportional between them. 1. Thus, as 5 on A : 1 on B : : 1 on A : 0,2 on B, then as 0,2 on A : 500 on B : : 1,8 on A (for 180 is off the rule) to 4500 on B, ſo (by Ex. 68.) 450000 = area. 2. Or (by Ex. 78) as 5 on A : 5 on D : : 5 on A (for 500 is off the rule) to 5 on D. So (by ex. 68) 50 is a mean proportional between 5 and 50. Then, as 1 (viz. the 10) on D : 180 on A : : 0,5 on D (for 50 is off the rule) : 45 on A ; ſo (by ex. 68) 450000 is the anſwer.

If the ⊥ Q P be given the ſide C D (by theorem 48) may be found, and the contrary. Thus, as ſine half the ∠ (C Q D) : half the oppoſite ſide (C D) : : co-ſine of that ∠ (viz. ſine ∠ Q C D or Q D C) : the ⊥ Q P ; now if we call the ſide C D 1 and ſo by this ⊥ find the area as before, we'll have a factor for all regular pentagons, (ſee theorem 36) and in this manner is this table calculated to the regular polygons mentioned in it.

Names of the polygons	number of ſides	half the ∠s at the center	areas, the ſides being unity.	theſe multipliers turned into diviſors
Trigon	3	60°,00′	,433013	2,3094
Square	4	45°,00′	1,00000	1,0000
Pentagon	5	36°,00′	1,72047	,5812
Hexagon	6	30°,00′	2,59807	,3849
Heptagon	7	25°,42′	3,6340	,2752
Octagon	8	22°,30′	4,8284	,2071
Nonagon	9	20°,00′	6,1818	,1617
Decagon	10	18°,00′	7,6942	,1299

107. To use the table. Multiply the square of the polygon's side by the tabular number belonging to that polygon, or divide it by the divisor, the product or quotient will be the area. So if the side of a regular hexagon be 10; here $10 \times 10 = 100$, then under areas, &c. and against hexagon stands 2,59807, so $100 \times 2,59807 = 259,807$ its area, or $100 \div ,3849 = 259,807$ the same.

Sliding-rule. As 1 on D : 2,59 on A : : 10 (see ex. 68) on D : 259,8 on A; or, as ,3849 on A : 10 on B : : 10 on A : 259,8 on B, the area. &c. for others.

To measure a circle.

Rule 14. Multiply half the periphery by half the diameter, or one fourth of their whole product is the area.

Ex. 108. Given the circumference A B C D = 314,16, and diameter A C = B D = 100, to find the area. Theorem 28. Fig. 43.

$$\begin{array}{rl} 157,08 & = \frac{1}{2} \text{ A B C D} \\ 50 & = \frac{1}{2} \text{ diameter} \\ \hline 7854,00 & = \text{area} \end{array}$$

Sliding-rule.

As 4 on A : 314,16 on B : : 100 on A : 7854 on B, the answer.

The diameter of a circle given, to find its area.

Rule 15. Multiply the square of the diameter by 0,7854, or divide it by 1,2732 (see ex. 42) so you'll have the area. Theorem 82.

$$\begin{array}{rl} \text{so} \left\{ \begin{array}{r} 100 \\ 100 \end{array} \right\} & = \text{diameter} \\ \hline 10000 & = \square \text{ diameter} \\ 0,7854 & \\ \hline 7854,0000 & = \text{area} \end{array}$$

Sliding-rule.

As 1,2732 on A : 100 on B : : 100 on A : 7854 on B; or, as 1 on D : 0,7854 on A : : 100 (see ex. 68) on D : 7854 on A the answer.

Note. 1 to 0,7854 (by prob. 173) will be found as 14 to 11; hence, as 14 : 11 : : the $\square$ of any circle's diameter to its area.

The circumference of a circle given to find its area.

Rule 16. Multiply the $\square$ of the circumference by ,07958, or divide the $\square$ thereof by 12,566, and you have the area. Theorem 83.

$$\begin{array}{rl} \text{so} \left\{ \begin{array}{r} 314,16 \\ 314,16 \end{array} \right\} & = \text{circumference} \\ \hline 98696,5056 & = \square \text{ circum.} \\ 85970, & = .07958 \text{ inverted} \\ \hline 7854,26 & = \text{area} \end{array}$$

12,566)98696,5056(7854,26 = area

Sliding-rule. As 1 on D : ,079 on A : : 3,1416 on D (for 314,16 is off the rule) : ,7854 on A, so (by ex. 68) 7854 = area; or, as 12,566 on A : 314,16 on B : : 314,16 on A : 7854 on B, as before.

Having the diameter of a circle, to find the periphery. Or, the periphery, to find the diameter.

Rule 17. Multiply the diameter by 3,1416 and the product is the periphery. (See theorem 82. But, by problem 173) as 1 : 3,1416 : : 7 : 22; or nearer, : 113 : 355 : : the diameter to the circumference. But 7 to 22 will do in most cases.

Ex. 109. If the diameter be 32, what is the periphery?

constant factor = 3,1416	or as 7 : 22 : : 32	or as 113 : 355 : : 32	
given diameter = 32	22	32	
reqd. per. = 100,5312	7) 704	or = 113) 11360	
	area = 100,57	or = 100,5309	

The reverse of any of these ways will find the diameter when the periphery is given; i. e. as 3,1416 : 1, or as 1 : 0,3184, or as 22 : 7, or (to be most exact) as 355 : 113 : : the periphery of any circle : its diameter, and on the sliding-rule may these proportions be wrought on A and B.

To measure any sector, semi-circle, or quadrant of a circle.

Rule 18. Half the product of the radius into the arch, or half the one into the other gives the area. Theorem 29.

Ex. 110. If the radius A B = A C be 50, and the arch B C = 218,56, what is the area? Fig. 9.

109,28 = $\frac{1}{2}$ arch B C
50 = radius A B
5464,00 = area

Sliding-rule.
As 2 on A : 218,56 on B : : 50 on A : 5464 on B, the answer.

To measure any segment of a circle.

It is evident that if from the area of the sector (fig. 153) S Q A B (found as before) you take the area of the △ S A B there will leave the area of the segment A Q B G; therefore,

Ex. 111. Let the arch B Q A = 28,656, radius S A = S B = 16, the versed sine or height of the segment = 6 = Q G, and chord A B = 24,98, to find the segment's area.

28,656 = arch A Q B
8 = $\frac{1}{2}$ radius S Q
229,248 = area sector S A Q B
124,9 = area △ S A B
104,348 = area segment A Q B

24,98 = chord A B
5 = $\frac{1}{2}$ S G.
124,90 = area △ S A B

Note. The area of any hyperbola, circular, and eliptical segments may be found to what exactness you please by the series in problem 189.

As 2 on A : 28,656 on B : : 16 on A : 229,248 on B = area sector, and as 2 on A : 24,98 on B : : 10 on A : 124,9 on B = area △, their difference is = 104,348 the answer.

But if the segment be greater than a semi-circle, you must find the areas of the sector W E D C (fig. 112) and △ W C E the same way, then its plain per fig. that their sum is = area of the segment W E D.

112. To construct the table of segments; suppose the radius of the circle (being unity) to be divided into 1000 = parts, and thro' every part chords be drawn, then the semicircle will be cut into 500 segments the versed sine of the first segment being = ,001. Now the half chord of any segment being a mean proportional between the versed sine of that segment and remaining part of the diameter, the said half chord and arch (see prob. 56, 57, and 58.) may be found, and so the area of this segment by the above method will be found = ,0000421, in like manner, the height of the second segment being = ,002 its area will be found = ,00011919 and so on for others. From such a table the area of any circular segment may be had by this,

Rule 19. To the given versed sine annex 3 cyphers, and divide it by the diameter of its circle, seek the quotient under V. S. or versed sine, and take out the number against it under seg. or area of segments, this number multiplyed by the □ of the diameter gives the area of the segment, for (by theorem 9.) As the diameter of any circle : any versed sine thereof : : the diameter of any other circle (viz. 1,000 that of the table) to its like versed sine, which proves the first part of the rule, and (by theorem 36.) As □ 1 the tabular diameter or radius : the area of any segment thereof : : the square of any other circle's radius to the area of its like segment, which proves the second part of the last rule.

Ex. 113. Let the versed sine be 6 and radius 16, as in ex. 111.

32) 6,000 (,1875	against which stands	1019
32	square diameter 32 =	1024
280		4076
256		2038
24 &c.		10190
	area =	104,3456

Sliding-rule, As 32 on A : 1 on B : : 6 on A : 18¾ on B against which in the table stands ,1019, then as 1 on D : ,1019 on A : : 32 on D (see Ex. 68) : 104,3 on A.

Because the radius of the table is 500, viz. 3 places of figures under V S. and this quote, is 1875 viz. 4 places, I therefore take the segment for 187 and say by the rule of three, as 187 : its segment : : 187,5 : 1019 its segment, and thus you may find a proportional part or segment where great exactness is required, otherwise the segment for 187 or 188 will do; if the segment be greater than half a circle you must find the lesser segment as above, and take it from the area of the whole circle. The area of a circular segment may be had nearly by theorem 84, and the area of an eliptical segment by theorem 85, but to go through these and other curve-lin'd segments or spaces would be almost endless, and when done would be to little purpose in practice, for such figures are seldom met with in measuring, and if they be, how must their forms be known, which must be had before their proper rules can be applyed, it is then certainly best to make use of a general rule which will come near any plane whatever, let it be what it will, See rule 24, and 25.

How to measure a parabola.

Rule, 20. The area is had by multiplying the axis and ordinate, the one by $\frac{2}{3}$ of the other, theorem 77.

Ex. 114. If the axis e u (fig. 27) be 10,5 and the greatest ordinate A B = 15,6, what is the area of the common parabola A u B ?

15,6 = A B	
10,5 = u e	Sliding-rule.
163,80	As 1,5 (viz. 3 ÷ 2) on A : 15,6 on
2	B : : 10,5 on A : 109,2 on B, the
3) 327,6	answer.
109,2 = area	

To measure an oval or ellipsis.

Rule 21. For the area, multiply the product of its two diameters by ,7854.

Ex. 115. If the transverse diameter T S (fig. 26) be 21, and the conjugate I G = 15,6, what is the area ? See theorem 80.

15,6 = G I	327,6 = prod.	Sliding-rule. As 1,27032
21 = T S	,7854 = factor	(viz. $\frac{1}{,7854}$) on A is to 21 on
327,6 = prod.	257,29705 = area	B so is 15,6 on A to 257,29 B.

A mean proportional between the two diameters of any ellipsis is = the diameter of a circle whose area is = that of the ellipsis, so that what rule ever gives the area of a circle by its diameter only, the same rule will give the area of an ellipsis by having its two diameters.

Having given the two diameters of an ellipſis to find its periphery.

Rule 22. To twice the ſquare root of the ſum of the ſquares of the two diameters, add $\frac{1}{3}$ of the ſhorter diameter, and the ſum will be the periphery nearly, ſo if the tranſverſe diameter be 40 and the conjugate 30 then □ 40 = 1600 and □ 30 = 900, their ſum is = 2500 whoſe □ root is 50 doubled is = 100, to which adding $\frac{1}{3}$ of 30 = 10 gives 110 for the required periphery nearly.

To meaſure an hyperbola,

Rule 23. To 80 times the tranſverſe axis add 39 times the abſciſſa, multiply that ſum by the ſquare root of the product of the ſaid abſciſſa and axis, and that product again by four times the product of the abſciſſa and conjugate axis, this laſt product is a dividend, then to 16 times the tranſverſe axis add 3 times the abſciſſa, this ſum multiplyed by 15 times the tranſverſe axis gives the diviſor, which diviſion being made the quote is the area, this is but changing the ſigns in theorem 80, which in the ſame words (by reading difference for ſum) will give the area of an elliptic ſegment, but to be very exact you muſt uſe the ſeries in prob. 189.

Ex. 116. Let the tranſverſe T u (fig. 28) be 30, abſciſſa or height u C = 10, and conjugate axis = 18, to aproximate the area.

```
      30 = T u                       30 = T u
      80                             16
  ----------                     ----------
    2400 = 80 × T u                 480
add  390 = 39 × u C           add    30 = 3 × u C
  ----------                     ----------
    2790                            510
mult. 17,32 = □ root 10 × 30  mult. 450 = 15 times 30
  ----------                     ----------
  48322,8 product               229500 diviſor
      720 = 4 times 18 × 10
  ----------
34792416 product = dividend
```

2295|00)247924|16(151,60| = area nearly.
2160 &c. the remainder

This area, by the ſeries problem 89, taking in 6027 terms would be 151,687; whence it appears that the above rule may ſerve in common practice, only differing in this example by (151,687 — 151,601 =) ,086.

Ex. 117. If 30 and 18 be the tranſverſe and conjugate diameters of an ellipſis and s a = 10 the height of a ſegment s b a b (fig. 136.) then by the laſt rule,

from	2400 = 80 times ſ T		from	480 = 16 times ſ T	
take	390 = 39 times ſ a		take	30 = 3 times ſ a	
	2010			450	
mult.	17.32 = □ root of 10 × 30		mult.	450 = 15 times 30	
	34813.2			202500 = diviſor	
mult.	720 = 4 times 18 × 10				
	25065504 = divid.			202500)25065504(123,78 = area nearly	

Ex. 118. If the diameter of a circle be 30, and the verſed ſine or height of a ſegment thereof be 10; then, by the laſt rule, all is the ſame as in the laſt example, only multiply by 4 times 10 inſtead of 4 times 18 × 10, and by 15 inſtead of 15 times 30.

ſo	34813,2	450	6750)1392528(206,3 = A nearly
mult.	40 = 4 times 10	15	
	1392528 = dividend	6750 = diviſor	

How to meaſure any ſegment or curve-lined ſpace.

Rule 24. Meaſure in a right line from one corner of the figure to another, and at right angles to this line take as many breadths as you chuſe, but eſpecially the greateſt and leaſt breadths, add all theſe breadths together and divide the ſum by the number of them which gives the mean breadth, this multiplied by the length gives the area, or find the area of every part by itſelf, thus multiply half the ſum of two breadths nerreſt together by their ⊥ diſtance (ſee theorem 26) and the product is the area of that part, do in the ſame manner with every part, and the ſum of all theſe areas will be the area ſought.

119. In meaſuring, the dimenſions muſt be taken ⊥ to each other, hence the area is the leaſt and moſt exact, becauſe ⊥ lines are the ſhorteſt.

Ex. 120. Let it be required to find the area of the ſpace D E m B, ſuppoſing D B = 12,245, E D = 6, m H = 5,49, z y = 3,88, and y b = y H = H D = 4,081. Fig. 168,

breadths add	0,	= br. at B
	6,	= E D
	5,49	= m H
	3,88	= z y
numb.	4) 15,37	ſum
	3,84	= mean breadth
multiply	12,245	= length
	47,0208	= area

By the ſecond part of the rule,

2)4.081	= y B
2.0405	= ½ y B
mul. 3,88	= y z
7,917140	= area △ B z y
19,119485	= m z y H
23.445345	= m E D H
50,481970	= area ſought

$$\begin{array}{rl} 3,88 & = y\,z \\ 5,49 & = m\,H \\ \hline 9,37 & = \text{sum} \\ \hline 4,685 & = \text{half sum} \\ \text{multiply} \quad 4,081 & = y\,H \\ \hline 19,119485 & = \text{area } y\,z\,m\,H \\ \hline 5,745 & = \tfrac{1}{2} \text{ of } E\,D \text{ and } m\,H \\ \text{multiply} \quad 4,081 & = H\,D \\ \hline 23,445345 & = m\,E\,D\,H \end{array}$$

But to be more exact, the following rule, taken from theorem 138, is to be used.

Rule 25. Take 4 breadths equally distant from one another, (the two outermost of which must be at the broadest and narrowest parts of the figure; and if the figure be very irregular, it must be taken at several times, that the sides or curves between the two outermost breadths may have a gradual curvature; this must also be observed in solids taken this way,) and to 3 times the sum of the two innermost breadths, add the sum of the 2 outermost ones, the last sum multiplied by $\frac{1}{8}$ part of the length gives the area.

Ex. 121. Let things be the same as in the last example.

$$\begin{array}{rl} 5,49 & = m\,H \\ 3,88 & = z\,y \\ \hline 9,37 & \\ 3 & \\ \hline 28,11 & = 3 \text{ times that sum} \end{array} \qquad \begin{array}{rl} 6 & = E\,D \\ 0 & \text{breadth at } B \\ \hline 6 & = \text{sum} \\ 28,11 & = \text{triple sum of inner. br.} \\ \hline 34,11 & \\ \text{multiply} \quad 1,53 & = \tfrac{1}{8}\, D\,B \\ \hline 52,1883 & = \text{area} \end{array}$$

Sliding-rule. As 8 on A : 34,11 on B : : 12,24$\frac{1}{2}$ on A : 52,19— on B, the area required.

The dimensions used in this and the former example, are the same with those in ex. 111, where by the true method the half segment turns out 52,174, which compared with these shews, that the first method by rule 24 is something wider than the second method by that rule; but the error in rule 25 appears inconsiderable, and as it comes so near a circular segment, it may be depended on as near in any segment whatever; consequently this rule is sufficient for the whole of superficial measure.

Superficial meaſure of ſolids.

To find the ſuperficial content of any cube, priſm, cylinder, &c.

Rule 26. Multiply the girt or periphery of the ſolid by its length, and the product is the ſuperficial content of all but the baſes, to which if you add the area of each baſe, viz. the double area of one baſe, you'll have the area or ſuperficial content of the whole ſolid. Theo. 132.

Ex. 122. If each ſide of a cube be 10,5 what is the whole ſuperficial content ?

10,5 = a ſide
multiply 4 ſuch ſides
42,0 = periphery
10,5 = length
441,0 = curve ſuper.
220,5 = area both baſes
661,5 = whole ſurface

10,5 = a ſide
10,5
110,25 = area 1 baſe
2
220,50 = area both baſes

Sliding-rule. Becauſe a cube has ſix = ſides, therefore the area of one ſide is a ſixth of the whole ſuperficial content. So as $\frac{1}{6}$ viz. 0,166 + on A : 10,5 on B : : 10,5 on A : 661,5 on B the anſwer.

Ex. 123. If each ſide of the baſe of a triangular priſm be 10 and its length 25,5, what is its ſuperficial content.

The area of any equilateral Δ or trigon is had by rule, or ex. 107. thus,

0,433013 tab. n° for a trigon
100 $\square$ of a ſide at baſe
43,3013 = area of either baſe
2
86,6026 = content both baſes

25,5 = length
30 = 10 $\times$ 3 = periphery
765 curve ſup.
add 86,6026 = baſes
851,6026 anſwer

Sliding-rule. As 1 on A : 30 on B : : 25,5 on A : 765 on B. And as 1 on D : 0,433 on A : : 1,0 on D : 43,3 on A (See ex. 68).

Ex. 124. If the length of a cylinder be 255, and periphery of its baſe 10, required its ſuperficial content ?

,07958 ſee rule 16
100 = $\square$ periphery
7,958 = area baſe
2
15,916 = area at both baſes
2550 = curve ſuperficies
2565,916 = the whole ſuperficies

255 = length
10 = periphery
2550 = curve ſuper.

Sliding-rule. As 12,56 on A : 10 on B : : 20 on A : 15,91 on B the double area. And as 1 on A : 255 on B : : 100 on A : 2550 on B, whose sum 2565,9 is the answer.

To find the superficies of any cone or pyramid.

Rule 27. The product of the slant height and periphery of the base divided by 2, or half the one multiplied by the other gives the curve superficies, to which adding the area of the base, the sum is the whole superficial content, theorem 131.

Ex. 125. If each side A B = B C = C D = D A (fig. 119) of the base of a □ pyramid be 10,5 and the slant height v A = v C 30, what is the superficial content?

10,5 = A B
4 such sides

42,0 = periphery base
15 = $\frac{1}{2}$ v A

630 = curve superficies
110,25 = area base

740,25 = whole surface

$\left.\begin{matrix}10,5\\10,5\end{matrix}\right\} = $ A B

525
1050

110,25 = base

Sliding-rule. As 2 on A : 42 on B : : 30 on A : 630 on B. And as 1 on A : 10,5 on B : : 10,5 on A : 110,25 on B, whose sum 740,25 is the answer.

Ex. 126. If the periphery of the base of some cone or pyramid be 45, and its slant height 380, what's the convex surface?

190 = $\frac{1}{2}$ slant height
45 = periphery at base

8550 = convex superficies

Sliding-rule. As 2 on A is to 380 on B so is 45 on A to 8550 on B, the answer.

To find the superficial content of any tapering streight sided solid.

Rule 28. Take the periphery of the two bases or ends in one sum, and multiply that sum and the solid's length one by half the other, the product is the curve superficies, to which adding the areas of the two bases you get the whole superficial content. Theorem 130.

Ex. 127. There is a conical frustum A B E F (fig. 127) the periphery at A B the greater base is 52,5, at the lesser base E F is 30,75, and length viz. slant length A F = E B = 40, required the superficial content of all except the greatest base.

52,5		30,75	
30,75		30,75	
83,25	sum	945,5625	square of 30,75
20 $\frac{1}{2}$	A F	85970	inverted factor
1665,00	curve superficies	66189	
752,47	= A L B	8510	
2417,47	answer	472	
		76	
		752,47	area lesser base

Sliding-rule. As 2 on A : 83,25 on B : : 40 on A : 1665 on B, and as 1 on D : ,07958 on A : : 3,075 on D (See ex. 68.) : 7,52$\frac{1}{2}$ nearly on A, so 752$\frac{1}{2}$ is the area of the lesser base, hence the answer is = 2417,5.

Ex. 128. If there be a cylinderoid, prismoid, frustum, &c. whose peripheries at the two ends is 502 and 35, and slant length 600, what is the convex surface.

502
35
―――
537
300 half the height
―――
161100 the answer

Sliding-rule. As 2 on A : 537 on B : : 60,0 on A : 16110 on B (See ex. 68) so 161100 is the convex superficies required.

To find the superficies of any globe or sphere.

Rule 29. (By theo. 133.) the surface of a globe is = to 4 times the area of its greatest circle, so any of the factors for the area of a circle (being multiplied by 4) will be factors for the surface of a globe when the like things are given, that is 3,1416 (= 4 times 0,7854) when the axis is given, or ,31832 (= 4 × ,07958) when the periphery is given, &c.

Ex. 129. If the axis of a globe or sphere be 10,5, what's the superficial content ?

$\left.\begin{matrix}10,5\\10,5\end{matrix}\right\}$ = axis
―――
110,25 square of the axis
22 see rule 16
―――
7) 2425,50
―――
376,5 surface

Sliding-rule.

As 1 on D is to 3,1416 on A so is 1,05 (for 10,5 is of the rule) on D to 3,465 on A so (by ex. 68.) 346,5 is the answer.

Having given the length or height of any part of a globe or ſphere to find its ſuperficial content, theo. 135.

Rule 30. Multiply the height of the given part, the axis of the globe and 3,1416 into one another, the laſt product is the curve ſurface, to which add the area of the end or ends for the whole ſurface.

Ex. 130. Given the axis B D (fig. 128.) = 10,5 of the globe B v D L and E v = 1,75 the height of a ſegment H v d, to find the curve ſurface.

10,5	axis
1,75	height ſegment
18,375	product
6141,3	inverted factor
55125	
1837	
735	
18	
11	
57,726	the anſwer.

Sliding-rule.

As 0.318 on A is to 10,5 on B ſo is 1,75 on A to 57,7+ on B the curve ſuperficies of the ſegment H v d, and is alſo = the curve ſuperficies of any part or zone m H d G whoſe length m H = d G is = 1,75

To find the ſuperficial content of any of the five regular bodies. See problem 157.

1. The tetraedron's ſurface being 4 = and equilateral △s; therefore, 4 times the area of one of theſe △s will be the ſurface of the tetraedron, or being a pyramid, its ſurface may be had by rule 27. Now if the ſide of an equilateral △ be 1, its area (by ex. 107) will be 0,433013 which multiplied by 4 gives 1,732051 for the ſurface of a tetraedron whoſe ſide is 1.

2. The ſurface of the octaedron being 8 = and equilateral △s, it is therefore double to that of the tetraedron, ſo 3,464102 is the ſurface of an octaedron, when each ſide thereof is unity.

3. The hexaedron is a cube, which ſee in ex. 122.

4. The ſurface of the icoſaedron is contained under 20 = and equilateral △s, and is therefore 20 + 433013 = 8,66026, when its ſide is unity.

5. The ſurface of the dodecaedron is contained under 12 = pentagons, ſo 1,720477, being the area of a pentagon whoſe ſide is unity, 12 times 1,720477 = 20,645724, will be the ſurface of a dodecaedron, whoſe ſide is unity, and thus you get this table, in which for the ſolidity's See Ex. 181.

If the side of the (body) be unity, its surfa. will be (superficies), and solidity (solidity):

If the side of the	be unity, its surfa. will be	and solidity
Tetraedron	1,732051	0,11785
Octaedron	3,464102	0,47140
Hexaedron	6,	1,
Icosaedron	8,660260	2,18169
Dodecaedron	20,645724	7,66312

Rule 31. Multiply the square of the side of the proposed body by the tabular superficies belonging to that body, and that product is its surface.

Ex. 131. If the side of a tetraedron be 10, what is its superficies?

1,732051
100
173,2051 answer

Sliding-rule.
As 1 on D is to 1,732 on A so is 1,0 on D to 1,732 on A, so (see ex. 68) 173,2. ans.

Ex. 132. If the side of a dodecaedron be 13, what is its superficial content.

20,645724
169 □ of 13
3489,127356 answer

Sliding-rule. As 1 on D is to 20,64 on A so is 1,3 on D to 34,89 on A, (see ex. 68) so 3489 is the answer.

In rule 30. Is the method of finding the surface of any segment or of any frustum of a globe, but to find the surface of any △ or polygon on the surface of a globe, this following rule is general.

Rule 32. Take the angles in degrees and decimal parts of a degree, to their sum add 360, from this sum take the product of 180 into the number of angles, multiply the remainder and the square of the sphere's axis and ,0044 (3,1416 ÷ 720) into one another, the last product is the area or superficies of the polygon. Theo. 76.

Ex. 133. and 134. If the axis of a globe be 16, what is the area of a spherical △ on that globe, whose 3 ∠s are 36° 8′ and 46° 18′ and 104°, viz. 36,1333 and 46,3 and 104. Also, what is the area of a spherical pentagon, whose 5 ∠s are 70, 80½, 130, 150, and 160, and axis of the globe 20.

1. For the Δ, (degrees)

	36,1333
	46,3
	104,
	186,4333
add	360,
	546,4333
ſubtract	540 = 180 × 3
	6,4333
	,0044
	,02830652
	256 $\square$ 16 the axis
	7,24646912 area Δ

2. For the pentagon.

	70 degrees
	80,5
	130
	150
	160
add	360
	950,5
180 × 5 =	900 ſubtract
	50,5
3,1416 ÷ 720	,0044
	,22220
$\square$ axis 20	400
area pent.	88,88

Ex. 135. But to find the ſurface of any ſolid which properly is not known.

Rule 33. Take 3 equi-diſtant girths or periphery's each at right *Ls* to the curve length, or ſlant length and to 4 times the middle girth add the other two girths, this ſum multiplyed by a ſixth of the curve length, gives the ſurface. Theo. 138.

Note. The two extreme girths muſt be taken the one at the greateſt bulge, and the other at the leaſt, in the figure, and the mean girth in the middle between them, and if the figure be very irregular, or there be ſeveral ſuch bulges it muſt be taken at ſeveral times as if it were ſo many ſeveral figures, and the ſum of all the contents will (nearly) be that required, or if you think 3 girths not ſufficient, you may take 4, and work by rule 25.

Required the ſuperficies of the ſolid Q V Q v (fig. 140) the periphery at V v the greateſt bulge, being = 40, at Q the leaſt = 0 at M in tho middle, between Q and V = 35, and the curve length Q V = Q v = 30,

Girths.
0 at Q
40 at N
140 = 4 × 35
180 their ſum
5 $\frac{1}{18}$ Q v
900

Sliding-rule. As 6 on A is to 180 on B ſo is 30 on A to 900 on B the ſurface of Q N v, and if the other part Q v N be = this part already meaſured, then twice 900 = 1800 the ſurface of the whole ſolid; or, as 6 on A is to 1800 on B ſo is 60 on A to 1800 on B.

SECTION V. *Artificers work.*

Artificers have different ways of measuring according to the custom of each craft, some give the content in feet, some in yards, some in squares, some in rods &c. they also differ according to the custom of the place, some give the content in one denomination, and some in another. Now as a general rule for all the various methods amongst these workmen, it is best to take the dimensions in feet and decimal parts of a foot (ex. 92.) and so find the content in feet which (by rule 6.) may be reduced to what name you please, for it is seldom that artificers use ought but superficial or flat measure, and because walls, doors, floors, windows, roofs, &c. are rectangles the areas of such are always found by rule 8, but if any other form come in practice it cannot but come under some of the foregoing rules, 2 or 3 examples in every case will clear up this section.

Of Bricklayers work.

The principals are walling, tileing, partitioning and chimney-work.

Bricklayers commonly measure their work by the rod square of 16 feet and an half, so that one rod in length and one in breadth contains (16,5 × 16,5) 272,25 square feet; but in some places they allow 18 feet to the rod i. e. (18 × 18) 324 square feet, and in others 21 feet is a rod with 3 feet height i. e. 63 square feet, and here they do not regard the wall's thickness, only moderates the price accordingly, so when you are to measure such work enquire which of these ways must be used, and then divide the product of the length and breadth in feet by the proper divisor, so will you have the content. But where brick walls are measured by the rod, and reduced to the standard thickness of 1½ brick thick, this is the

Rule 34. Multiply the superficial content of the wall in feet by the number of half bricks it is in thickness, one third part of that product is its content in feet, at the standard thickness.

Ex. 136. If a wall be 72½ feet long, 19¼ feet high, and 5½ bricks thick, how many standard rods doth it contain?

	19,25 feet
multiply	72,5
	1395,625
½ br. thick =	11
3)	15351,875
272)	5117,291
rods =	18,8

Note. In reducing feet to rods, it is usual to reject the odd ,25 and divide only by 272, but the divisors 3 and 272,25 may be reduced to one factor or divisor, and so be a constant gauge point.

Thus, 272,25 × 3 = 816,75

which divided by 11 quotes 74,19, which is a conſtant diviſor for $5\frac{1}{2}$ bricks thick and in this manner is this table contructed, for the thickneſs between 1 and $4\frac{1}{2}$, and may be continued at pleaſure.

bricks thick	multipliers	diviſors
1	,00245	400,27
$1\frac{1}{2}$	,00367	272,25
2	,00489	204,19
$2\frac{1}{2}$	,00662	163,35
3	,00734	136,12
$3\frac{1}{2}$	,00857	1 6,68
4	,00884	102,1

Sliding-rule.

As 74,19 (the gauge point for the given thickneſs $5\frac{1}{2}$) on A : 19,25 on B : : 72,5 on A : 18,8 rods on B. the anſwer.

The uſual way to take dimenſions of a building is to meaſure half round it (at the middle) on the out-ſide, and half round it in the in-ſide with a cord, this gives the true compaſs of the building in which the thickneſs of the wall is conſidered and if it be all of a height, meaſure its height at any place from the bottom of the foundation, but if the height be unequal, you muſt take ſeveral heights, and their ſum divided by the number of heights gives the mean height, the pykes on the end walls being △s are meaſured as ſuch, or two = ones being put together makes a rectangle of the ſame height and breadth of the building.

Ex. 137. If each ſide wall of a building be 45 feet long on the out-ſide, each end wall 15 feet broad on the in-ſide, the height of the building 20 feet, and the pyke at each end wall 6 feet high, the whole, 2 bricks thick, how many ſtandard rods, as alſo how many ſquare yards not minding the thickneſs, as if it were a ſtone building, &c.

Feet.

90 ſum ſide walls
add 30 that of the ends

120 compaſs
20 height

2400 area without pikes
15 × 6 = 90 pikes add

2490 □ feet in the whole
mult. ,00489 fact. 2 bricks thick

12,17610 rods, anſwer.

For the ſquare yards.

9)2490 = area in feet

$276,\frac{2}{3}$ □ yards

Sliding-rule, for the rods. By the table, the diviſor for two bricks thick is 204,19, ſo as 204,19 on A : 300 on B : : 90 on A : the rods excluſive of the pikes, and as 204,19 on A : 15 on B : : 6 on A : the rods contained in the pikes.

Sliding-rule for the yards. As 9 on A is to 120 on B ſo is 20 on A to $266\frac{2}{3}$ on B the yard's area pikes excluded; and as 9 on A is to 15

on B ſo is 6 on A to 10 on B the pikes; ſo $266\frac{2}{3}$ added to 10 gives $276\frac{2}{3}$ the area in yards as required.

Note. Stone buildings are ſometimes reckoned amongſt maſon's works.

Of Chimneys.

In meaſuring a chimney, the uſual way is, if it ſtand alone without leaning againſt a wall, &c. girt it about below the mantle, and take that for the length, and the height of the room, or chimney ſo far as it keeps the ſame girt, the product of theſe two is the content, but if it ſtand againſt a wall you muſt meaſure it round to the wall for the girt, and take the height as before; for arches, ſome take the breadth and half breadth of the arch, and multiplies that ſum by the length of the arch, for the content; ſome make deduction for windows, doors, and the vacancy in chimneys between the hearth and mantle, &c. and ſome make none.

Of Tileing, and Slating.

Tileing is meaſured by the ſquare of 100 feet, viz. 10 feet long and 10 feet broad. Slating ſometimes by the yard ſquare, and ſometimes by the rod of 49 ſquare yards, &c. To take the dimenſions, meaſure with a cord the length of the ridge, then having a ſmall weight at one end of the cord, put it over the ridge and let it go down to the eave, and take the other end to the other eave, ſo you have the breadth of the roof, which multiplied by the length of the ridge gives the content.

Note. Double meaſure is commonly allowed for hips, vallies, gutters, &c. And in tileing it is common to allow double meaſure at the eaves ſo much as the projecture is over the plate, which is generally about 18 or 20 inches.

Ex. 138. If the breadth of a roof with the uſual allowance at the eaves be 24,5 feet and the length of the ridge 45 feet, how many yards ſquares, and rods are in that roof?

24,5 feet breadth
45 length
9) 1101,5 □ feet
49) 1225 □ yards
2,5 roods

100) 1102,5 □ feet
11,025 □s

Sliding-rule. As 9 on A : 45 on B : : 24,5 on A : $122\frac{1}{2}$ on B the □ yards area, and by ſetting 100 and 441 (viz. 9 times 49) on A inſtead of 9 you'll get the anſwer in ſquares and in roods.

Note. Some make an allowance in roofs for the ſpaces taken up by chimneys, &c. but others make none.

Of Carpenters work.

Carpenters work, as flooring, partitioning, and roofing, are meaſured by the ſquare of 100 feet like tileing, and ſome places by the ſquare yard.

Ex. 139. If a floor be 57,25 feet long, and 28,5 feet broad, how many ſquares?

feet 57,25 length
28,5 breadth
100)1631,625 □ feet
16,31625 ſquares

Sliding-rule.
As 100 on A : 57,25 on B : : 28,5 on A : $16\frac{1}{3}$ nearly, anſwer

Ex. 140. If a partition between two rooms be in length 80,5 feet, and in height $12\frac{1}{4}$, how many ſquares are contained therein?

12,25 feet
80,5
100)986,125 □ feet
9,86125 ſquares

Sliding-rule. As 100 on A : 12,25 on B : : 80,5 on A : 9,86 on B, the anſwer.

Ex. 141. If a houſe within the walls be 40,5 feet long and 20,5 feet broad, how many ſquares?

40,5 feet
20,5 breadth
100) 830,25 □ feet in the flat
8,3025 ſquares in the flat
add 4,1512 half the flat
12,4537 ſquares in the roof

Sliding-rule.
As 66,66 (viz. $\frac{2}{3}$ of 100) on A : 40,5 on B : : 20,5 on A : 12,45 on B, the anſwer.

Note. If the roof of a building be truly pitched, the area and half area of the flat or floor will be the area of the roof, or as 2 is to 3 ſo is the area of the floor to that of the roof, on which footing the laſt ex. is wrought.

Of Plaiſterers work.

Plaiſtering, rendering, and cieling, are all meaſured by the ſquare yard, in ſuch works it is beſt to take dimenſions with a decimally divided yard, when a room is plaiſtered or rendered, take its compaſs in the inſide girting in chimneys, &c. where ever the work comes, this multiplied by the height of the room gives its content.

Ex. 142. If the compaſs of a room be 47,2 yards, and its height 4,3 yards, what is the content?

yards 5,41 height
45.8 circuit
247,778 content

Sliding-rule. As 1 on A : 47,2 on B : : 4,3 : 203 nearly.

Note. Whiting and colouring are both measured by the yard like plaistering. Deductions or allowances are to be made according to the different customs of places.

Of Joiners work.

This work is also given in square yards, in taking dimensions, they measure round the room, or round the floor upon which it stands for the breadth of their work, and for the height, they measure with a cord the height of the room denting in the line where the plane comes, and girting over cornishes, pannels, mouldings, &c.

Ex. 143. If a room of wainscot (being girt over the mouldings &c.) be 5,41 yards, and the compass thereof 137,4 feet, what is the content?

yards 5,41 height
45,8 circuit
247,778 content

Sliding-rule.
As 1 on A : 45,8 on B : : 5,41 on A : 247.7 on B, the answer.

Note. Doors, window-shutters, and and all such like as are wrought on both sides are called work and half work; in such cases find the content as before, take half thereof and add to it, which gives the content; or, as 2 : 3 : : the content before found : the content at work and half work.

Ex. 144. If the window shutters about a room be 70 feet broad (viz. all their breadths in one sum = 70 feet) and the height of each shutter 6,5 feet, what's the content at work and half?

6,5 feet
70.
2)455.0 work
227.5 half work
9)682,5
75,8 yds. work and ½

Sliding-rule, As 6 (viz. 9 times 2 divided by 2) on A : 70 on B : : 6,5 on A : 75,8 on B, the content at work and half.

In such examples as this, if all the heights be = you may take all the breadths in one sum, if all the breadths be = you may take all the heights in one sum, which is easier than to work them by 1 at a time, and equally true.

Of Painters work.

Painters work is the same with that of joiners, both in taking dimensions and measure.

Ex. 145. If a globe be painted whose greatest circumference is 10 feet, how many yards of painting is thereon?

10 periphery
100 □ periphery
,31832 see rule 19.
9)31,832 □ feet
3,537 □ yards

Sliding-rule.
As 9 on A : 100 on B : : 0,318 on A : 3,53+ on B, the answer.

Of Masons work.

Masons work is sometimes measured by the foot square, and sometimes by the running foot, (viz. only so many feet in length) where observe that for every foot in length, running measure, there ought to be a foot in breadth, or girt if it be a solid wrought about, and this makes running measure agree with superficial measure, as it always ought to do; if a stone be hewed into any solid form you may by some of the rules in the last section find its area. Masons like joiners, &c. always measure so far as their tools come. Stone buildings are sometimes reckoned amongst masons work. Paving is also reckoned in with masons work.

Ex. 146. If a wall be 112,25 feet long, and 16,5 feet high, how many rods at 63 square feet to a rod.

112,25 feet
16,5
6)1852 125
29,4— rods

Sliding-rule. As 63 on A : 112,25 on B : : $16\frac{1}{2}$ on A : 29,4— on B.

A person that cannot even multiply may easily find the □ feet. &c. in any rectangle; thus, this wall being $16\frac{1}{2}$ feet high, if you take its length $112\frac{1}{4}$ feet and measure it on your rule 16 times and a half, you will find $1852,\frac{1}{8}$ feet for the area of the wall.

Of Glaziers work.

Glaziers measure by the square foot, and commonly take dimensions nearer than any of the foregoing, for they'll go to the 8th, or 10th of an inch.

Ex. 147. If there be 4 panes of glass each 4,23 feet high, and 1,25 feet broad, how many square feet?

4,23
5 = 4 times 1,25
21,15 □ feet required

Sliding-rule.
As 1 on A : 4,23 on B : : 5 on A : ~~25,38~~ 21·15 on B, the answer.

Ex. 148. If the diameter of a round window be 2,23, what is the content?

2,23
2,23
4,9729 □ feet

Sliding-rule.
As 1 on A : 2,23 on B : : 2,23 on A : 4,97 on B the answer.

Note. Round and oval windows are measured as if they were full □ that length, &c. because glass wastes in cutting for such forms.

To measure Board and Plank.

Board and plank are sometimes sold by foot running measure, which is had by measuring the planks length along its middle, and by the flat foot, which is had by multiplying the length, taken as before in feet and decimal parts of a foot, by the breadth in the middle taken in inches, that product divided by 12, gives the superficial or flat feet, and sometimes by the solid foot, which is most reasonable, because it considers length, breadth and thickness, viz. one multiplyed into the product of the other two, and if they all three be inches, the product divided by 1728 (the solid inches in a cubical, or solid foot) gives the solid feet, but if any one of these 3 dimensions be in feet as generally the length is, then divide the last product by 144, or if two of these 3 be in feet, then divide by 12. But it is easiest, and will require no dividing, to take all the 3 dimensions in feet and decimal parts thereof, see the following examples.

Ex. 149. If a board be 16 inches broad and 13 feet or 156 inches long, how many square or flat feet is therein?

vulgarly		common method	decimally
inches	156 length	13 feet	13 feet
	16 breadth	16	1,333
	144)2496 □ inches	12)208	17,329
	$17\frac{1}{3}$ □ feet	$17\frac{1}{3}$	

1. As 144 on A : 16 on B : : 156 on A : $17\frac{1}{3}$ on B,
2. As 12 on A : 16 on B : : 13 on A : $17\frac{1}{3}$ on B,
3. As 1 on A : 13 on B : : $1\frac{1}{3}$ on A : $17\frac{1}{3}$ on B,

} the answer, in square feet.

Any of these 3 ways (as also by feet and inches) may be taken, but the last being most expeditious and exact too, if the dimensions are taken decimally, it is most in use.

Ex. 150. If the length of a plank be 15,2 feet, its breadth 0,32 parts of a foot, and thickness 0,25 of a foot, what is its content superficial and solid.

	15,2 length	4,864 superficial content
mul tiply	,32 breadth	,26 thickness
	4,864 superficial content	1,216 solid content

Sliding-rule. As 1 on A : 15,2 on B : : ,32 on A : 4,86 on B the flat feet, for the solid feet find a mean proportional between any two of the 3 dimensions (the breadth and depth are commonly taken for this purpose). Then it will be as 1 (or the square root of any other divisor) on D : the length on A : : the mean proportional on D : the content on A, or turn som one of the 3 dimensions into a divisor (see

ex. 106) ſo here, as ,25 on A : ,25 on D : : 32 on A : ,28 on D the mean proportional between ,25 and 32 ; then as 8 on D : 15,2 on A : : ,28 on D : ,28 on D : 1,22 feet on A the ſolid content.

Ex. 151. If a board be 5 inches broad, how much in length will make a flat foot ?

Rule 35. Since the product of the length and breadth gives the area, 'tis evident that if the given area be divided by one of them, the quotient will be the other of them.

5)144 inches in a flat foot
anſwer 28,8 inches in length

Sliding-rule. As 5 on A , 144 on B : : 1 on A : 28,8 on B.

Ex. 152. If a board be ,82 feet broad, how much in length will make $2\frac{1}{2}$ flat feet ?

,82)2,50 flat feet
anſ. 3,05 feet length

Sliding-Rule. As 0,82 on A : $2\frac{1}{2}$ on B : : 1 on A : 3,05 on B.

Theſe two examples conſiders the board to be an = breadth throughout, but if it be broader at one end than at the other, you may take a breadth as near as you can gueſs in the middle of the part to be cut off, and with that breadth find the length as in the laſt ex. and then find the content by ex. 150, which if it be too much, or too little, you may try a leſs, or a greater breadth, &c. ſo by a few trials may come near the length required, but to do it exactly and at once, ſee Queſt. 19.

Of ſquare Timber.

Square timber meaſure is the ſame with ſolid plank, therefore the length of the tree, the breadth and depth, or thickneſs, taken in the middle thereof, and multiplied into one another gives the true ſolid content if the baſes or ends are equal, (for then it is a priſm) but in tapering timber it gives the content ſomewhat too little as appears by ex. 172.

Ex. 153. If a piece of ſquare timber be 13.2 feet long, and 1,52 feet ſquare, (viz. 1,52 broad and 1,52 deep) how many feet of timber ?

1,52 breadth
1.52 depth
2,3104

2,3104 area in the middle
13,2 length
30,49728 anſwer

Sliding-rule. As 1 on D : 13,2 on A : : 1,52 on D : 30,5 on A the ſolid feet required.

Ex. 154. If a piece of ſquare tapering timber be 15 foot long 1 foot broad, and 0,7 of a foot deep at the leſſer end, 2 feet broad and 1,4 feet deep at the greater end, how many ſolid feet is therein?

Note. If the tree (whether ſquare or round timber) be not contained under ſtreight planes from baſe to baſe, ſo that you judge its breadth and depth taken in the middle not exact, you may take theſe dimenſions at each baſe or end, and half their ſum will give that in the middle, ſo in this ex. half the ſum of the two breadths 1 and 2 is 1,5 and half the ſum of the two depths 0,7 and 1,4 is 1,05. Then,

1,5 breadth } taken in the middle of the tree.
1,05 depth }

1,575
15 length

23,625 ſolid feet.

Sliding-rule, ſee ex. 106. and 150. As 1,05 on A : 1,05 on D : : 1,5 on A : 1,25 on D a mean proportional between 1,05 and 1,5. Then as 1 on D : 15 on A : : 1,25 on D : $23,62\frac{1}{2}$ on A. the anſwer.

Otherwiſe. As 15 on A : 1 on B : : 1 on A : ,066 on B, which is 15 turned to a diviſor. Then as ,066 on A : 1,5 on B : : 1,05 on A : $23,62\frac{1}{2}$ on B the anſwer.

Ex. 155. If a piece of ſquare tapering timber be 18 foot long, 6 inches deep and 1 foot 6 inches broad in the middle, how many ſolid feet?

1,5 breadth
,5 depth

,75
18 length

13,50 ſolid feet.

Sliding-rule, here 0,5 turned to a diviſor is 2. Then as 2 on A : 18 on B : : 1.5 on A : 13,5 on B. the anſwer.

Note. Some meaſurers of timber take half the ſum of the breadth and depth taken in the middle of the tree, for the ſide of a mean ſquare which ſquared and multiplied by the length they ſay gives the content. But it gives it too much, and if the difference between the breadth and depth be great, this error is ſo too, as in the laſt example, the depth ,5 feet added to the breadth 1,5 feet gives $2\frac{1}{2}$ feet whereof is 1 foot, which multiplied by 18 feet the length, gives 18 feet for the content, too much by 4,5 feet.

Ex. 156. If a piece of = baſed ſquare timber be 6 inches ſquare at each end, how much in length makes $1\frac{1}{2}$ ſolid feet.

Rule 36. Because the product of the length, breadth, and depth gives the content; therefore, if the given content be divided by the product of any two of them, the quotient will be the third.

$$\text{feet}\ \left\{\begin{matrix} ,5 \\ ,5 \end{matrix}\right\} = \text{g. side}$$

$$\underline{,25)1,5}\ \text{g. content}$$

$$6\ \text{feet answer}$$

Sliding-rule. As 0,5 on D : 1,5 on A : : 1 on D : 6 on A the answer.

Note. If the timber be tapering, see question 20, section 9.

Of round Timber.

The usual way is to girt the tree in the middle with a small cord, then one fourth part of that girt squared and multiplied by the length gives the content, but if the tree be uneven in the middle so that a true girt cannot be taken there, then you must girt it at each end, and take half the sum of these two girts, which answers to a girt taken in the middle, $\frac{1}{4}$ whereof must be used as before, when you've taken the girt with a small string, double it and measure that double on the inches on a sliding rule, &c. half this is a fourth of the girt in inches, which squared and multiplied by the length in feet and divided by 144 gives the content in feet, or as a general rule upon the sliding rule. As 12 on D : the length in feet on A : : a fourth of the girt in inches on D : the content in feet on A, but if you measure the girt upon a foot rule decimally divided, you need not divide by 144, and on the sliding-rule it will be as 1 on D : the length in feet on A : : a fourth of the girt in feet and decimal parts of a foot on D : the content in feet on A. This is the method generally practised because of its ease and expedition, but it always gives the content too little; as is thus proved, if the circumference of a circle be 1 then $\frac{1}{4}$ of 1 is = 0,25, which squared gives ,0625 for the area of the girth but (by rule 16) the true area of such a circle is ,07958, so that if the timber be = based, (viz. a cylinder) the true content will be to that found this way, as ,07958 : ,0625 which is nearly as 22 to 18, so those that buy round timber by this measure, have nearly one fifth part allowed for chips, and more if the timber be tapering as you'll see in ex. 174. But if the diameter of a circle be 12 inches its area (by rule 15) is = 113,0976, whose square root is 10,635, so that if instead of 12 on the line D you take 10,635 on it, or by the pen divide by 113,0976, instead of 144 you'll have the true content nearly. But custom being so much in favour with the erroneous gauge point 12 that it is needless to offer any other method.

Ex. 157. If a piece of round timber be 20 foot 3 inches long, and 6 inches square in the middle, how many feet of timber is therein?

Note. That by ſaying round timber is ſo much ſquare means that $\frac{1}{4}$ of the girt is that much.

common way		decimally	
inches	6, 6 } $\frac{1}{4}$ girt	feet	,5, ,5 } $\frac{1}{4}$ girt
	36 □ $\frac{1}{4}$ girt		,25 area girt
	20,25 length		20,25 length
	144)729,00 area girt		5,0625 feet, content
	5,06 feet content		

Sliding-rule. 1. As 12 on D : 20$\frac{1}{4}$ on A : : 6 on D : 5,06 on A the ſolid content in feet. 2. As 1 on D : 20,$\frac{1}{4}$ on A : : ,5 on D : 5,06 on A as before.

Ex. 158. In a piece of round timber 18 feet long, 0,92 feet girt at the leſſer end, 1,8 feet girt at the greater end, how many ſolid feet?

Here the ſum of 0,92 and 1,8 is 2,72 (ſee ex. 154) half of which 1,36 for the girt in the middle.

feet ,34 } = $\frac{1}{4}$ of 1.36
,34 } girt in the middle

,1156 = □ $\frac{1}{4}$ girt
mult. 18 feet, length

2,0808 anſwer

Sliding-rule.
As 1 on D : 18 on A : : 34 on D : 2,08 on A the anſwer.

Ex. 159. If the length of a walking-ſtick be 3,5 feet and $\frac{1}{2}$ an inch ſquare, what's the content?

feet 3,5 length
,25 □ $\frac{1}{4}$ girt in inches

144),875

,0061 = feet content
1728 in. in a ſolid foot

10,5408 ſolid inches anſwer

Sliding-rule. As 12 on D : 3,5 on A : 5 (for 0,5 is off the rule) on D : 0,61 on A ; ſo (by ex. 68) ,0061 part of a foot is the anſwer.

After theſe methods the following table is calculated.

Length in feet.	$\frac{1}{4}$ girt in inches.	Content in feet.
20,25	6	5,06
18	4,08	2,08
20	7	6,8
32	8	14,2
15	6,5	4,4
30	18	67,5
30	12	30
18	24	72
29	21,5	64,2

Note. To meaſure a growing tree is the ſame thing when the dimenſions are taken, which may be had by a ladder and a long ſtaff, with a cord to take the girt, or you may take the height by problem 130 &c to 135, and find the girt by problem 162, 163, &c.

To find the ſolidity of any cube. Fig. 123.

Rule 26. Multiply the ſide by itſelf, and that product multiplied by the ſame ſide gives the ſolidity. Theorem 86.

Ex. 160. If each ſide of a cube be 62,06, what is its ſolid content?

60,2	given ſide
60,2	
3624,04	its ſquare
60,02	given ſide
218167,208	ſolidity

Sliding rule.

As 1 on D : 60,2 on A : : 0,602 (for 60,2 is off the rule) on D : 21,8 + on A, ſo (by ex. 68) 218000 ÷ is the anſwer.

To find the ſolidity of any parallelopipedon, priſm, or cylinder.

Rule 37. Multiply the area of the baſe by the length of the ſolid, and the product is the ſolid content. Theorem 86.

Ex. 111. If the length of a □ priſm be 15, and each ſide of its □ baſe 7,2, what is the ſolidity?

7,2	a ſide of the baſe
7,2	
51,84	area of the baſe
15	length
777,60	ſolidity

Sliding-rule.

As 1 on D : 15 on A : : 0,72 (for 7,2 is off the rule) on D : 7,776 on A ſo (by ex. 68) 777,6 is the anſwer.

Ex. 162. If a piece of = baſed hewn timber or ſtone, &c. be 25 feet long, 9 inches deep, and 25 inches broad, how many ſolid feet?

common way		decimally		by feet and inches		
inches 25	breadth	2,0833 feet		2	1	
9	depth	,75		0	9	
225	area baſe	1,562475	area baſe =	1	6	9
25	ft. height	25	multiply	25		
144)5625 (	39,061875 content			25	0	0
				12	6	0
				1	0	6
				0	6	3
				39	0	9 anſw.

Sliding-rule. 1. As 9 on D : 9 on A : : 25 on A : 15 on D, a mean proportional between 9 and 25. 2. As 12 on D (becauſe 12 is the ſquare root of 144), 15 on A : : 15 on D : 39,00 ſolid feet on A the anſwer. Otherwiſe. See ex. 106. 1. As 0,75 on A : 1 on B : : 1 on A : 1,33 on B, 0,75 in a diviſor. 2. As 1,33 on A : 25 on B : : 2,083 on A : 39,06 on B the anſwer.

Ex. 163. If there be a triangular prifm, the bafe of its triangular bafe being $13\frac{1}{2}$ inches, and ⊥ thereof 12 inches, and the folid's length 20 feet, how many folid feet is contained therein ?

common way		decimally		feet and inches	
13,5	inches	1,125	feet	1 1 6	
6 = $\frac{1}{2}$	of 12	1		1	
81,0	area bafe	1,125		1 1 6	
20	feet height	$\frac{1}{2}$ = 10		10	$\frac{1}{2}$ height
144) 1620		11,25	anfwer	11 3 0	
11,25	anfwer				

Sliding-rule. 1. As 1 on A : 1,06 on D : : a mean proportional between 1 and 1,125. 2. As 1 on D : 10 ($\frac{1}{2}$ of 20) on A : : 1,06 on D : $11\frac{1}{4}$ on A the anfwer. Otherwife. 1. As 6 on A : 6 on D : : 13,5 on A : 9 on D, a mean proportional between 9 and $13\frac{1}{2}$. 2. As 12 on D : 20 on A : : 9 on D : 11,25 on A the anfwer.

Ex. 164. If the length of a cylinder be 8 and the diameter of each bafe 2,1, what's the folid content ?

2,1	
2,1	
4,41	□ diameter
,7854	
3,463614	area bafe
8	height
27,708912	folidity

Sliding-rule.

,7854 turned to a divifor is 1,2732, whofe fquare root is 1,128 ; therefore, (fee ex. 68) as 1,128 on D : 8 on A : : 2,1 on D : 27,7+ on A the anfwer.

To find the folidity of any cone or pyramid.

Rule 38. Find the area of the bafe according to its form, by fome of the rules in fection 4, which area multiplied by a third part of the folid's axis gives the folid content. Theorem 89.

Ex. 165. If A v C D (fig. 119) be a fquare pyramid, whofe axis v P is 15, and each fide A B = B C = C D = D A of its fquare bafe A B C D 10,1, what's its folid content ?

10,1	
10,1	
102,01	area bafe
5 = $\frac{1}{3}$	axis
510,05	folidity

Sliding-rule.

As 1 on D : 5 on A : : 1,01 (for 10,1 is off the rule) on D : 5,1 on A ; fo (by ex. 68) 510 is the anfwer.

Ex. 166 If there be a pyramid whoſe baſe is a regular heptagon each ſide whereof is 10 and the axis of the pyramid 68,1, what is its ſolidity.

$\left.\begin{matrix}10\\10\end{matrix}\right\}$ a ſide of baſe

100 □ that ſide
3 6339 ſee ex. 107

363,39 area baſe
22,7 $= \frac{1}{3}$ axis

8249,08 ſolidity

Sliding-rule.

Set the ſquare root of ,2751 (viz. 3,6339 or 3,6340 in a diviſor) on D: 22,7 on A then againſt 10 on D ſtands 8249 on A the anſwer.

Ex 167. If the axis v D of a cone v A B (fig. 127) be 68,1, and A B the diameter of its baſe 10, what is the ſolid content?

$\left.\begin{matrix}10\\10\end{matrix}\right\}$ diam. baſe

100 its □
,7854

78,54 area baſe
22,7 $= \frac{1}{3}$ v D

1782,858 ſolidity

Sliding-rule.

See ex. 164. As 1,128 on D : 22,7 on A : : 1 (for 10 is off the rule) on D : 17,83— on A, ſo (by examp. 68) 1783— is the anſwer.

Ex. 168. If the cone be elliptical, (viz. of an elliptical baſe) the greater diameter of the ellipſis 15,2, the leſſer 10, and cone's axis 22, what's the ſolidity?

15,2
10

152
22 axis

3344 product

multiply 3344
,7854

3)2626,3776

875,4592 ſolidity

Sliding-rule. As 1,128 on D : $\frac{22}{3} = 7{,}34$— on A : : 12,38 (a mean proportional between 15,2 and 10) on D : 875,4 on A the anſwer.

To find the ſolidity of a fruſtum of any pyramid or cone.

Rule 39. For the fruſtum of a pyramid, whoſe baſes are ſimilar regular polygons, to a ſide of the greater baſe add a ſide of the leſſer baſe, from the ſquare of that ſum take the product of the two ſides, multiply that difference by the tabular number (ex. 107) which reſpects the form of its baſes, this laſt product multiplied by a third part of the fruſtum's ⊥ height gives its ſolidity, Theorem 107.

Rule 40. To the product of two ſides of the two baſes, add $\frac{1}{3}$ part of the □ of their difference, this ſum multiplied by the tabular number (as before) and then by the fruſtum's ⊥ height or length gives the ſolidity. Theorem 106.

Rule 41. Multiply the difference between two sides of the two bases by 0,52 that product added to a side of the lesser base, this sum squared and multiplied by the tabular number and frustum's length as before, gives its solidity nearly. Theorem 124.

Note. These 3 rules hold true in conical frustums by using the diameters of the bases as sides, and ,7854 as the tabular number, or if you take the peripheries instead of the diameters you must take ,07854 instead of ,7854, and if the bases of the frustums be not regular polygons, as rectangles ellipsis &c. the two following rules are general.

Rule 42. Multiply the area of the greater base by the area of the lesser base, to that product add the said two areas, this produces the solidity after being multiplied by a third part of the frustums height, let the bases be in any form whatsoever provided they be similar and parallel. Theorem 108.

Rule 43. To four times the area taken in the middle of the solid parallel to its two bases, add the area of each base, this sum multiplied by one sixth part of the solids length gives its solidity. Theorem 138. or 140.

Note. This last rule holds true in all streight sided frustums, whether of cones or pyramids, as also in cylinderoids and prismoids, and very near in any solid whatever, as appears in prob. 190, whence this rule (like rule 25) is in itself sufficient for the whole of solid measure, and there is no difficulty in it, but taking the said middle area, which in practise may be done as easily as an area at each end; and in curve-lined solids it cannot be harder to come at than the figures form which is to be known before you can find its content by the rule adapted to such a form &c. But in streight lined solids (by theo. 9) half the sum of any two like sides at the bases is equal to a similar side in the middle between these two bases, so in these solids it is had without measuring. In all cases of solid measure the length is to be taken perpendicularly to the breadth or breadths, as is evident from the theory of mensurations. Part second.

Ex. 169. If A b C D be the frustum of a □ pyramid whose axis p P is 24, A B, a side of the greater base 13, and a b, a side of the lesser base 8, what is its solidity? Fig. 119

First, by rule 39.

13 = A B
add 8 = a b
21 sum
441 □ sum
ſubtract 104 = 13 × 8
337
8 = $\frac{1}{3}$ p P
2969 ſolidity

Secondly, by rule 40.

13
8 } = {
104 product
add $8\frac{1}{3}$ = $\frac{1}{3}$ of 25, □ differen.
$112\frac{1}{3}$
24 = p P
2696 ſolidity

Thirdly, by rule 41.

5 difference of ſides
0,52 multiply
2,60
add 8 = a b
10.60 mean ſide
112,36 its □
24 = p P
2696,64 ſolidity

Fourthly, by rule 43.

21 ſum of A B and a b
441 = 4 times ar, mid.
169 = □ A B
64 = □ a b
674 ſum
4 = $\frac{1}{6}$ p P
2696 ſolidity

The beſt way to work theſe examples by the ſliding-rule is deduced from rule 43; thus, as 1 on D : 4 ($\frac{1}{6}$ of 24) on A : : 21 (the ſum of 8 and 13) and 13 and 8 on D : 1764 and 676 and 256 on A reſpectively, the ſum of theſe 3 numbers gives 2696 for the anſwer.

Each of theſe methods gives the ſame content except that by rule 41, which differs from truth by 0000,64, the reaſon is becauſe 0,52 is not a true factor nor can there be any one multiplier fixed for this purpoſe, as is proved in prob. 187, yet the ſaid factor never can exceed an error of ,08, therefore, if you chuſe to work by it, it may ſerve with the word nearly.

Ex. 170. Suppoſe the laſt mentioned fruſtum to be one of a trigonal pyramid (viz. a trigon) whoſe axis or length p P is 24, a ſide of the greater baſe 13, and one of the leſſer 8 as before, to find its ſolid content.

Sliding-rule. If you divide 6 by 0,433013 (the tabular number for a trigon, ſee ex. 107) the ſquare root of the quotient will be 3,72 a conſtant gauge-point for ſuch fruſtums; then, as 3,72 on D : 24 the whole axis on A : : 21 (twice a ſide in the middle) and 13 and 8 (a ſide at each end) on D : 3 ſuch numbers on A as being added together gives 1167,368 for the anſwer. It is to be noted, that 0,433 may ſerve as the factor in common uſe inſtead of 0,433013, as is done in this example.

By rule 23.

13
8
21 ſum
441 its □
ſubtract 104 product
337
,433 tab. number
145,921
8 = $\frac{1}{3}$ axis
1167,368 ſolidity

Ex. 171. If each ſide of the greater baſe of the fruſtum of a hexagonal pyramid be 13, each ſide of the leſſer baſe 8, and length 24, what's the ſolidity?

By rule 41.

	0,52	multiplier	112,36	□ mean ſide
	5	difference	2.598	tabular number
	2,60	product	391,91128	product
add	8	a ſide leſſer baſe	24	axis
	10,6	mean ſide	7005,86972	ſolidity nearly
	112,36	its □		

Sliding-rule. By rule 41. As 0,62 (ſquare root of 0,3849, ſee ex. 107) on D : 24 on A : : 10,6 (ſee ex. 68) on D : 7005+ on A, the anſwer nearly.

Otherwiſe, as in the laſt ex. If you divide 6 by 2,5984, the tabular number for an hexagon, the quote will be 2,309, whoſe ſquare root is 1,518, a conſtant gauge point for ſuch fruſtums. Then, as 1,518 on D : 24 on A : : 21 and 13 and 8 on D : three ſuch numbers on A as being added together gives 7004,208, (ſee ex. 68) the ſolidity.

Note. Any of theſe 3 examples foregoing, may be wrought by the pen, by any, or all of the 5 foregoing rules laſt laid down.

Ex. 172. If there be a fruſtum of a rectangular pyramid, whoſe length is 15, breadth at the greater end 2, depth there 1,4. breadth at the leſſer end 1, and depth there 0,7, to find its ſolid content?

Note. This ex. is the ſame with ex. 154, where the content is found = 23,625. But here the true content is found = 24,5 ; hence in ex. 154, the breadth and depth (in meaſuring hewn tapering timber the uſual way) ſhould not be taken in the middle, but ſomething nearer the greater baſe, if you would come near the true content.

By rule 42.

1 breadth	2	2,8 area at greater base
,7 depth	1,4	1,4 square root
,7 areas	2,8	,7 area at less base
2,8		4,9 sum
1,96 (1,4 square root		5 = $\frac{1}{3}$ length
1		24,5 solidity
24) ,96		
96		

By rule 43,

2,8 } area { greater base
,7 } area { lesser base
6,3 = 4 times area middle
9,8 sum
2,5 = $\frac{1}{6}$ length
24,50 solidity

Sliding-rule. If you take 4 times the area in the middle and the area at each end in one sum, the set on the rule will be easy. Thus, as 6 on A : 15 on B : : 9,8 on A : 24,5 on B the answer.

Otherwise. Find 3 mean proportionals, one between 1 and 0,7, one between 2 and 14, and one between 2,1 and 3, (the sum of the two breadths 1 and 2, and the two depths 0,7 and 1,4 which answers to twice a side of a square in the middle) then it will be as 1 on D : 2,5 a sixth of the length on A : : these 3 mean proportionals on D : 3 such numbers on A as taken in one sum gives the content.

Ex. 173. If there be a frustum ABEF (fig. 127) of a cone whose length Dy is 24, diameter AB = 13 and EF = 8, at the two bases, to find the solidity.

First, by rule 40.

13 = AB = 13
8 = EF = 8
104 | 5 their difference
add 8,33 | 3)25 its square
112,33 | 8,33 = □ diff.
,7854 mul,
88,223982
24 length
2117,375568 the content

Secondly, by rule 41.

,52
5
2,60
add 8, = EF
10,6 } mean diam.
10,6 }
112,36 its square
mult. ,7854
88,247544
24
2117,941054 content

Sliding-rule, by rule 43. Here 6 ÷ ,7854 = 7,639 whose square root is 2,76, a constant gauge point, conical frustum, so as 2,76 on D : 24 on A : : 8 and 13 and 21 on D : 201,26 and 532,9 and 1383,2 on A whose sum is the answer viz. 2117,36.

Sliding-rule, by rule 41. As 1,128 on D : 24 on A : : 10,6 on D the mean diameter : 2117,9 on A the content nearly.

Ex. 174. Let there be a frustum of a cone whose length is 18, and the periphery at the $\left\{\begin{array}{c}\text{greater}\\ \text{lesser}\end{array}\right\}$ base $\left\{\begin{array}{c}1{,}8\\ {,}92\end{array}\right\}$ what's the solidity?

by rule 43.

1,8	peripheries
,92	
2,72	per. mid. twice
7,3984	its square
3,24	sq. 1,8
,8464	sq. ,92
11,4848	sum
mult. ,07958	
,913960384	
3	$= \frac{1}{6}$ length
2,741881152	solidity

Sliding-rule. by rule 43.

If you divide 6 by ,07958 the quotient is 75,89, whose sq. root is near 8,7, a constant gauge point for conical frustums, when the peripheries of the bases are used; so, as 8,7 on D : 18 on A : : 2.72 and 1,8 and ,92 on D to 3 such numbers on A, as being added together gives 2,74 the answer. This ex. is the same with ex. 158, where the content the false way is but 2,08.

To find the solidity of any prismoid, or of any cylinderoid.

Rule 44. To twice the length of the greater base, add the length of the lesser base, multiply that sum by the breadth of the greater base; also, to twice the length of the lesser base, add the length of the greater base, multiply that sum by the breadth of the lesser base, now the sum of these two products multiplied by 0,7854 and then by a sixth of the ⊥ length gives the solid content. Theorem 139.

Note. Length and breadth of the bases here means these two things which multiplied together and then by any factor, will give the areas of the bases. Hence, if either base be a square, or a rectangle you need not multiply by ,7854.

Ex. 175. If there be a cylinderoid whose length is 20, with two parallel, and elliptical unlike bases, the transverse diameter of the greater base 13, conjugate 8, transverse diameter of the lesser base 10, and its conjugate 5,2, what is its solidity?

13	10
2	2
26	20
10	13
36	33
8	5,2
288 1st prod.	171,6
171,6 2d prod.	
459,6 sum	

,7854	factor
mult. 459,6	sum
360,96984	
20	= long
6)7219,39680	
1203,2328	solidity

Sliding-rule. The sum of the 2 aforesaid products 459,6 being had the set is easy. Thus, as 1,2743 on A : 459,6 on B : : $\frac{1}{6}$ of 20 = 3,33 on A : 1203,2 on B. See ex. 82.

Ex. 176. Let A d C D (fig. 169) be a priſmoid whoſe height c P is 20, the greater baſe A B C D a ſq. each ſide whereof is 13, the leſſer baſe a b c d, a rectangle, whoſe length is 12 and breadth 5, what's the ſolidity?

Note. When the baſes are ſloping as in this fig. the ⊥ length of the ſolid will fall from a corner, or ſide of one baſe upon the plane of the other baſe produced, and if the baſes are not parallel, it muſt be meaſured at twice, by meaſuring a hoof &c. from one of the baſes, to make them parallel, ſo that the figure may be a priſmoid &c.

26 = twice A D
12 = a d
38 ſum
13 = D C
494 firſt product

24 = twice d a
13 = D C
37 ſum
5 = d c
185 ſecond prod,
494 = firſt product
679 ſum

679 ſum product
20 = c P
6)13580
2263 $\frac{1}{3}$ ſolidity

Sliding-rule. As 6 on A : 679 on B : : 20 on A : 2263,3 on B the anſwer. See the laſt.

To find the ſolidity of a hoof, of a cone or pyramid.

Rule 45. For the greater of two elliptic hoofs, from the ſquare of the diameter of the hoofs baſe, take the ſquare root of the product of the two diameters, multiply the remainder, by the leſs diam. of the hoofs baſe and its height, and 0,2618 into one another, the laſt product divided by the difference between the two diameters, gives the ſolid content. But for the leſſer hoof, multiply the ſquare root of the product of the two diameters, by the greater diameter, from that product take the ſquare of the diameter of the hoofs baſe, multiply the remainder, the diameter of the hoof's baſe, and its height, and 0,2618 into one another, the laſt product divided by the difference between the two diameters, gives the ſolidity. Theorem 116.

Ex. 177, and 178. Required the ſolidities of the two elliptical hoofs T S n (fig. 136) and T S m, the heights being S R = 24, diameters T n = 13, and S m = 8.

1ſt, For the greater hoof T S n.
13 = T n
8 = S m
104 product

2d, For the leſſer hoof T S n.
10,198 = ſquare prod. 8 and 13
13 = T n
132,574 product

10,198 ſquare of 104
8 = S m
81,584
ſubtract 169 ſquare of 13
87,416
multiply 13 = T n
1136,408
multiply 24 = S R
27273,792
multiply ,2618
5)7140,1+
1428,02+ ſolidity gr. hoof

132,574 product
64, = ſquare S m
68,574
multiply 8 = S m
548,592
multiply 24 = S R
13166,208
multiply ,2618
5)3446,75+
689,35+

The ſliding-rule requires ſo many mean proportionals (ſee ex. 82.) that it is eaſier by the pen.

Note. The dimenſions here are alſo thoſe in ex. 173, where the fruſtum is found = 2117,37+, and here

the { greater / leſſer } hoof is { 1428,02+ / 689,35+

proof, ſum 2117,37+

Ex. 179, and 180. If T m s n (fig. 136) be the fruſtum of a ſquare pyramid whoſe height S R is 24, a ſide T n of the greater baſe 13, and a ſide s m of the leſſer baſe 8, the ſolidities of the two hoofs T S n and T S m, may be found (by theorem 115) thus, to 26 (twice 13) add 8, multiply the ſum 34 by 13, and the product 442, multiplied by 4 (a 6th of 24) gives 1768 the ſolidity of the greater hoof. Alſo, to 16 (twice 8) add 13, multiply 29 that ſum by 8 and 232 that product multiplied by 4 (a 6th of 24) gives 928 for the ſolidity of the leſſer hoof, by comparing this with ex. 169 you'll find theſe two hoofs to make juſt 2696, the whole fruſtum.

Sliding-rule. 1. As 24 the length on A : 6 on B : : 1 on A : 0,25 on B. (See ex. 106) Then, As 25 on A : 13 on B : : 26 (twice 13) and 8 on A : 13,53 and 4,15 on B whoſe ſum is 17,68. And, as 25 on A : 8 on B : : 16 (twice 8) and 13 on A : 5,12 and 4,16, whoſe ſum is 9,28, each of theſe multiplied by 100, becauſe we uſed the diviſor 25 inſtead of 0,25 (See ex. 68) gives 1768 and 928, for the two hoofs as before. The parabolic and hyperbolic hoofs may be had by theorems 117 and 118, or any hoof nearly by rule 43. But as hoofs are of little uſe in practice except the elliptic ones in finding the drip of a tun &c. I think theſe ſufficient.

To find the ſolidity of any of the five regular bodies.

These bodies being made up of pyramids whose vertexes all meet in the center of the body, it is but finding the solidity of one of their pyramids (they being in each body equal) and multiplying it by the number of pyramids which compose the body, so have you its solidity, and thus are the solidities got in the table. (ex. 131) whose use is

Rule 46. (by theorem 37) Multiply the cube of the side and the tabular solidity belonging the body, the one into the other so you have the solidity.

Ex. 181. If each side of a dodecaedron be 20, what's its solidity?

7,66312 tab. number
8000 cube side
61304,96 solidity

Sliding-rule. 1st, 7,66+ turned to a divisor is 0,1305, whose square root is 0,36; then, as 36 on D : 20 on A :: 20 on D : 6,13 on A, so (by ex. 68) 61300+ is the answer, &c. for any other of these bodies.

To find the solid content of any globe or sphere.

Rule 47. As 1 : 0,5236 :: 21 : 11 (nearly) so is the cube of any globe's axis to the globe's solidity, theorem 93, or multiply the circumference and square of the axis, one by the other, a 6th part of that product is the solidity. Theorem 134.

Ex. 182. If A P the axis of a globe (fig. 170) be 20, what is the solidity?

,5236 constant factor
8000 cube of axis
4188,8 solidity

Sliding-rule. As 1,382 (the square root of $\frac{1}{,5236}$) on D : 20 on A :: 20 on D (see ex. 68) : 4188,8 on A the answer.

To find the solidity of any segment of a globe.

Rule 48. From three times the axis of the whole globe, take twice the segments height, multiply the remainder, the square of the segments height, and 0.5236 into one another, the last product is the solidity, this is when the segments height and axis of the globe are given, but when the segments height and diameter of its base are given; then, to three times the square of the half diameter of the segments base, add the square of its height, multiply that sum, and the said height and 0,5236 into one another, the last product is the solidity. Theo. 93.

Ex. 183. Let K P L (fig. 170) be the segment of a globe, whose height P D is 4, and K L the diameter of its base 16, or A P the globe's axis 20, to find the solidity.

1ſt, When K L and D P are given.	2dly, when A P and P D are given.
8 = K D = L D	20 = axis P A
64 the ſquare of it	3
3	60
192 3 times that ſquare	ſub. 8 = 2 D P
add 16 ſquare of P D	52
208	16 = ſquare D P
4 = P D	832
832	,5236 factor
0.5236 factor	435,6352 ſolidity
435,6352 ſolidity	

Sliding-rule. As 1,382 on D : 4 on A : : 8 and 4 on D : 134,04 and 33,51 on A, ſo 3 times 13404 = 402,12 added to 33,51 gives 435,63+ the anſwer, (ſee ex. 91) by which it will be eaſy to ſet the ſecond operation alſo.

To find the ſolidity of a fruſtum, or middle zone of a globe.

Rule 49. To twice the ſquare of the axis or greateſt diameter, add the ſquare of the leaſt diameter, multiply that ſum, and the length of the zone, and 0,2618 into one another, the laſt product is the ſolidity. Theorem 91.

Ex. 184. Required the ſolidity of the half zone H K K (fig. 170) whoſe length C D is 6, diameters H I 20, and K L 16.

20 = H I
400 its ſquare
2
800 twice ſquare H I
add 256 ſquare K L
1056
,2618 factor
276,4608
6 = C D
1658,7648 ſolidity

Sliding-rule. Firſt, 0,2618 turned to a diviſor, and then the ſquare root taken is 1,954; ſo, as 1,954 on D : 6 on A : : 28,2 ($\sqrt{2 \times 20}$) and 16 on D : two ſuch numbers on A, as being added together gives 1658,76 the anſwer.

Note. The ſame rule ſerves for the fruſtum or middle zone of a ſpheroid.

If the ſolid be leſs than a half zone as the part K L n m. (fig. 170) Then,

Rule 50. To half the ſum of the ſquares of the two diameters, add $\frac{2}{3}$ to the ſquare of the height, that ſum multiplied by the height, and by 0,7854 gives the ſolidity. (by theo. 141) But if it be the like part of a ſpheroid, you muſt multiply the ſaid $\frac{2}{3}$ of the ſquare of the height by the ſquare of the ſpheroids greateſt diameter, and divide that by the ſquare of the ſpheroids axis, and then add the quotient and work as before. Theorem 141.

Ex. 185. Suppose fig. 170 to be a spheroid whose axis A P is 30, greatest diameter H I 20, K L the greatest diameter of the part K L n m to be measured 16, least diameter n m 10, and height D E 4, to find the solidity, of K L n m.

256 square K L	16 square DC
100 square n m	2
2)356	32
178 half sum	mult. 400 square 20
add 4,74	9|00)128|00 900=square 30
182,74	3)14222
4 = DE	4,74 = $\frac{2}{3}$ square 4 multiply square 20, and divide square 30.
730,96 product	

730,96 product
,7854 factor
574,095984 solidity

☞ The pen is easier for this example than the sliding-rule.

Ex. 186. If fig. 170, be a globe, and the solidity of the part K L n m, be required, K L, n m and D E, being as in the last example. Then,

178 $\frac{1}{2}$ sum squares K and nm	754,64 product
add 10,66 + = $\frac{2}{3}$ square D E	:7854 factor
188,66	522,69+ solidity
4 = DE	
754,64 product	

Sliding-rule. As 1,128 (see ex. 164) on D : 4 on A : : 16 and 10 and 4 on D : 3 such numbers on A, that if to $\frac{1}{2}$ the sum of the two first you add $\frac{2}{3}$ the last, that sum will be 592,69 the answer. See ex. 91.

To find the solidity of any spheroid, right or oblate.

Rule 51. Multiply the axis, the square of the greatest diameter, and 0,5236 into one another, the last product is the solidity. Theo 92.

Ex. 187, and 188. If the axis of a right spheroid be 30, and diameter of its greatest circle 20, or the axis of an oblate spheroid 20, and diameter of its greatest circle 30, what's the solidity of each.

1st, For the right spheroid.	2d, For the oblate spheroid.
400 = □ diameter	900 = □ diameter
30 = axis	20 = axis
12000	18000
,5236 factor	,5236 factor
6283,2000 solidity	9424,8000 solidity.

Sliding-rule. (See ex. 182) 1. As 1,382 on D : 30 on A : : 20 on D : 6283,2 on B the right spheroid. 2. As 1,382 on D : 20 on A : : 30 on D : 9424,8 on A the solid content.

To find the ſolidity of any ſpheroidical ſegment.

Rule 52. With the axis of the ſpheroid, and height of the ſegment, find the ſolidity exactly by rule 48, then it will be as the ſquare of the ſaid axis is to the ſegment thus found, ſo is the ſquare of the ſpheroids greateſt diameter to the ſolidity required. Theo. 90.

Ex. 189. Suppoſe K L P (fig. 170) to be the ſegment of a ſpheroid, (either right or oblate) wherein P D = 4, D L = 16, A P = 20, as in ex. 183 and let H I the greateſt diameter of the ſpheroid be 15, what is the ſolidity. 1. By Ex. 183 the ſolidity of the ſegment, if it were the ſegment of a globe, is found = 435,6 ; then as 400 (□ 20) is to 435,6 ſo is 225 (□ 15) to 245, the ſolidity ſought. By the ſliding-rule. As the axis 20 on D : 435,6 on A : : 15 the greateſt diameter on D : 245 on A the anſwer. The middle zone of a ſpheroid is had exactly by rule 49.

Ex. 190. Let the ſame things be given as in ex. 184, then by the ſliding-rule. As 1,954 on D : the height 6 on A : : the two diameters 20 and 16 on D : two ſuch numbers on A ; that if to twice the firſt you add the ſecond, the ſum will be 1658,75 the anſwer, and is a better way then that, by the ſliding-rule to ex. 184, for a part of this zone. See Ex. 185.

To find the ſolidity of a parabolic conoid.

Rule 53. Multiply the area of the baſe by half the axis. Theo 87.

Ex. 191. If E u F (fig. 139) be a parabolic conoid, whoſe axis u P is 60, and E F the diameter of its baſe 40, what's the ſolidity?

40	= E F
1600	= □ E F
30	= $\frac{1}{2}$ U P
48000	
,7854	factor
37699,2	ſolidity

Sliding-rule.

As 1,59 (the ſq. root of one divided by half ,7854) on D : 60 on A : : 4,0 on D (See ex. 68) : 37699 on A, ſo 37699 is the anſwer.

To find the ſolidity of a parabolic conoid's fruſtum.

Rule 54. To the ſquare of the greateſt diameter, add the ſquare of the leaſt, multiply that ſum and the conoid's fruſtum's height, and 0,3927 into one another. Theorem 102.

Ex. 192. If E F T S (fig. 139) be the fruſtum, whoſe height I P is 38,4 diameters E F 40, and S T 24, what's the ſolidity?

1600 = □ E F
add 576 = □ S T
2176
mult. 38.4 = I P
83558,4
,3927 factor
32813,38368 ſolidity

Sliding-rule.
As 1,595 (the ſq. root of ,$\frac{1}{3927}$ on D : 38,4 on A : : (See ex 68) 40 and 24 on D : 24127 and 8686 on A whoſe ſum is 32813 the anſ.

To find the ſolidity of a parabolic ſpindle.

Rule 55. Every parabolic ſpindle being $\frac{8}{15}$ of its circumſcribing cylinder, multiply 0,41888 (viz. $\frac{8}{15}$ of 0,7854) and the axis, and the ſquare of the greateſt diameter, into one another and the laſt product is the ſolid content. Theorem 101.

Ex, 193. If Q V Q v (fig. 140) be a parabolic ſpindle, whoſe length Q Q is 60, and V v the diameter of the greateſt circle 40, what's the ſolidity?

1600 ſquare V v
mult. 60 axis Q Q
96000
,41888 factor
40212,48 ſolidity

Sliding-rule As 1,545 (the ſquare root of 1 ÷ 0,41888) on D : 60 on A : . 4,0 on D (ſee ex. 68.) : 402,1248 on A, ſo 40212,48 is the anſwer.

To find the ſolidity of the fruſtum of a parabolic ſpindle.

Rule 56. To twice the ſquare of the greateſt diameter, add the □ of the leaſt diameter, from that ſum take $\frac{4}{10}$ of the ſquare of their difference, multiply the remainder and the length, and 0,2618 into one another, the laſt product is the ſolidity. Theo. 100.

Ex. 194. If M M m m (fig. 140) be the middle zone of a parabolic ſpindle, whoſe length n n is 40, greateſt diameter V v 32, and leaſt diameter M m = 24, what is its ſolidity?

32 V v
1024 ſquare V v
2
2048 tw. that ſq.
add 576 ſquare M m
2624 ſum

32 V v
24 M m
8 difference
64 ſquare diff.
0,4
25,6 $\frac{4}{10}$ ſq. diff.

2624 ſum
ſub. 25,6 = 0,4 ſq. differ.
2598,4
40 = n n
103936,0
8162, factor inverted
207872
62361
1039
832
27210,4 ſolidity

Sliding-rule. As 1,954 on D : 40 on A : : 32 and 24 and 8 on D : 10723,3 and 6031,9 and 670,2 on A respectively, so twice 10723,3 = 21446,6 added to 6031,9 gives 27478,5 from which take $\frac{4}{10}$ of 670,2 = 268,08, and there leaves 27210,42 the solidity.

Ex. 195. Let it be required to find the content of the last mentioned frustum by rule 43 ; in order to do this we must have a diameter in the middle between V v and M m, which by theo. 65, or by mensuration, is found = 30 because rule 43 is general for all solids, from that rule may be deduced this general one for measuring any solid by the sliding-rule, viz.

1024	= square V v 32
3900	= square twice 30
576	= square M m 24
5200	= sum
,1309	= one sixth of 0,7854
6806,8	
40	= n n
27227,2	solidity
27210,4	the true solidity
16,8	difference

Rule 57. As 2,764 the square root of 6 ÷ ,7854 is to the height or length on A so is a diameter taken at each end and one in the middle, on D, to 3 such numbers on A, that their sum is the answer. Or if for the diameters you take the peripheries, then the gauge point is 8,7 (See ex. 174) to be used with the peripheries as 2,764 is with the diam.

Ex. 196. Let ex. 183 be wrought by rule 43, where a diameter m n taken in the middle between D and P is 9, height D P 4, greatest diameter K L 12, least diameter at P = 0.

144	= square K L 12
324	= sq. twice n m 9
0	= sq. diameter P O
468,0	sum
9031,	inverted factor
4680	
1404	
000	
42	
61,26	
4	= D P
245,04	solidity

Sliding-rule. By rule 57. As 2,764 on D : 4 on A : : 12 and 18 (twice 9) on D : 75,04 and 170 on A whose sum is 245,04, the same as turns out by the particular rule ex. 183, which rule 43, will also give the true solidity of any of the solids in this section except ex. 177, 178, and 194 ; and in these the error would be less than what might arise from guessing at the form of the solid ; for in the last you see the error is but 16,8 in so great a number as 27227,2, I judge it needless to give any rules &c. for the circular and elliptical spindles, the

second segments, slices, &c. which may be easily done from their respective theorems, if curiosity require it, practice will not call for any such difficulties, if what is said of rule 43 be observed.

SECTION VII. *Of Surveying.*

What is here meant by surveying, is the measuring, plotting mapping or protracting, and dividing of ground, and if what goes before be understood this will be easy; for in prob. 129 the chain and other instruments for this purpose are described and applied to practice. Also, land being always computed by superficial measure, you have that done already in sect. 4; where besides the general method rule 25, you have general rules for all the useful forms of figures particularly exemplified, so that let a field be in what form it will, its content may be had by some of these rules. In this section I shall give such examples wrought by the said rules, as are most useful in surveying, and be more tedious in mapping dividing &c.

A Table of Long Measure.

	Link	Foot	Yard	Perch	Chain	Miles
Inches	7,92	12	36	198	792	63360
	Links	1,515	4,56	25	100	8000
		Feet	3	16,5	66	5280
			Yards	5,5	22	1760
				Perches	4	320
					Chains	80

That is, 36 inches, or 4,56 links, or 3 feet, make 1 yard; also, 792 inches = 100 links = 66 feet = 22 yards = 4 perches, each = 1 chain, &c. Chains and links are set down and wrought in chains and decimal parts of a chain. See prob. 129, deff. 21.

Examples in Long Measure.

Ex. 197. In 15 chains 25 links, how many feet?

15,25
,66 feet in one chain
1006,50 feet answer

Ex. 198. In 29 chains how many feet?

29
66
1914 feet, answer

Ex. 199. In 27,30 chains how many poles roods or perches?

4

109,2 roods &c. anſwer.

Ex. 200. In 2580 links how many chains?

100)2580

25,80 = 25 chains 80 links

Ex. 201. In 25,8 chains how many links?

25,8

100

2580 links, anſwer

Note. Links are turned into chains by pricking off 2 decimals.

A TABLE of Square Meaſure.

	Inches	Links	Feet	Yards	Perches	Chains	Acres	Miles
Inches	1							
Links	62,764	1						
Feet	144	2,295	1					
Yards	1296	20,755	9	1				
Perches	39204	625	272.25	30,25	1			
Chains	627246	10000	4356	484	16	1		
Acres	6272460	100000	43560	4840	160	10	1	
Miles	4014480600	6400000	2787400	3097600	102400	6300	940	1

This table reads thus, 1 □ inch = 1 □ inch, 62,764 □ inches = 1 □ link, 144 □ inches 2,295 □ = links = 1 □ foot, 1296 □ inches = 20,775 □ links = 9 □ feet = 1 □ yard, &c.

Examples in ſquare meaſure.

Ex. 202. In 1671925 ſquare links, how many ſquare chains?

10000) 1671925

167,1925 = 167 chains 1925 links anſ.

Ex. 203. In 1232,52 ſquare chains, how many ſquare links?

10000

12325200 ſquare links anſwer

Ex. 204. In 152 ſquare links, how many ſquare chains?

,0152 ſquare chains anſwer.

Ex. 205. In 152 ſquare links, how many ſq. parts of an acre?
,0152 acres, or rather parts of an acre anſwer.

Ex. 206. In 1278642 ſquare links, are 12,78642 acres.

Note. Square links are turned into ſquare chains, by pricking off 4 places towards the right hand for decimals, or ſquare links, and the reſt are ſquare chains; alſo ſquare chains are turned into ſquare links by annexing four cyphers, and acres into ſquare links by annexing 5 cyphers, or ſquare links into acres by the contrary viz. pricking off 5 decimals, and ſquare chains into acres by pricking off one decimal, or removing the decimal point a place nearer to the left hand, &c. ſo,

Ex. 207, In 2560276 ſquare links, or 256,0276 ſquare chains, are 25,60276 acres &c.

Ex. 208. In 20 acres, how many ſquare yards?

	4840	ſquare yards in an acre
	20	
anſwer	96800	yards

Ex. 209. 99220 ſquare yards, how many acres?
4840) 99220 (20,5 acres anſwer

Ex. 210. In 358,75 ſquare chains, how many acres?

	358,75	chains
acres	35,875	
	4	= roods in one acre
roods	3,500	
	40	poles in one rood
roods, poles or perches	20,000	

Ex. 211. In 9,9 ſquare chains, or in 0,99 acres, how many roods and perches?

0,99	acres
4	
3,96	roods
40	
38,40	poles

Ex. 212. In 172 ſtatute acres, how many cuſtomary acres of 6 yards to the pole. As 144 (ſq. 12 the $\frac{1}{2}$ yards in 6 yards) is to 172 ſo is 121 (ſq. 11, the $\frac{1}{2}$ yards in a ſtatute pole) to 144, $,\frac{76}{144}$ acres cuſtomary.

so 76
4 roods in one acre
144) 304 (2 roods
288
16
40 poles in any acre
144) 640 (4 poles
576
64

Answer 144 acres 2 roods 4 $\frac{64}{144}$ poles, at 18 foot or 6 yards to the pole, are = to 172 acres statute at $5\frac{1}{2}$ yards per pole.

Note. Some land measurers, work altogether by acres roods and perches, but it is readiest to work decimally, and to set down chains and links as a mixed decimal number (vide def. 21, problem 129.) as you'll find done in this section.

The two foregoing tables and these examples, respect the statute pole, of $5\frac{1}{2}$ yards in length. But in several parts of England there are poles of different lengths, called customary, some 18, some 21, some 24, &c. yards in length; but in all places 4 roods make an acre, and 40 square poles a rood, so that the difference is in the pole only; now to reduce one of these measures to the other, observe this general

Rule 58. As the square of the feet in a customary acre is to any number of statute acres so is the square feet in a statute pole to the customary acres, as by ex. 212.

In Ireland 7 yards in length is their statute pole or perch, in some places the inhabitants know not the length of their pole, but say that so many gallons of oats will suffice one of their acres for seed. Now in many places it is reckoned that 60 gallons of oats will sow a statute, or plantation acre. Therefore, it will be as 60 gallons it to 121 (the square of 11, the $\frac{1}{2}$ yards in a statute pole) so is the number of gallons that sow a customary acre, to the square of the $\frac{1}{2}$ yards in that customary pole, as by the last rule.

Ex. 213. If 97 gallons of oats be sufficient for sowing some customary acre, how many yards are to the pole of length in that acre?

gall. □ $\frac{1}{2}$ yards gall.
If 60 — 121 — 97?
97
6|0) 1173|7
195,6166 —whose square root is 14, nearly, the half yards required.

Ex. 214. If 120 gallons ſow an acre, how many yards to the pole in length in that acre?

$$\begin{array}{l} \text{If } 60 - 121 - 120? \\ \qquad\qquad 120 \\ \hline 6|0)\,1452|0 \\ \hline \qquad 242\,(\,15{,}5 = \tfrac{1}{2} \text{ yards anſwer} \\ \qquad 1 \\ \hline 25)\,142 \\ \quad 125 \\ \hline 305)\,1700 \\ \quad 1525 \\ \hline \quad 175 \text{ remains} \end{array}$$

Ex. 215. What proportion doth an Engliſh ſtatute acre, bear to an Iriſh ſtatute one? Firſt the half yards in an Engliſh pole are 11 whoſe ſquare is 121, and in an Iriſh pole are 14 half yards, whoſe ſquare is 196; therefore, As 121 : 196 : : an Engliſh to an Iriſh acre, which is nearly as 11 to 17.

Ex. 216. In 52,04 cuſtomary acres at 6 yards to the pole, how many ſtatute acres?

$$\begin{array}{l} \text{Firſt, } \square\, 5{,}5 = 30{,}25 \text{ yards. Second, } \square\, 6 = 36 \text{ yards.} \\ \text{Then, If } 30{,}25 - 36 - 52{,}04 \\ \qquad\qquad 52{,}04 \\ \hline 30{,}25)\,1873{,}64\,(61{,}9 + \text{anſwer. by rule } 58 \\ \hline \qquad 2839 \text{ remainder} \end{array}$$

Theſe are the principle examples in reduction of land meaſure, if any other caſe is wanted, it may alſo be done from the two foregoing tables

Of meaſuring Land.

Ex. 217. If the ſide of a ſquare field, be 7 chains 60 links, what acres roods and perches are contained in that field?

Ex. 218. In a long ſquare piece of ground 10 ch. 75 links long, and 2 chains broad, how many acres roods and perches?

(217)	(218)
7,6 } by rule 7	10,75 } by rule 8
7,6	2
57,76 ſquare chains	21,50 ſquare chains
5,776 acres	2,15 acres
4	4
3,104 roods	,60 roods
40	40
4,160 perches	24,00 perches
anſ. 5 acres 3 roods 4,16 perches	anſ. 2 acres 24 perches

Note. When chains and links are multiplied together, if you cut 4 figures off towards the right hand, they will be ſquare links or decimal parts of a chain, but 5 figures pricked off are acres and decimal parts of an acre.

Ex. 219. If a field in form of a rhomboides, length 2 chains 1 link, perpendicular breadth 90 links, how many acres, roods, and perches?

Ex. 220. In a triangular field whoſe baſe is 7 ch. 10 links, perpendicular 2 ch. 30 links, how many acres, roods and perches?

(219)	(220)
2,1 } by rule 9	3,55 = half 7,1 } by rule 10
,9	2,3
,189 acres	,8165 acres
4	4
,756 roods	3,266 roods
40	40
30,240	10,64 perches
Anſwer, 0 A. 0 R. 30,24 P.	Anſwer, 0 A. 3 R. 10,64 P.

Ex. 221. In a trapezia, whoſe diagonal is 28 ch. 20 links, perpendiculars 10 ch. 50 links, and 8 ch. how many acres, roods, &c.?

10,5 by rule 12.
8

18,5 ſum of the perpendiculars
14,1 = half the diagonal

26,085 acres
4

,340 roods
40

13,600 perches

Anſwer, 26 A. 0 R. 13,6 P.

Ex. 222. In a trapezia with 2 parallel ſides 3202 links and 4008 l. and the perpendicular diſtance between them 2500 links, and the other two ſides 3400 links and 2800 links, how many acres, &c.?

First way.

40,08 30,02	two ſides
2)72,10	ſum, (by theo. 26)
36,05	half ſum
25	perpendicular
88,125	true area

Second way.

28 34	other two ſides
2)62	ſum
31	half ſum
mul. 36,05	half ſum of the ſides
111,755	falſe area
ſubtract 88,125	true area
23,620	error

Some Surveyors, do no more with 4 ſided fields but multiply by half the ſum of two oppoſite ſides, by half the ſum of the other two oppoſite ſides, for the area, which is a groſs error, and always gives the content too much, as appears by the laſt example wrought both ways, and ſhews that the error in 88 acres is 23 acres, the true way is to take a diagonal and two ⊥s, or if any two of the 4 ſides be parallel, it is equally true, (by theo. 26) if you multiply half the ſum of theſe two parallel ſides by their ⊥ diſtance aſunder, as per laſt ex.

Ex. 223. If the diameter of a circle be 20,02 yards, required its area in acres roods and perches. Here, you may find the content in ſquare yards, and then reduce it to acres &c. or you may turn the diameter from yards to chains and links, thus. As 22 yards is to 1 chain or 100 links ſo is any number of yards to the chains and links anſwering, then find the area by rule 15. See both ways.

First.

20,02 20,02	yards diameter
400,8004	its ſquare
,7854	factor
314,78863416	area in yards

4840)314,79(0	acres
4	
4840)1259,16(0	roods
40	
4840)50366,40(10,406	perches

Here, becauſe 314,79 is nearly = 314,78863416 the area in yards, I uſe 314,79 inſtead thereof.

. K

Secondly. If 22 — 100 — 20,02

$$\begin{array}{rl} 100 & \\ \hline 22)2002 & \\ \hline 91 & \text{diameter in links} \\ \hline 8281 & \text{its square} \\ ,7854 & \text{factor} \\ \hline 6503,8974 & \text{area in links} \\ \hline ,65038974 & \text{area in chains} \\ ,065038974 & \text{area in acres} \\ 4 & \\ \hline ,260155896 & \text{roods} \\ 40 & \\ \hline 10,406235840 & \text{perches} \end{array}$$

Ex. 224. To meaſure hills, vallies, &c. (fig: 104) meaſure round the bottom z y of the hill, which ſuppoſe 15,88 chains, and round its ſummit or top T D 10,12 chains; then take the length of the longeſt ſide z D 14,1 chains, and alſo the length y F 10 chains of the ſhorteſt ſide, half the ſum of theſe two ſides is = 12,15 chains for the mean length; alſo half the ſum of the two peripheries 15,88 and 10,12 is 13 chains for the mean periphery; then 12,15 × 13 = 157,95 ſquare chains = 15,795 acres, the ſuperficial content of the hill, which is true if the peripheries were taken parallel to each other, and the hill not too rugged; but if it be very uneven, you ſhould take more peripheries and work by rule 24 or 25. This way will do for ſmall hills, but for large ones it is better to meaſure them as planes, as taught in the following examples.

Note. What is ſaid of hills alſo holds true in vallies, they being but as hills ſurveyed in the inſide, but in juſtice the content of a hill or valley, ſhould be no more than the plane on which it ſtands, for ſince graſs &c. always grows ⊥ to the horizon, its evident that if you ſuppoſe a hill to be raiſed upon any plane, that the ſame quantity of graſs on the plane, will by ſprouting thro' the ſides of the hill be no ways increaſed, becauſe growing upright, it will be a ſlope to the ſides of the hill.

Ex. 225. Let ABCD (fig. 171) be ſome piece of bending ground whoſe content is required. Firſt, meaſure along its middle E F, taking breadths at right angles thereunto as you go on, and ſuppoſe the dimenſions to be as ſet down in the figure: (ſee deff. 30, prob. 129.)

number of breadths	breadths
1	1,2
2	2,2
3	3,08
4	4,02

divide by num. breadths = 4) 10,50 ſum

See rule 24 2,625 mid. breadth
multiply 16,71 = E F

area in acres = 4,386375

Note. In ſuch figures as this, where the ends ſlope, &c. ſuch ends muſt not be taken for breadths, becauſe they are not ſquare to the lengths F F, in ſuch caſes the breadths at ends muſt be taken within the ends as you ſee from B and D, for the length E F, if it be taken in the middle, it gives the true mean length, however the ends ſlope, provided they be ſtreight. This method in ſuch forms holds pretty true, and is much eaſier than to divide ſuch figures in trapezia's, Δs, &c. See ex. 119.

These examples may ſuffice on this head, the next is to take the plans &c. of fields.

To take the plan of any field by the plane table.

Ex. 226. Let it be required to take the true form of the field A B D E F G, (fig. 172) upon paper.

1. Cauſe marks or ſtaffs to be ſet up at every ∠ or corner of the field, as at A, B, D, E, F, G, and alſo if there be any place H, in the field that you would have in your map, cauſe a ſtick to be ſet up there at H alſo.

2. Chuſe any point C, in the field, from whence you can eſpy all the marks ſo ſet up, there place your inſtrument parallel to the horizon, with a ſtreight ruler, or index moveable upon its center, under which index, upon the table muſt be faſtened a ſheet of white paper, then thro' the ſights upon the index, or along its ſtreight edge, eſpy every mark one by one, and meaſure the diſtance between the center at C, of your table and each mark, and from a diagonal ſcale from C, along the edge of the ruler pointing to the ſaid mark, lay this meaſured diſtance, this do for every corner in the field, and where the diſtances end on your paper as at A, B, D, E, F, G, join with lines, ſo will you have a true plan of the field before ever you go out of it, this method of taking a plan by the plain table (if the weather is dry ſo as not to wet the paper) is as eaſy and expeditious as any, for in a manner, you have the map on paper, and the dimenſions all at once; your own judgement will direct you, that ſuch places as H, whether brook, tree,

house, &c. is not to be joined in with the marks which compose the out, or ring hedge A B D E F G, and if this hedge be much bended you must take so many more marks or take offsets between the principal marks as in ex 236.

To take the plan of a field by the semi-circle, or theodilite, at one station.

Ex. 227. Required the true map of the field A B D E F, fig. 173.

1. Having set up marks at every angle in the field, and at all the objects you would have in your map, at any point C, where you can see all these marks, place your instrument flat, and screw it fast so that along the diameter where the degrees begin you may espy any one of the marks, suppose that at A, then turn the index espying every of the other marks thro' its sights observing what angles are made, or degrees cut, by looking at these marks (the same as in prob. 137) which let be as set down in the figure.

2. Next measure the distance between C, the center of the instrument and each mark, which let be as set down along the lines, C A, C B, &c,

To lay down this plan.

1. With a chord of 60° sweep a circ le, on whose periphery lay the chords of 74°, 90°, 62°, &c. as per figure, and by these chords draw the lines C A, C B, C D, &c. on which from a diagonal scale lay 4 C, 3,8 C, 3,3 C, &c. and where these ends as at A, B, D, &c. draw the black lines A B, B D, D E, &c. so have you a true plan of your observations.

Ex. 228. If you measure but one distance as C A, and measure all the hedges A B, B D, D E, &c. it will be the same for to lay down the plan by these dimensions; first lay down all the angles at the center C, as before, and the line C A, then from the same diagonal scale take the line A B, and with one foot in A cross the line C B in B, join A B, also with the line B D in your compasses and one foot in B, cross the line C D, join B D, and thus go on till all the hedges be laid down.

To take the plan of a field at one station by the chain only.

Ex. 229. This is the same as before, only you take all the ∠s at C, with the chain instead of the theod. &c. thus, set down a staff at O, over which put the ring at the end of the chain, where stand, and let your assistant go with the other end towards A, till you see him and the mark A, in the same right line, there stick down a stick which is at b (fig. 173) then direct him towards B, and there set down another stick which is

at a, then meaſure the diſtance a b, which let be 1,2 C, ſo is the angle A C B meaſured by the chain, and thus you may meaſure all the angles, at the ſtation C.

To lay down this plan.

1. From the diagonal ſcale take one chain, with which in your compaſſes and one foot in C, ſweep a circle on whoſe periphery from the ſame diagonal ſcale lay down all the ∠s as you ſee them in your rough draught (def. 30. prob. 129) and thro' theſe points draw the line CA, CB, CD, &c. (meaſured in the field as per ex. 227) at pleaſure, on which from a diagonal ſcale lay their lengths &c. and join A B, B D, D E, &c. ſo have you a true map of your obſervations, or you may meaſure out one line between your ſtation C, and any of the marks A, B, D, &c. and meaſure all the hedges, as directed in the laſt ex.

To take the plot of a field at two ſtations, by only meaſuring the diſtance between the two ſtations, and obſerving the ∠s.

Ex. 230. Let A B C D E (fig, 174) be a field to be mapped.

1. Chuſe two convenient points F and S for your two ſtations, then work as in problem 139 which is the very ſame with this example, only here for variety, I take the two ſtations within the field, which ſhould be pretty far aſunder, that ſo the lines of obſervation F A, F B, &c. and S A, S A, &c. may not meet in very ſmall angles A, B, &c. for if they do, it is not ſo plain to ſee where the angular points are.

2. Place your inſtrument as at F, letting the diameter point to S your ſecond ſtation, there having it flat, ſcrew it faſt, and thro' the ſights upon the index eſpy all the marks or corners of the field E, A, B, C, D, marking down the angles of obſervations which let be S F E 10° E F A 84°, A F B 78°, B F C 140°, C F D 46°; then meaſure the ſtationary diſtance F S, which ſuppoſe 80,2 C, there at S place your inſtrument and make obſervations to F, and every mark as before, and ſuppoſe the ∠s A S F 42°, A S B 44°, B S C 80°, C S D 84°, D S E 54°, and E S A 56°.

To lay down this field on paper.

1. On a large ſheet of paper draw a line S F, on which from the diagonal ſcale, lay the ſtationary diſtance 80,2, then on F and S, with the chord o 60° ſweep two circles, make ∠ S F E = 10° drawing F E at pleaſure, on S make the ∠ (E S A + A S F) F S E = 96°, drawing S E to meet F E in E, ſo is E one of the marks or corners in the field, again make ∠ E F A = 84°, and ∠ E S A = 56°, meeting in A, ſo is A, another mark &c. and thus go on till you have all the corners or marks, which join with lines E A, A B, &c. and its done. This ex.

requires that when you are at either ſtation, you muſt ſee the other ſtation and all the marks, but when that cannot be done, you may ſet lines from your ſtation to the unſeen marks &c. leaving ſtaffs ſet up in thoſe lines to go by. Otherwiſe, make obſervations to all the marks in ſight, and to new ones, towards thoſe unſeen, which is all a one as to take ſeveral fields adjoining to one another, &c. as is plain to underſtand.

To take the plot of a wood, moraſs, &c. by going about the outſide of it.

Ex. 231. Let A B C D E F G, be ſuch a wood &c. (fig. 275)

1. Set up marks at every corner of the wood &c. and at every object you would have in your map.

2. Place your inſtrument (viz. any thing that wall take, or meaſure an angle) at any of theſe marks, ſuppoſe at A, there obſerve the two neareſt marks B and G, and ſuppoſe the index between looking at B and G, moves over 100°, which is the meaſure of the ∠ B A G; then remove your inſtrument to another mark B, meaſuring the diſtance, or hedge A B 21 ch. 8 links, there look out for the next two neareſt marks A and C, and mind what degrees are included, ſuppoſe 76° = ∠ A B C, remove to C, meaſuring the hedge B C 15 chains 20 links, there as before, take the ∠ B C D &c. by going thus round, take all the ∠s and meaſure all the hedges. Alſo, if there be any place as H, out of the hedges that you would have on your plan, make two obſervations, to it as one from A and another from B. See prob. 137.

To lay down theſe obſervations.

1. Draw a line A B making it by the diagonal ſcale = 21,08 chains, upon A make an ∠ of 100°, and upon B one of 76°, make A G by the diagonal ſcale = 25,06 chains, and B C = 15,2 chains, and thus lay down all the ſides and ∠s as you find them (def. 30. prob. 129) in your paper, ſo you'll have a true map.

Ex. 232. Note. When the two laſt lines in a map are ſet off, being produced they will meet without laying down their lengths, or the ∠ which their meeting makes, ſo if this ∠ and theſe two lines be meaſured and compared with the like ∠ and two hedges taken in the field, it will prove, if the plan is truly made.

Ex. 233. To know if the angles in the field are truly taken.

Take 2 from the number of the inward angles, and multiply the remainder by 180, this product, if the work is right, will be equal to the ſum of all the angles in the figure, if there be outward angles, you add their complements, ſo as in the laſt figure, the number of ∠s (exclud-

ing $\angle$ C) are 6, then $180 \times 4 (6 - 2)$ is $= 720 = 695 + 25$, the ſum 695 of all the inward $\angle$s added to 25, the comp. of the outward $\angle$ C, to 180°. This is eaſily proved, ſince the ſum of the three $\angle$s of any plain $\triangle$ is known to be 180°, as alſo that the ſum of the external $\angle$s of any right lined figure is $= 360°$.

Ex. 234. The laſt ex. may be done by the chain only. Thus (in fig. 176) ſet up marks at every corner A B C D, &c. in the field, then begin at any of theſe marks as at A, and meaſure the hedge A B 50 ch. 8 links, then hold you one end of the chain at B, and let your aſſiſtant go with the other end untill he ſee himſelf in a line with the marks B and A, there ſtick down a ſtick which is at c, then let him move the end of the chain, until you ſee him and the mark C in one line, there ſet down another ſtick at d, and meaſure the diſtance d c 1,21 chain, again meaſure the hedge B C 75,29 chains and holding one end of the chain at C, or putting the great ring over a ſtick there, move the other end to e, in a line with B C, and then to f, in a line with C D, then meaſu re the ſtreight diſtance e f, 1,7 ch. next meaſure the hedge C D 70,5 chains, and upon D take the $\angle$ g D h 1,4 ch. as before, and thus may all the ſides and $\angle$s be taken by the chain only.

To plan this figure.

1. Draw a line A B at pleaſure, laying 50,08 chains from A to B, then with the radius one chain on B, ſweep the arch c d, on which lay 1,21 chain from c to d, thro' d draw B C at pleaſure, and lay 75,29 ch. from B to C, upon C with the radius 1 chain, ſweep the arch e f, lay 1,7 ch. from e to f, thro' f draw C D &c. as is plain by the figure, all theſe numbers are taken from the diagonal ſcale. where,

Note. That one ſcale muſt be uſed to all the ſides, but you may uſe a larger ſcale to the $\angle$s if you pleaſe, for great care and exactneſs ſhould be uſed in the $\angle$s both in taking, and laying them down, for a ſmall matter in an $\angle$ where the hedges are long, will cauſe a great error. Here as in ex. 232, the laſt line laid down D A, will meet B A in A and be $= 62,11$ ch. if the work be right. Alſo, in the next ex. are an $\angle$ and two ſides, to prove the plan.

Ex. 235. To take the plot of a field A B C D E (fig. 175) by going the inſide of it, with the chain only.

This is performed nearly as in the laſt ex. by meaſuring the hedget and at every corner, or ſet up mark, for inſtance, at A put the ring as the chains end over a ſtick, and ſtretch out the other end to a, in a line with A E, and alſo to b, in a line with A B meaſure the diſtance ſtreight from a to b, 1 ch. 29 links, and thus do at every corner A, B, C, D, &c. as you meaſure the lengths of the hedges, and ſuppoſe the hedges and angles to be as ſet down in the figure.

To lay down this figure on paper.

This is ſo eaſy, from the numbers on the figure, and what is ſaid in the laſt ex. that I think it needs no other directions.

Ex. 236. To take the plan of a field A B C D E (fig. 177) by diagonal lines.

To do this you muſt meaſure ſo many diagonal lines as will divide the field into △s, and then it will be eaſily laid down (by prob. 14, or 16) in all the foregoing examples, the hedges are ſtreight, but in this ex. for variety, I have taken them bended, and what is here done in crooked hedges is to be obſerved in any other method of plotting &c.

Having taken a view, and a rough draught of the field, meaſure in a line from A to B taking offsets as you go on, and ſet them (ſee def. 30 prob. 129) at the end or along the offset line which muſt always be ⊥ to the ranging line A B. Alſo, along the ſaid ſtreight line A B, ſet pown the diſtance between every offset, and always take care to offſet to every turn, or bend in the hedge, thus go round the field, or rather the trapezia which you are about to take, and the greater trapezias you take the better, for the fewer the parcels the truer the content in this reſpect, alſo the ſhorter the offsets the better; becauſe without much labour it is difficult to get them ſquare to the ranging line, &c. when you have thus gone round the trapezia, take a diagonal A C, ſo will you be able (by prob. 16) to lay down the trapezia A B C D, for by your aforeſaid draught the ſide A B is $= 3 + 1 + 1{,}5 + 2 = 7{,}5$, the ſide B C $= 2 + 5 + 8 = 15$, the ſide C D $= 3 + 2 + 2 + 2 + 1 = 10$, and the ſide D A $= 2 + 2 + 3 + {,}5 + 1 = 8{,}5$ the diagonal, A C $= 12{,}94$, you may alſo meaſure a diagonal from B to D in the field, which applied to your plan will be proof thereunto, for the offsets, they are ſet off as directed in prob. 36, from the ſame diagonal ſcale, that you take the other lines A B, B C, A C, &c. from. Thus, ſet 3 chains from A towards B upon A B, there raiſe a ⊥, making it = 1 chain, at one chain further raiſe a ⊥, and make it = 1,3 ch. and ſo on. Then over the tops of theſe ⊥s draw the curve, or bended line, which will form the thing required.

Ex, 237. Any four ſided figure (right lined) may be readily laid down by having its four ſides and one ∠, (ſuppoſe ∠ A) given; thus make the ∠ A (fig. 177) = what you took it in the field, drawing A B and A D till they be = what you took them, then upon B, with B C in your compaſſes ſtrike an arch, and upon D with D C, croſs it in C, join B C and D C, and its done, on which you may ſet the offsets, if any, as before.

Ex. 238. There are ſeveral other methods for taking the plot of a field, &c. but they all end in the taking of ſides and ∠s &c. thoſe here laid down are the moſt eaſy, plain and uſeful, and if underſtood, it will be eaſy not only to underſtand, but even invent other methods; that in ex. 236, as is true as any to depend on where it can be practiſed, becauſe it meaſures diagonal lines, which cannot eaſily fail in taking the field truly, if it be uneven, becauſe in ſome degree they muſt go over theſe uneven places, &c. If you have a pocket compaſs you may ſet it to any line you are meaſuring, and ſo have the compaſs or bearing of the places in your map. Alſo by the help of a compaſs, you may take the ∠s in a field, by ſetting it at one mark, and obſerving the bearing of the next two neareſt marks as is eaſy to underſtand, for if one bear north, and the other weſt, then its plain the ∠ at the mark where you ſtand is 90°. &c. &c.

Ex. 239. How to take a perpendicular in the field. (fig. 14)

This may be done with any inſtrument that will take an ∠, thus, as you are meaſuring along any diagonal A G, try now and then with your inſtrument, by looking at the marks A and I, till you find the ∠ A a I = 90°, ſo will a I be the ⊥ which meaſure, this may alſo be done eaſily by a maſon's ſquare; thus, as you meaſure along A G, try now and then with the ſquare, till along one ſide of it you ſee the marks A or G, and the ſame time along the other ſide of it, ſee the mark I, and that is the point where the ⊥ will riſe, which in this figure the ∠ of the ſquare will be at a, there ſtop and meaſure the ⊥ a I, then come back to a, and as you meaſure along a G, take in like manner the ⊥ e H, now having ſet down in your rough draught the diſtances A a and a e or e G, and the ⊥s a I and e H, you may from them lay down the figure. Thus, draw a line A G, on which lay A a on a, raiſe the ⊥ a I, on it lay a I, lay a e from a to e, on e raiſe the ⊥ e H on it lay e H, then join the points A, I, G, H, which will be a true plan of the right lined trapezia A I G H. If there be more trapezia's than one you may take, and lay them down (by one at a time joining to one another as they lie in the field, taking offsets if the out-hedges are bended, as in ex. 236) either by this method, if ⊥s can be eaſily had, or by ex. 237.

Ex. 240. How to meaſure ſtreight.

Either let him that goes with the fore end of the chain, ſtick down his ſtick, in a line with him that goes behind, and the mark behind the ſaid hindmoſt man. Otherwiſe, he that goes behind may direct him that goes before, by ſome mark in the line, called a foremark, the other, being called a back mark.

Ex. 241. To ſet out a line, when both ends cannot be ſeen at once.

Set up a long pole as at A, (fig. 14) and one at G, and let there be ſo many men between A and G, as may ſee to direct one another into the ſame right line A G, then where they thus ſtand, ſet up marks within ſight of one another, you may run a line by back marks at a venture, and by it find the true line you want. Thus, ſuppoſe you want a line from A to C, and by reaſon of ſome obſtacle you cannot ſee between A and G. (fig. 14) Firſt, meaſure as near the line as you can gueſs, ſuppoſe from A to I, where you can ſee the mark G, there take the ∠ A I G, which if it be a right ∠, the ſquare root of the ſum of the ſquares of A I and I G, will be = A G, or if it be any other ∠, you may by trigonometry find the ſide A G, having firſt meaſured I G, for then you'll have two ſides and their contained ∠, when a field is to be divided into ſhares, then its content ſhould be had very near, then it is beſt to uſe ſuch methods for plotting, as will take diagonal lines &c. in or through the field. (See ex. 238) But to plot a townſhip, county &c. it will be ſooneſt done if you go round it taking the ∠s &c. as before directed, and as you go on meaſure the ſides with a parambulator, or ſurveying wheel, which is done by driving a wheel before you, made for that purpoſe, with which you may ſurvey as faſt as you can walk, for the pointers on this wheel ſhew what diſtance at any time it has moved.

Ex. 242. To find the content of any plot when laid down.

If a field is ſurveyed by ex. 236. its content may be caſt up by rule 11, or if by ex. 239, its area may be had by rule 10, ſo that in theſe two caſes, if you want only the area the field need not be laid down on paper, in all caſes, the contents of the offſets are had by rule 24. Let it be required to find how many acres is contained in fig. 177.

Offſets between A and B,	1,5	=	$\frac{1}{2}$ of 3 × 1.
	1,15		1 + 1, 3 × $\frac{1}{2}$ of 1.
	1,575		1,3 + ,8 × $\frac{1}{2}$ of 1,5.
	,84		,8 + ,04 × $\frac{1}{2}$ of 2.
Offſets from B to C,	2,00		2 × $\frac{1}{2}$ of 2.
	8,25		3,5 × $\frac{1}{2}$ of 5.
	6,00		1,5 × $\frac{1}{2}$ of 8.
Offſets from C to D,	1,50		3 × $\frac{1}{2}$ of 2.
	3,00		1 + 2 × $\frac{1}{2}$ of 2.
	2,50		1 + 1, 5 × $\frac{1}{2}$ of 2.
	2,00		1,5 + ,5 × $\frac{1}{2}$ of 2.
	,25		,5 × $\frac{1}{2}$ of 1.
	39,25	=	△ A B C, found by rule 11.
	45,29		△ E D A,
	114,105		Total
Subtract	10,5		Area of D E A.
	10,3605		Acres, the anſwer

$$\text{Offsets from D to A,}\left\{\begin{array}{r}1,5\\3,5\\4,5\\,5\\,5\end{array}\right\}=\left\{\begin{array}{l}1,5\times\tfrac{1}{2}\text{ of }2.\\1,5+2\times\tfrac{1}{2}\text{ of }2.\\2+1\times\tfrac{1}{2}\text{ of }3.\\1+1\times\tfrac{1}{2}\text{ of },5.\\1\times\tfrac{1}{2}\text{ of }1.\end{array}\right.$$

Sum 10,5 = the area of D E A.

Theſe offset areas are all found by rule 24; and as the content of any plot is thus found, it ſeems unneceſſary to give any more examples. Becauſe by the figure, the hedge D E A bends inward from the ranging line D A, it is plain the content of the part D E A muſt be taken from that of the field. In any of the other methods, when the field is laid down on paper, be careful to meaſure diagonals and perpendiculars on the ſame diagonal ſcale, by which you'll eaſily find the content.

Ex. 243. How to divide a plot when laid down.

When any plot is truly laid down by a good diagonal ſcale upon a large ſheet of paper, i. e. your paper ſhould be ſo large as that every line may in your map, have its true length in chains and links ($\frac{1}{2}$ or $\frac{1}{4}$ links be ſeldom or never regarded) ſee prob. 23. This done, you are taught in the firſt part of this book, to divide or cut away any plot by art every way poſſible. But as there is yet another eaſy, general and practical method, I ſhall here give an example thereof. It is thus, when the plot is laid down, and its content found, (in doing this you need have no lines on your plot but the black ones or out hedges, for the pricked lines in the foregoing plots, being only for illuſtration, may in your plot be drawn dry, and not pricked, for needleſs lines ſerve only to blind a figure or plot) and you know what way the dividing hedges muſt range, and the quantity of ground every new incloſure is to contain, then meaſure off ſuch parts in your plan, and ſee what diſtance is thereon between every new hedge, which mark down, then go into the field and meaſure out the ſame diſtances upon the ſame lines, as on your plan, there cut out little holes or ſtrike in marks in the ground, and meaſure every parcel thus laid down in the field, and if the areas of theſe new incloſures do not ſuit thoſe in your plan, or are not the areas they ſhould be, you muſt vary the lines, till they do agree.

Note. Since moſt ground is uneven, it meaſures to more in parcels, than it does altogether.

Let A B C D, be a field of 120,55 acres, to be divided between two men, z to have 20,55 acres, and y to have the reſt, (viz.) 100 acres, to be hedged off parallel to the hedge D C. Fig. 178.

1. Draw a line d e parallel to D C, as near good as you can guess, then measure the trapezia d e C D, which suppose 90,21 acres, this is too little, by 9,79 acres, so you must move the line d e nearer to A B, until you find it cut you off a content of 100 acres, which suppose it does when at E F, then measure the distance D F and C E by your diagonal scale, then go and measure out the same distances with your chain in the field, from D towards A, and from C towards B, and measure the trapezia D C E F, and also the figure E F A B, and if the are = 100 and 20,55 acres, respectively, its done. Otherwise, you must remove the line E F in the field till you find by measuring that it answers. To find nearly, how far the line d e must shift, suppose it = 10 chains, because it cuts off an area too little by 9,79 acres or 97,9 square chains, and content divided by length gives breadth, therefore 97,9 divided by 10, gives 9 chains 79 links, and so much may you remove d e towards A B, on your plan, and then measure and see what the content is, which if it differ a little, you may divide and try again &c.

Ex. 244. Note. If you are to cut off a share next to any bended hedge A z B (fig. 178) it is best to add the offsets in A z B to the share and then lay it off as before from the streight line A B, but if the hedge bend out-wards, the offsets must be taken from the said share as is plain by the figure. Also if the hedges which the dividing line cuts, be bended, it is best to work with those cut offsets by themselves, and then add or subtract from the right line part, as the fig. directs.

Ex. 245. Let A B C D E, (fig. 179) be a field containing 27 acres, to be divided equally amongst 3 men, and all the hedges to meet at a pond, or watering place g in the field.

1. Since no point in the out hedges is given to begin the new hedges from, draw a line A g, from the angle A to the well at g, which let be one of the new hedges. Also, join C g, and measure the trapezia A g C B 6,65 acres, now if this 6,65 had been 9 acres viz. $\frac{1}{3}$ of 27 acres the fields area, then C g would have been another dividing hedge, but 6,65 wants 2,35 of 9, so if you add a △ of 2,35 to A g C B 6,65 you'll have the second dividing hedge, therefore, take the nearest distance 8 chains 62 links from g to the streight line C D, then because the area of a △ divided by half its base gives the ⊥ &c. therefore, 2,35 divided by 4,31 (half of 8,62) gives 5 ch, 54 links, which set from C to H, and join g H the second dividing hedges, in like manner by joining g D, you'll find the area of the △ g D H will want the area of the △ g D I, to make it 9 acres, so I g will be the third and last dividing hedge, now measure the distances E I and C H on your

plan by the diagonal ſcale, that you may know what chains and links, to meaſure from the like corners in the field, where make marks at I and H, and meaſure the 3 parts A g H C B, H D I g, and I E A g, which if the plan and work be true will be very near good, but if there be a ſmall difference you may rectify it, by regulating the lines, as before. In this manner you may divide land at pleaſure. where

Note. That in uneven ground, you muſt take dimenſions, ſo, if you can, as that they may go over theſe uneven places, ſo will your plan be the truer, and conſequently leſs varying the diviſional hedges in the field.

Rectangular, and triangular pieces of ground, are eaſily divided by rule 35, and prob. 111, &c. as in the two following examples.

Ex. 246. If a rectangle be 20 ch. long, how many chains in breadth will make 5 acres, or 50 ſq. chains, alſo how many yards in breadth will make 2 acres.

chains	yards
20) 5|0 given area	4840 yards in one acre
anſwer 2,5 chains	2
	44|0) 968|0 given area
	anſwer 22 yards.

Becauſe 22 yards is 1 chain, 22 × 20 = 440, the yards in 20 chains.

Ex. 247. Let there be a triangular piece of ground A B C (fig. 89) whoſe area is 300 chains, and baſe B C 40 chains to be divided into 3 = ſhares by lines drawn from the vertical A, to the baſe B C.

1. By rule 10, the ⊥ d A muſt be 15 chains, becauſe 40 × 15 and ÷ 2 = 300 the given area, a third of which is 100, this ÷ 15 the ⊥ gives 6 chains 66 links, for the baſe of 100 chains the third ſhare, or by prob. 111, you need only divide the baſe B C into 3 = parts, and draw lines between each of theſe parts and the ∠ A, and its done.

Ex. 248. If a field be 80 graſſes, and its content 192,57 acres, how many acres are there to a graſs.

As 80 graſſes is to 192,57 acres ſo is 1 graſs to 2,407 + acres anſ. ſo that if any number of graſſes be multiplied by 2,407, the product will ſhew how many acres are in that number of graſſes. As for

Ex. 249. How many acres muſt be to 5 and 1½ and 6¾ graſſes reſpectively.

2,407	2,407	2,407
5	1,5	6,75
acres 12,635 = 5 G.	ac. 3,6105 = 1½	ac. 16,24725 = 6¾ graſſes

Ex. 250. If one half of the laſt mentioned field, muſt abate 1 rood per acre, and the other half thereof have added 1 rood per acre, how much to a graſs in each.

Becauſe a rood is the fourth of an acre, and every acre is to abate a rood, therefore, every whole is to abate $\frac{1}{4}$ of itſelf, ſo the $\frac{1}{4}$ of 2,407 is 0,60175, which taken from, and added to 2,407 gives 1,80525 per graſs for the abating part, and 3,00875 per graſs for the increaſed part.

Note. In caſes of abating &c. you muſt take care that the ſum of the abatements be = the ſum to be added, for, if you abate more than you add, you'll have land to ſpare, and the contrary if you abate leſs then is to be added, as you may eaſily perceive.

SECTION VIII. *The practice of Gauging.*

Ex. 251. Gauging, is to know what quantity of liquor any veſſel will hold, or how much liquor is in it when it is quite full or partly ſo. All liquid meaſures are computed by ſolid inches, whence gauging is no more but ſolid meaſure and therefore may be all done by ſection 7 only, there the dimenſions are taken to the outſide of the ſolid, but here they muſt be taken within, to get the ſolidity of the cavity, or part holding the liquor, as is evident.

Ex. 252. Gauging (for eaſe ſake) is almoſt all performed by the ſliding-rule, and may be done by Coggleſhal's ſliding-rule as by ſect. 7. But ſince there is one on purpoſe for gauging, it may not be amiſs to deſcribe it here.

Ex. 253. This rule is in form of a parallelopipedon, three faces of which have each a ſlider, tho' there are other ſorts, but this is what I uſe here; on one face, ſtands A, at the end of the rule, and B at the end of the ſlide, theſe are the lines mentioned in ex. 69, here is alſo a line marked M D which is the ſame with the lines A and B, but reads contrary way, and begins at 21,5042, and ending at 2,15042; now if from the logarithm of 10, or 100 you take the log. of 2,15042 or of 21,5042 the remainder ſhews the diſtance of unity or 1 from the end of the line M, D, and is exactly the ſame with the diſtance of the points M B, (on the line A) from the end of the line A, the line mark N, on the ſlider is the ſame with the line B, above it; on this face are ſeveral other marks as A at 282, the ſolid inches in a gallon of ale, W at 231, theſe in a gallon of wine, C at 3,14 the periphery of a circle whoſe diameter is unity, S I at 0,707 the ſide of a ſquare inſcribed in that circle, diameter unity s e at 0,886 the ſide of a ſquare = area of a circle, diameter unity, At each of 0,785 area of a circle diameter unity and ,0795 periphery unity, ſtands a black point.

Ex 254. On a ſecond ſide of this rule ſtands two lines, the one marked D, being the ſame with that line on the common ſliding-rule (the other marked D on the ſlider is alſo the ſame) the other marked E is a line of a triple radius to the line E or its radius is $= \frac{2}{3}$ of the lines A, B, or C, being all three the ſame, ſo that if the line A be made, all the other lines B, C, D, E, &c. M D may be made from it, on the line D are ſeveral marks as W G, A G, M S, M R, at 17,15, 18,95, 46,36, and 52,32, being the gauge points for wine and ale gallons, and for malt buſhels, in ſquare, and round veſſels, T P, the gauge point for a pound of tallow nett weight and ſtands at 6,32.

Ex. 255. On a third ſide of this rule is a line on the ſlider marked N, and is the ſame with the lines A, B, C, and on the rule is two lines, the one marked S L. or ſegment, l y, or ullaging a lying caſk, the other marked S S, or ſegment, s t for ullaging a ſtanding caſk. Theſe lines may be conſtructed thus, firſt for the line S L, take a caſk which agrees neareſt with the forms of the common ones in practice, and ſuppoſe its bung diameter to be divided into 100 = parts, then planes paſſing thro' each of thoſe parts parallel to the axis of the caſk, will divide it into ſo many unequal parts or ſlices. Now its evident if you find the contents of theſe ſlices in gallons, and to the content of the firſt, add that of the ſecond, you'll have a number for ,01 and ,02, and adding the content of the third ſlice to the laſt ſum you'll have a number for 03, &c. by continually adding a ſlice more to the laſt ſum you'll have a table of meaſures, by which the line S L may be laid down. In like manner, if for a ſtanding caſk, you imagine the axis to be divided into 100 = parts and planes to paſs thro' each of them parallel to the caſks bung diameter, it by this means will be divided into 100 unequal parts, whoſe contents being ſeverally found, and the laſt being always added to ſome of the preceding ones, you'll have a table of meaſures for the lines. Becauſe ale gallons are moſt common in practice, theſe lines are fitted to that meaſure.

Ex. 256. The line A L, (a new line for ullaging) is made thus, let s = any ſegment, under ſegment in the table of ſegments, d = diameter of any cylinder, and l = its length, $\frac{ddls}{282} = m$, the meaſure of a ſlice thereof parallel to the axis, in ale gallons, whoſe end is ſimilar to the ſegments, or $\frac{ddls}{231} =$ the content of the ſame ſegment in wine gallons, But if $\frac{ddls}{282} = m$, then $\frac{282}{s} = \frac{ddl}{m}$, whence ariſes this pro-

portion on the ſliding-rule viz. as $\sqrt{\frac{282}{s}}$, on D : l on C : : d, on D : m on C, ſo that if 282 be divided by every number under ſegment in a table of ſegments made to a radius of a 1000, or 1000 verſed ſines (but the table in this book taken from Mr. Shirtcliffe's book of gauging is only made to 500 ſegments or half the circle) and half the log. of each quotient, taken from the ſame ſcale of = parts, that the line N was taken from, and thoſe diſtances laid on the line A L, you'll have the line required.

Ex. 257. On the fourth and laſt ſide of this rule, is a line of inches divided into tenths, under it ſtands 3 other lines marked ſpheroid. 2d variety, and 3d variety, for finding the mean diameters of caſks when the difference between the head and bung diameters is under 8, inches and are made by theorems 121, 122, and 123. See ex. 337.

Ex. 258. On the inſide of the ſliders B and C, are placed two lines, the one numbered from 13 to 36 inches, and the other from 0,42 to 3,60 gallons, which is only as a table to ſhew the content of any cylinder at one inch deep in ale gallons, whoſe diameter is between 13 and 36 inches; on the inſide of the little ſlider are two lines for reducing the 4th variety of caſks to a mean diameter, as per ex. 257, conſtructed by theo. 124. What is ſaid in the foregoing ſections of the lines A, B, D, on the common ſliding-rule, are here to be underſtood of the lines A, B, C, D, and N, they being the ſame lines on both rules, only, the line D on this rule begins at 1 and ends at 10, yet their uſes is the ſame, both in meaſuring and in gauging. The uſe of all theſe in gauging follow.

Ex. 259. Extraction of the cube root by the ſliding-rule.

The cube root is extracted by the lines D and E, as the ſquare root is by D and A (ſee ex. 77.) therefore, as the firſt 1 on E is to the firſt 1 on D ſo is any number on E to its cube root on D, ſo if the cube root of 2197, were required, it will be as 1 on E : 1 on D : : 2,197 on E : 1,3 on D, ſo 13 is the cube root of 2197.

Note. As in taking the ſquare root, you divide the number given, by the ſquare of 10, ſquare of 100, &c. ſo in the cube root, when the number given is too large you muſt divide it by the cube of 10, or of 100, &c. for what number ever you divide by, the anſwer muſt be multiplied by the root of that number; ſo here I divide 2197 by the cube of 10 viz. by 1000. If I were to divide it by 100, or by 10, theſe have no perfect cube roots to multiply 1,3 the cube root of 2,197, by, (See ex. 68.

Ex. 260. Having given any 3 numbers (2, 8 and 9) to find a fourth which may be to the third, as the cube of the ſecond is to the cube of the firſt. By what is ſaid in the laſt ex. it is plain, that as 2 on D is to 9 on E ſo is 8 on D to 576 on E the anſwer. Again, if the three given numbers were 2, 80, and 9. Here when 2 on D is ſet to E, 80 is off the line D, but againſt 8 (viz. 8,0) on D ſtands 576 on D which multiplied by (1000) the cube of 10 (becauſe 80 was divided by 10) gives 576000 the term required.

The lines of numbers A, B, C, N, and D, are uſed in gauging veſſels by the directions in ſect. 2, as you'll find in the following examples. For the uſe of the line M D, and the gauge points M R, and M S, ſee malt gauging, S L, and S S, ſee ullaging, &c.

Ex. 261. Deſcription and uſe of the gauging rule, or rod.

This rule is commonly 4 feet long, and for convenience, is made to fold in four joints, and hath 4 ſides or faces, on the firſt face are 2 lines, the one marked A G, for ale gallons, the other W G, for wine gallons, called the diagonals. Becauſe when the end which is cut a ſlope, is put in at the bung of the caſk, and to the bottom of the head (try to both the heads to know if the bung be in the middle) the number cut at the bung, ſhews how many ale or wine gallons this caſk will hold, and is near good in the French wine hogſhead, or London beer barrel, the lines being made for ſuch caſks. Thus, as the content of any caſk is to the cube of its diagonal, ſo is the content of any ſimilar caſk to the cube of its diagonal, whoſe cube root will be a number on the diagonal line for that meaſure, whence veſſels gauged this way ought to be ſimilar and of the ſame variety. Otherwiſe, its not true.

On a ſecond face is a line of inches from 1 to 48, decimally divided and alſo, upon the ſame ſide ſtands Oughthred's gauge line, being a line of one third of areas of circles in wine gallons, by which you may gauge a caſk. Thus, let the bung diameter be 0,71 wine gallons, then its double is 1,42, to this add the head diameter, ſuppoſe 0,58, the ſum 2 gallons multiplied by 30 inches the caſks length, gives 60 wine gallons for its content, this holds true in a ſpheroidical caſk.

On a third ſide of this rule is a line of = parts from 1 to 96, which with Oughthred's gauge line, makes a table of circular areas in ale gallons. As for example, if the diameter of a circle be 30 inches, the area of that circle upon the other edge of Oughthred's line is a little above 1 gallon, the diameter being found on the line of = parts ; on the fourth ſide is a line of numbers, the ſame with the lines A, B, C, and N, on the ſliding-rule, on which are the ſame gauge points, A G, W G,

&c. which works with a pair of compaſſes inſtead of a ſlider. Thus if the length of any caſk or veſſel be 30 inches and its mean diameter 26 inches, then ſet one foot of your compaſſes in the gauge point AG, or W G, and extend the other foot to 26 the mean diameter, with that extent and one foot in 30 the length, turn the compaſſes twice over and you'll find the laſt moved foot to fall upon 57 ale, or 60 wine gallons the content. With this line of numbers, there is alſo a line of ſegments (S L) for ullaging a lying caſk. Thus, extend from the bung diameter on the line of numbers to 100, or radius on the line of ſegments, that extent the ſame way reaches from the dry inches on the line of numbers, to a reſerved number on the line of ſegments, or, which is better, on the line of numbers extend from the bung diameter to the dry inches, then on the line of ſegments that extent reaches from 100 to the reſerved number. Then extend on the line of numbers from 1 to the caſks content, that extent the ſame way reaches from the reſerved number to what the caſk wants of being full. But if you uſe the wet, inſtead of the dry inches, you'll have the ale gallons in the caſk. There are other four feet gauging rods made for London caſks, &c. Thus, ſet the veſſel level and pour in a gallon of water, and putting in your rod down right make a mark for 1 gallon where the ſurface of the water cuts the rod; pour in one gallon more, ſo you'll have a point on your rod to mark for two gallons, and ſo by putting in the rod for every gallon poured in you'll have marks for any number of gallons in the veſſel whether ſtanding or lying. And ſo when you come to gauge ſuch a veſſel, you have nothing to do but put in your rod down right, and the wet part will ſhew what gallons are in that veſſel. The black dots on theſe rods made for quarts, are made by pouring in a quart at a time, and at the end of every quart mark a dot on your rod, and as you did a figure for a gallon. Theſe rods are made up for the following veſſels, viz. 1 a line for a butt ſtanding, 2 a line for a butt lying, each of which contains 108 gallons beer meaſure, 3 a line for a hogſhead of 54 gallons, 4 a line for the barrel of 36 gallons, 5 one for the kilderkin of 18 gallons, 6 one for firkin of 9 gallons, all beer meaſure; ſo that you may eaſily know which of theſe veſſels you are gauging by putting in your rod, for if the top of the veſſel cuts 9 gallons it is a barrel, if 54 gallons the veſſel is a hogſhead, &c. The London coopers are obliged to make theſe veſſels = and alike. For the ſame purpoſe there are other lines put on ſuch gauge rods for wine meaſure, as for a tun, butt, puncheon, hogſhead, tierce, barrel, rundlet, and anchor, of 252, 126, 84, 63, 42, 31½, 18, and 9, gallons.

Ex. 262. The English wine gallon by act of parliament, is to contain 231 cubic or solid inches. By which is measured all wines, brandy's, spirits, strong waters, mead, perry, cyder, vinegar, oil, and honey. And as 1 pound troy weight is to 1 pound averdupoize weight, so is the cubic inches in a gallon of wine to those in a gallon of ale. Now (by Ward's mathematics, page 35) 1 pound averdupoize is = 14 oz. 12 p, w. troy, therefore, as 12 oz is 14 oz 12 p.w. so is 231 to 281$\frac{1}{4}$ ear = 282 solid inches as the ale or beer gallon is now setled. The standard, or Winchester corn bushel, to contain 8 gallons of wheat, every gallon to weigh 8 pounds troy, each pound to weigh 12 ounces, each ounce 20 penny-weights, and each penny-weight to weigh 32 grains of wheat taken out of the middle of the ear. Now such a bushel is found to contain 2145,6 solid inches, whose fill of common spring water is found to weigh 1131 ounces 14 penny-weights troy. But a cylinderical vessel 8 inches deep and 18,5 inches diameter, is 2150,42 solid inches content, which tho' it exceed the standard bushel of 2145,6 by 4,82 solid inches, a cylinder of these dimensions is now fixed on for the standard bushel; there being no other dimensions so convenient without running further into decimals. Now the eighth part of 2150,42 is 268,8025, the solid inches in a corn or malt gallon, but the decimal ,0025 being but small is left out and 268,8 taken for the standard corn gallon, therefore, let the shape of any vessel be what it will, if its solid content be 2150,42, or 268,8, or 282, or 231, solid inches it will hold a statute bushel of malt, or a gallon of malt, or a gallon of ale, or beer, or a gallon of wine.

Note. The Irish gallon of wine, oil, or ale, contains 217,6 solid inches.

A TABLE of Ale Measure in London.

	Sol. In.	Pints	Qts.	Gall.	Fir.	Kil.	Bar.	Hd.
Hogshead	13536	384	192	48	6	3	$1\frac{1}{2}$	1
Barrel	9024	256	128	32	4	2	1	
Kilderkin	4512	128	64	16	2	1		
Firkin	2256	64	32	8	1			
Gallon	282	8	4	1				
Quart	$70\frac{1}{2}$	2	1					
Pint	$35\frac{1}{4}$	1						

A TABLE of Wine Meaſure.

	Sol. In.	Pints.	Qrts.	Gall.	Rund.	Bar.	T.	Hd.	Pnn.	But.	Ton
Ton	58212	2016	1008	252	14	8	6	4	3	2	1
But	29106	1008	504	126	7	4	3	2	1½	1	
Puncheon	19404	672	336	84	4,66	2,66	2	1,33	1		
Hogſhead	14553	504	252	63	3½	2	1½	1			
Tierce	9702	336	168	42	2,33	1,33	1				
Barrel	7276½	252	126	31½	1,75	1					
Rundlet	4158	144	72	18	1	0					
Gallon	231	8	4	1	0						
Quart	57¾	2	1	0							
Pint	28⅞	1	0								

A TABLE of Beer Meaſure in London.

	Sol. In.	Pints.	Quarts.	Gall.	Fir.	Kil.	Barrel.	Hd.	Butt.	Ton
Ton	60912	1728	864	216	24	12	6	4	2	1
Butt	30456	864	432	108	12	6	3	2	1	
Hogſhead	15228	432	216	54	6	3	1½	1		
Barrel	10152	288	144	36	4	2	1			
Kilderkin	5076	144	72	18	2	1				
Firkin	2538	72	36	9	1					
Gallon	282	8	4	1						
Quart	70½	2	1							
Pint	35¼	1								

A TABLE of Ale and Beer Meaſure in the Country.

	Sol. In.	Pints	Qrts.	Gall.	Fir.	Kil.	Bar.	Hd.
Hogſhead	14382	408	204	51	6	3	1½	1
Barrel	9588	272	136	34	4	2	1	
Kilderkin	4794	136	68	17	2	1		
Firkin	2397	68	34	8½	1			
Gallon	288	8	4	1				
Quart	70½	2	1					
Pint	35¼	1						

Note. That one pound of	tallow grofs	is equal to	30,28	solid inches.
	tallow nett		31,4	
	hard foap		27,14	
	green foft foap		25,67	
	white foft foap		25,56	
	raw ftarch		34,8	

It is common in gauging to take the dimenfions in inches (in the infide of the veffel) and give the content in gallons, which done may be reduced to other meafures as you pleafe by thefe tables, thus (fee rule 6.)

263. In 5768 gallons of beer, how many barrels country meafure?

34) 5768 (169 barrels
17) 22 (1 kilderkin
5 gallons
Anfwer 16 bar. 1 kil. 5 gal.

Ex. 264. In 68502 gallons of ale, how many hogfheads country meafure?

51) 68502 (1343 hog. 9 gall. Anfwer
9 remains

Ex. 265. In 2520 gallons of wine, how many tons?

252) 2520 (10 tons Anfwer

Ex. 266. In 0,5768 parts of a gallon of ale, how many quarts, pints, gills and noggins?

,5768
4 quarts in one gallon
2,3072
2 pints in one quart
,6144
2 gills in one pint
1,2288
2 noggins in one gill
,4576

Anfwer 2 quarts 0 pints 1 gill 0,4576 nog.

Ex. 267. In 67854,2 folid inches, how many gallons of ale, and rundlets of wine?

	282) 67854,2		4158) 67854,2
gallons	240,61773	rundlets	16,318949
	4		18
quarts	2,47092	gallons	5,741082
	2		4
pints	0,94184	quarts	2,964328
	2		2
gills	1,88368	pints	1,928656
	2		Anſwer 16 R. 5 G. 2 Q.
noggins	1,76736		1,9 Pint in wine.

Anſwer 240 Gall. 2 Q. 0 P. 1 G. 1,76 Nog. in ale.

Ex. 268. In 1 gallon of wine, how many gallons corn?

	Inches	Gall.	Inch.	Gall.
As	268,8	: 1 ::	231 :	0,8594

1

268,8) 231,000

0,8594

Ex. 269. In 1 ale gallon, how many wine gallons?

As 231 : 1 :: 282 : 1,22

That is, a wine gallon is to a corn gallon, as 1 to ,8594, which is nearly as 1 to ,86, ſo if any number of wine gallons be multiplied by 0,8594 the product will be corn gallons, and for the like reaſon (ex. 269) if any number of ale gallons be multiplied by 1,22, the product will be wine gallons, &c. for others.

Ex. 270. Having in the foregoing tables ſet down the diviſors, viz. how many ſolid inches makes any of the meaſures therein mentioned, if you ſuppoſe the dimenſions in ſection 6 to be taken in inches, then any of the ſolidities in that ſection, divided by 282, or by 231, &c. will give the content of the ſolid in ale gallons, or in wine gallons, or in malt buſhels if you divide by 2150,42, &c. but if you reduce the factors in that ſection and theſe diviſors 282, 231, &c. into one factor or diviſor, the work will be much ſhortened, and yet performed by the ſame rules as you'll find in the following examples, where obſerve, that when you gauge a ſuperficies it is ſuppoſed to have one inch of depth for a plane ſurface can hold no liquor. Alſo, all ſuch figures as are meaſured by uſing the circular factor 0,7854, &c. are called circular figures, and all other figures are called right lined figures

Ex. 271. All, or any of the aforeſaid diviſors may (by ex. 46.) be turned into multipliers. Thus,

282) 1,0000 (,003546	is a multi-plier for	ale	gallons in right lined figures.
231) 1,0000 (,004329		wine	
268,8) 1,0000 (,003720		corn	bushels.
2150,42) 1,0000 (,0004650		malt	

See exp. 289. Or, if you make ,7854, or more exactly ,785398 your dividend instead of 1,000, you'll have multipliers for circular areas; or, making ,785398 your divisor to the dividends 282, 231, &c. you'll have divisors for circular figures; thus,

282	,785398	,002785	a multi-lier for	ale gall.	in circular figures.
231		,003399		wine gall.	
268,8		,002921		corn gall.	
2150,42		,0003652		corn bush.	

And	,785398	282	359,05	are divisors for the same.
		231	294,12	
		268,8	342,25	
		2150,40	2738,	

Now to gauge by the sliding-rule. Take the square root of any of these divisors, and set it upon the line D, (see ex. 82,) and it will be a constant gauge point for that measure.

So the square root of	282	is	16,79
	231		15,19
	2150,42		46,37
	359,05		18,94
	294,12		17,14
	2738		52,32

and is marked on D with
- M S. malt bushels, right lined figures
- A G. ale gallons, } circular figures because most
- W G. wine gallons, } vessels are such.
- M R. malt bushels, circular figures.

272. In this manner you may find multipliers, gauge points &c. for any other measure, and mark such gauge points on the line D, as you please.

How to gauge superficial figures.

Ex. 273. If each side of a square be 7,6 inches, what is its content in ale and wine gallons, at one inch deep.

7,6 } by rule 7.
7,6 }

57,76 area in inches
,003546 multiplier for ale

,20482396 ale gallons

Or thus, by division

282) 57,76

,2048 ale gallons
1,22 See ex. 269

,249856 wine gallons

Sliding-rule. As $\left\{\begin{matrix}16,79\\15,19\end{matrix}\right\}$ on D : 1 on C : : 76 (for 7,6 is off) on D : $\left\{\begin{matrix}20,48\\24,98\end{matrix}\right\}$ on C, (which divided by square of 10, See ex. 68.) is $\left\{\begin{matrix},2048 \text{ ale}\\,2498 \text{ wine}\end{matrix}\right\}$ gallons.

Ex. 274. Required the content of a rectangle in ale gallons, whose length is 100,5 inches and breadth 20 inches.

20
2010,0 area in In. by rule 8
,003546 multiplier for A. G.
7,127460 ale gallons, Ans.

Sliding-rule. As 282 on A : 20 on B : : 100,5 on A : 7,13 nearly on B the answer. Or as 16,79 on D : 44,83 (a mean propor. between 100,5 and 20) on D : : 1 on C : 7,13 on C the ale gall. required.

Ex. 275. If the base of a plane triangle be 260 inches and its perpendicular 110 inches, what is its area in corn gallons, and pounds of tallow nett.

$\left.\begin{matrix}130\\110\end{matrix}\right\}$ see rule 10
14300 area in inches
,00372 mult. for C. I.
53,196 corn in. Ans.

31,4) 14300 (455,4 pounds tall. nett
130 remains

Sliding-rule. As 268,8 on B : 110 on A : : 130 on B : 53,2 fere on A, the C. G. And as 31,4 on B : 130 on A : : 110 on B : 455,4 on A the tall. pounds.

Ex. 276. In a trapezia, the diagonal 2820 inches, the perpendiculars 105 and 80 inches, whats the content in corn gallons, and tallow pounds, gross.

$\left.\begin{matrix}105\\80\end{matrix}\right\}$ see rule 12
185
1410 = ½ diagonal
260850 area in inches
,00372 mult. for corn gall.
970,36200 corn gallons

30,28) 260850 (8621,2 tall. lb. gross
6120 remains

Sliding-rule. As $\left\{\begin{matrix}268,8\\30,28\end{matrix}\right\}$ on B is to 1410 on A so is 185 on B to $\left\{\begin{matrix}970,36\\8621,2\end{matrix}\right\}$ on A the answer.

When you take dimensions of a trapezia &c. stretch a diagonal line the longest way corner-wise, with a cord, and there let it lie fast, then with another cord, take the nearest distance between the two opposite angles and the said line, so you'll get the perpendiculars.

I think it needless to add any more examples in right-lined surfaces, for few such forms are to be met with in the practice of gauging, and if they were, the rules in section 4 are sufficient; for if the dimensions there be in inches, then any area there divided, or multiplied, as directed in ex. 270, will give the content at one inch deep, &c.

Ex. 277. If the diameter of a circle be 40 inches, what is its area in ale and wine gallons.

40 } see rule 15 40 }	1600 ,002785 mul. for A. G.	1600 [cir. fig. ,003399 mul. for w. g.
1600 sq. diam.	4,456000 ale gallons	5,438400 wine gallons.

Sliding-rule. As { 18,94 or ale gallons / 17,14 or wine gall. } on D : 1 on G : : 40 on D : { 4,45 / 5,44 } on C, the anſwer.

Otherwiſe. As 359,05 or 294,2 on B : 40 on A : : 40 on B : 4,45 or to 5,44 nearly on A, the anſwer.

Ex. 278. If the diameters of an oval be 72 and 50 inches, what is its area in ale gallons.

72 } see rule 21 50 }	3600 ,002785
3600 prod. diam.	10,126000 ale gallons

Sliding-rule. As 359,05 on A : 72 on B : : 50 on A : 10,126 on D the anſwer. Otherwiſe, as the gauge point A G, on D : 1 on C : : 60 (a mean proportional between 72 and 50) on D : 10,126 on C anſ.

Ex. 178. If there be a ſegment of a circle, whoſe verſed ſine is 10 inches, and diameter of the circle 50 inches, what is its area in ale gallons.

This may be done by ex. 118, but that by ex. 113, being ſhorter is commonly uſed.

Thus 50) 10,000 (200 againſt which under V. S. in the table of ſeg.

ſtands	,111823	
mult.	2500	ſq. diam.
	279,5575	area ſeg.
	,003546	mul. for A G.
	,9913 +	anſwer

Sliding-rule. As 50 on A : 1 on B : : 10,000 on A : 200 on B, againſt which under V. S. ſtands ,1118 +, then as 282 on A : ,1118 on B : : 2500 on A : 0,9913 on B the anſwer.

Ex. 279. If the area of any figure whatever be 521883, as in ex. 121, what is the area in malt buſhels, at an inch deep, the area 521883 inches, being every where the ſame, or the veſſel of the ſame breadth in that inch of depth.

2150,4) 521883 (242,2 malt buſhels, Anſwer
4862 &c.

Sliding-rule. As 2150,4 on B : 1 on A : : 521883 on B : 242,2, (ſee ex. 68) on A the anſwer.

Thus you ſee, it is but dividing the area in inches by the diviſor for that meaſure you want, let the figure be what it will, or multiply the ſaid area by the multiplier for the meaſure wanted, ſo you'll have the anſwer.

Ex. 280. By theſe examples (and ſect. 4) you may gauge any plane at one inch deep, and conſequently any ſimilar and = baſed ſolid, for the content at one inch deep, multiplied by any number of inches, muſt give the content of that number of inches in ſuch a ſolid. As for example, if a cylinder lie parallel to the horizon, whoſe length is 100 inches, diameter of each baſe 50 inches, and ſo much liquor in this veſſel, as that putting a ſtick into it ⊥ to the horizon the wet inches may be 10, by ex. 278, the content of this ſegment is found to be 0,9913 which multiplied by the length 100 inches, gives 99,13 ale gallons of liquor in this cylinder.

Ex. 281. Again, if there be a floor of malt whoſe content is 242,2 buſhels, at one inch deep, and the floor be 5 inches deep, then 242,2 multiplied by 5 gives 1211, for the malt buſhels in this floor, &c. for any other.

Ex. 282. Becauſe 16,79 multiplied by 16,79 gives 282 the inches in an ale gallon right lined figures, its evident, that if you divide a ſtreight rule into = parts 16,79 inches to each part, and each of theſe parts into 10 or 100 = parts, ſuch a rule will take the dimenſions in gallons, and decimal parts of an ale gallon, by which you may have the content without uſing the factor or diviſor, as in the foregoing examples is uſed. This is plain, if we conſider, a board &c. 12 inches ſquare to be = to one a foot ſquare, for the area of both is the ſame, viz. 144 ſquare inches = 1 ſquare foot &c; therefore, the area is 1 foot whether you take dimenſions in inches or feet. Alſo, 18,94 inches, being the gauge point for circular figures A, G. If you put 18,94 inches to a diviſion or gallon, on your rule, and divide it decimally as before, you'll have a rule to take the dimenſions of circular figures in ale gallons, by which the area of a circle in ale gallons is = to the ſq. of its diameter, that of an oval = the product of its two diameters, and ſo may ſuch rules be made for any other meaſure, or ſuch meaſures may be marked at their proper places upon an inch rule, and the contents much ſooner had by them than by the inches as is evident.

Ex. 283. If each ſide of a regular hexagon be 10 wine gallons (viz. 10 times 15,19 inches) what is its content at 1 inch deep. By ex. 107 the required content is 259,807 wine gallons.

Ex. 284. If the diameter of a round ciſtern be 100 W. G. (viz. 100 times 15,19 inches) what is its content at 3 inches deep. By rule 15, the content at one inch deep is 7854 wine gallons, which multiplied by 3 the depth, gives 23562 W. G. anſwer. But if you take the diameter in circular wine gallons, it will be 88,6 + (viz. 88,6 times 17,14 the circular wine gauge point), and its ſquare will be ($88,6 \times 88,6 =$) 7854 as before nearly. Hence, if you ſuppoſe the dimenſions in ſect. 4, to be taken in theſe meaſures the areas there, will be the contents in ſuch meaſures, at 1 inch deep.

How to gauge ſolids, or rather veſſels.

If any of the ſolidities in ſection 6 be in inches, and be divided by any of the foregoing diviſors, or multiplied by the multipliers anſwering thereunto, the quote, or product thence ariſing will give the meaſure in gallons &c. (as by ex. 270) I ſhall here, therefore, only inſtance in the moſt common veſſels.

Fx. 285. If each ſide of a cube be 30 inches, how many gallons of wine and pounds of hard ſoap will it hold?

$$\left.\begin{array}{r}30\\30\end{array}\right\}\text{ſee rule 36}$$

$$\begin{array}{r}900\\30\end{array}$$

231) 27000 ſolidity

116,88 wine gallons

27,14) 27000,00

994,43 pounds of ſoap

Sliding-rule. As $\left\{\begin{array}{c}15,19\\5,2\end{array}\right\}$ on D : 30 on C : : 30 on D : $\left\{\begin{array}{c}116,88\\994,43\end{array}\right.$

$\left.\begin{array}{l}=\text{gallons of wine}\\=\text{pounds of hard ſoap}\end{array}\right\}$ on C.

Ex. 286. If the length of a rectangular priſm, be 81 inches, breadth 25 inches, and depth 26 inches, what is the content in ale, wine, malt, &c.

$$\left.\begin{array}{r}81\\25\end{array}\right\}\text{ſee rule 37}$$

$$\begin{array}{r}2025\\26\end{array}$$

282)52650(186,7 ale gallons
231)52650(227,92 wine gallons
2150,4)52650(24,48 malt buſhels
31,4)52650(1676,75 pounds tallow nett
30,28)52650(1738,77 pounds tallow groſs

27,14)52650(1939,94 pounds hard ſoap
25,67)52650(2051,03 pounds green ſoft ſoap
25,56)52650(2059,85 pounds white ſoft ſoap
34,8)52650(1529,31 pounds raw ſtarch

Sliding-rule. Firſt, a mean proportional between ſome two of the 3 dimenſions, as 81 and 25 is found to be 45. (See ex. 82.) Then,

As	16,79		186,7 A. G.
	15,19		227,9 W. G,
	46,37	the ſq. root of the diviſor,	24,48 &c.
	5,6	on D : 26 on C : : 45 on D	1667,7
	5,5	the mean propor. between	1738,9
	5,2	81 and 45 : the anſwer on	1939,9
	5,06	C. (viz,)	2051,0
	5,05		2059,8
	5,9		1529,3

Ex. 287. If the diameter of a cylinder be 56½ inches, and its length 96 inches, what is the content in ale, wine, malt, tallow, &c,

diam. 56,5 } ſee rule 37
56,5

3192,25
len. 96

359,05)306456(853,52 ale gallons
294,118)306456(1041,94 wine gallons
2738)306456(111,92 malt buſhels
32,68)306456(9238,29 green ſoft ſoap
38,55)306456(7949,57 pounds tallow groſs
39,98)306456(7665,23 pounds tallow nett
34,56)305456(8867,36 pounds hard ſoap
32,54)306456(9417,82 pounds white ſoft ſoap
44,32)306456(6914,62 pounds raw ſtarch

As	18,94	Sliding-rule.	853,52
	17,14		1041,94
	52,32	the gauge point, or ſq. root	111,92
	5,7	of the circular diviſor on D	9377,47
	6,2	: 96 the length on C : : the	7949,57
	6,3	diam. 56,5 on D ; the anſ.	7665,23
	5,9	on C. (viz,)	8876,36
	5,7		9417,82
	6,66		6914,62

Theſe two examples (from Leadbetters gauging) are ſet down to all the moſt uſeful meaſures and weights, on purpoſe that you may have ready the gauge points &c. if wanted. (See ex. 270, and 271.)

Ex. 288. If there be a frustum of a square pyramid, each side of the greater and lesser bases 13 and 8 inches, and depth 24 inches, what is the content in ale and wine gallons. This is the same with ex. 169, which I shall work by rule 43.

13	169 sq. 13	
add 8	64 sq. 8	9,560016
21 twice a mid. side	441 sq. 21	1,22
	sum 674	11,66321952 W. G.
$\frac{1}{6}$ of ,003546 is =	,000591	
	,398334	
	24 height	
	9,560016 ale gall.	

Sliding-rule, for ale. If you multiply 282 by 6 the square root of that product will be 41,13 a constant gauge point for the frustums of square pyramids in ale gallons. Then, as 41,13 on D : 24 on C : : 13, 8 and 21 on D : three numbers on C whose sum is 9,56 ale gallons answer.

Or if you set the common right lined gauge point 16,79 on D to a sixth of the length on C, the rest of the work will be the same.

Ex. 289. The reason why I take the square root of 6 times 282, is because 6 and 282 are both divisors to the solidity, viz. 282 for the gallons and 6 for the $\frac{1}{6}$ of the length. And here observe, that in all such cases, any number of divisors multiplied into one another, the last product is one divisor for them all. Also, if you multiply any number by the product of any number of multipliers, the answer will be the same as if you multiply that number by these multipliers one after another, hence, a multiplier divided by a divisor, gives one multiplier for both, and a divisor divided by a multiplier gives one divisor for both, &c.

Ex. 290. If several numbers are to be multiplied together continually, it is no matter which you take first, or which last, the answer will be the same, so in ex. 288, I have 674 and ,000591 and 24 to multiply into one another, you'll find the product 9,56 +, multiply them in what order you please. Also, if a part of some one of these numbers is to be taken, you may take that part of any one of them, that suits best. Thus, in ex. 288, I should by the rule 43, take $\frac{1}{6}$ of the length 24, but I take $\frac{1}{6}$ of the factor ,003546, and so multiplies by the whole length, and the product is 9,56 +, take $\frac{1}{6}$ of which number 674 or 24 or ,003546, you please, this is all evident from algebra.

Ex. 291. There is a frustum of a rectangular pyramid, the length of the greater base 27 inches, its breadth 12 inches, length of the lesser base 18 inches, its breadth 8 inches, and the frustum's length 100 inches, how many ale gallons will it hold? First, the sum of 27 and 18 is 45 = twice the length in the middle, and the sum of 8 and 12 is = 20, twice the breadth in the middle, so per rule 43.

900 = 45 × 20 area mid. 4 times
324 = 27 × 12 area greater baſe
144 = 18 × 8 area leſſer baſe

1368 ſum
100 axis

136800 ſix times ſolidity, in inches
,000591 = ⅙ of ,003546

80,8488 ale gallons anſwer

Sliding-rule. Find 18 a mean proportional between 27 and 12, the length and breadth of the greater baſe, and alſo 12, a mean proportional between 18 and 8, the length and breadth of the leſs baſe, which mean proportionals 18 and 12 in one ſum gives 30, twice a mean proportional in the middle. Then as 41,13 on D : 100 on C, or as 16,79 on D : $\frac{100}{6}$ on C : : 18 and 12, and 30 on D : 19,15 and 8,51 and 53,19 on C, whoſe ſum is 80,85 the ale gallons required.

Ex. 292. There is a fruſtum of a cone whoſe length, or depth is 100 inches, diameter at the greater baſe 18 inches, and diameter at the leſſer baſe 12 inches, how many ale or beer gallons will it hold?

Firſt the ſum of 18 and 12 is 30 twice a diameter in the middle, then by rule 43.

144 ſq. 12 leſs diameter
324 ſq. 18 greater diam.
900 ſq. 30 twice mid. diam.

1368 ſum
100 length

136800
,000464 = ⅙ of ,002785

63,475200 ale gallons anſwer

Sliding-rule. If you multiply 359,05 by 6 the ſquare root of the product 2154,3 is 46,41 a conſtant gauge point for all conical fruſtums ale gallons, ſo as 46,41 on D : 100 on C : : the two diameters 18 and 12, and their ſum 30 on D : 14,9 and 6,7 and 41,9 on C whoſe ſum is 63,5 ale gallons for the anſwer.

Ex. 293. There is a fruſtum of an elliptical cone, length 100, the tranſverſe and conjugate diameters of the greater baſe 27 and 12, and of the leſſer baſe 18 and all in inches, whats the content in ale gallons See ex. 291.

144 = 18 × 8
324 = 27 × 12
900 = 45 × 20

1368 ſum
100 length

136800
,000464 factor

63.4752 ale gall. anſwer

Sliding-rule. Firſt find 18 a mean proportional between 27 and 12, as alſo 12, one between 18 and 8, then, as 46,41 on D : 100 on C : : the two mean proportionals 18 and 12 and 30 their ſum, on D : 41,9 on C, whoſe ſum is 63,5 the anſwer.

Ex. 294. If the axis of a ſphere be 20 inches, how many ale gallons will it hold?

1. The greateſt diameter in a ſphere is that thro' its center, which is equal to its axis (viz. H I = A P fig. 170.) and the leaſt diameter (as at A or at P) = 0, and G F or K L a diameter in the middle between theſe two, now K L doubled and ſquared, or 4 times ſq. K L or G F is (by the property of a circle or by meaſuring) = 3 times the ſq. of the greateſt diameter H I or axis A P. So by rule 43.

0 = ſq. leaſt diam.
10000 = ſq. great diam.
30000 = 4 times ſq. mid. diam.

40000 ſum
,000464 factor

18,560090
100 length or axis

1856 ale gallons anſwer

Sliding-rule. By the operation you ſee that the ſum is = 4 times the ſq. of the axis, i. e. = twice the axis ſquared, ſo it will be, As 46,41 on D : 100 the axis or length on C : : 200 twice the diameter or axis on D : 1856 on C. anſwer.

Ex. 295. If K L P (fig. 170, or fig. 180) be a bowl, bottom of a kettle, &c. whoſe depth D P is 15 inches, diameter K L 60 inches, diameter at P = 0, and middle diameter m n 40 inches, what ale gallons will it hold? By rule 43.

6400 = ſq. 2 m n, or = 4 times ſq. m n
3600 = ſq. K L
0 = ſq. at P

10000 ſum
15 = D P

150000
,000464 factor

69,600000 A G. anſwer

Sliding-rule. As 46,41 on D : 15 on C : : 60 and 80 (twice 40) on D : 25,1 and 44,5 on C, whoſe ſum is 69,6 ale gallons for the anſwer.

296. I have wrought each of theſe examples by rule 43, which has turned out the very ſame anſwers as if you had uſed their particular rules in ſection 6. This ſaid rule 43 is as extenſive in gauging as in meaſuring. See ſection 6.

297. You may find the wine and malt, or any other gauge points, as the ale one is found in ex. 288 and 292, which if marked on the line D, inſtead of the common gauge points A G, W G, M R, &c. would be of general uſe in gauging, for any round veſſel what ever may be done by this general

Rule 59. As the gauge point on D is to the depth on C so is a diam. at each end and twice one in the middle on D, to 3 such numbers on C, as being added together, gives the required content, or you may take the diameters in gallons (see ex. 282) and the depth in inches, and then the gauge point on the line D will be unity, and so will be easier. In short, take dimensions either this way or all in inches. This noble rule is sufficient for the whole of gauging, for the error arising by guessing at the vessels form, may be much greater than can happen by this rule; in the most common sorts of vessels it gives the content exactly, and in any form very near as is proved in prob. 190.

How to gauge malt.

The Duty upon malt is charged at sixpence the Winchester bushel of 2150,42 solid inches, so that what form soever the surface of the malt lie in, whether floor, cistern, couch, &c. find the content of that form at one inch deep in malt bushels, which multiplied by the depth in inches gives the content in such bushels. And here observe, that malt being a dry substance, will not like liquor have an even surface, also the bottom of the floor &c. may be uneven. Now to remedy all this you must take the depth or thickness of the malt in 6, 8, or 10 places or more if needful (particularly, observe to take it where you judge it to be thickest, and also where thinnest, see rule 24,) then the sum of these depths divided by their number, will give the mean depth.

Ex. 298. There is a rectangular floor of malt, length 72 inches, breadth 48 inches, and the $\left\{\begin{array}{c}1\\2\\3\\4\\5\end{array}\right\}$ depth is $\left\{\begin{array}{c}4,7\\5,4\\5,6\\4,9\\4,4\end{array}\right\}$ inches, what is the cont. in malt bushels?

number of depths $= 5)25,0$ sum depths
5 mean depth

72 length
48 breadth
3456
5 mean depth
17280 solid inches

17280 content in inches
,000465 mul. for M B.
8,035200 malt bushels answer

By the sliding-rule. First, as 48 on D : 48 on C : : 72 on C : 58,7 on D a mean propor. between 48 and 72, then as the gauge point MS 46,37 on D : 5 the mean depth on C : : 58,7 on D : 8,04— on C, the answer. Otherwise, Set the length 72 on B to the breadth 48 on M D then against the mean depth 5 on A stands the content 8,04— bushels

on B. This line M D, with the lines B A, are for gauging rectangular floors of malt, for malt lies moſtly on ſuch floors, its uſe (viz. the uſe of the line M D) in ſuch caſes turn one of the dimenſions into a diviſor malt meaſure. (See ex. 106) Now when the figure 1 on M D denotes 1, then the braſs p in M B over againſt it on A denotes 2150,42 the cubic inches in a malt buſhel, by which means the leaſt number that can be on A is 100 (viz. the firſt 1 on A is 100) but if this firſt one be 1, then the line A muſt be divided by 100, and when A is divided by 100 then B and M D muſt be divided by two ſuch numbers as being multiplied together may produce 100, i. e. the diviſor for A, and the product of the diviſors for B and M D being equal, the fourth number found on B will be the content in malt buſhels as is plain by ex. 253. Then this is the

Rule. As the length on B is to the breadth on M D ſo is the depth on A to the malt buſhels on B, the dimenſions being all in inches.

Ex. 299. If the breadth of a couch, floor &c. be 56,2 inches, length 270, and mean depth 5,2 inches, how many malt buſhels is in that priſm.

Sliding-rule. Here that 5,2 the depth, may be found on A, the firſt 1 on A may be either 1 or 0,1, ſuppoſe it = 1, then the ſecond M B on A is divided by 100 viz. 2150,42 becomes 21,5042, ſo the length 270 and breadth 56,2 being each divided by 10 (becauſe 10 times 10 is 100 the diviſor of A) will be 27 and 5,62. Therefore, as 27 on B : 5,62 on M D : : 5,2 on A : 36,7 on B anſwer.

Ex. 300. If the breadth be 72, length 140, and depth 18,2. Here if the firſt 1 on A denotes 1, then the ſecond M B 2150,42, on A will be 21,5042, ſo the diviſor of A being = 100, the product of the diviſors of the length 140 on B and breadth 72 on M D muſt alſo be 100, ſo let each of 140 and 72 be divided by 10, and they'll be 14 and 7,2 then as 14 on B : : 7,2 on M D : : 18,2 on A : 85,3 on B the buſhels required.

Ex. 301. Let the length be 1250, breadth 360 and depth 9 all in inches.

Let the firſt 1 on A be 0,1, then the ſecond M B 2150,42 on A will be 2,15042, ſo A is divided by 1000. Therefore, let 1250 be divided by 10 and 360 by 100 (becauſe 10 × by 100 = 1000) and they'll become 125 and 3,6. Then as 125 on B : 3,6 on M D : : 9 on A : 1884 buſhels on B the anſwer. Theſe three examples are ſufficient to ſhew the uſe of the line M D, they or any ſuch like may alſo be wrought as ex. 298, by the right lined malt gauge point M S.

Ex. 302. There is an elliptical ciſtern of malt, whoſe longeſt diameter is 72 inches, ſhorteſt diam. 48 inches, and mean depth 5 inches, how many malt buſhels.

```
       72
       48
      ----
     3456
  ,000365 factor
  -------
  1,26044
        5 depth
  -------
anſ. 6,3022 M B.
```

Sliding-rule. As M R 52,32 on D : 5 on C : : 58,7 (a mean propor. between 72 and 48) on D : 6,3 buſhels on C, anſwer.

303. When barley is wet, or ſteeped in the ciſtern, it ſwells out about a fifth part (or more if its very good corn) it is then called a couch and bears that name till it hath been 30 hours out of the ciſtern, but if it has been out above 30 hours, then on account of its ſprouting &c. it is double to what it was before being wet, it is then called a floor, and ſo continues till it come to the kiln to be dried. Now from the ciſtern to the couch there is an allowance of $\frac{1}{5} = 0,2$, which taken from 1 leaves 0,8. Hence if any number of couch buſhels be multiplied by 0,8 it will give the nett buſhels. But from the couch to the floor there is allowed $\frac{1}{2}$. Therefore any number of floor buſhels multiplied by 0,5 gives the nett buſhels.

The law having thus given the allowance of 4 buſhels in 20, on the ciſtern and couch gauges, and 10 in 20 on the floor ; if therefore an officer do not advance in his gauges according to theſe proportions he may expect his ſupervisor will diary him for ſuch a fault, for this you muſt write a reaſon for, damaged barley &c. for ſuch barley will not hold out to ſuch allowances, tho' good barley will exceed them.

Ex. 304. If a ciſtern's gauge of dry barley (or however very ſoon after it is wet) be 13,8 buſhels, and the beſt couch gauge you can get be 15 buſhels, whether does it anſwer the allowance or not.

Here the allowance $\frac{1}{5}$ of 13,8 is $=$ 2,76, and 13,8 from 15 leaves 1,2 which ſhould be 2,76, therefore it is too little by 1,56, i. e. 2,76 added to 13,8 gives 16,56 for the couch gauge, inſtead of 15 as it was gauged to.

Ex. 305. If a floor gauge be 100,8, and the beſt couch gauge be 63,6 which will afford the moſt duty? Here (by ex. 303,) a floor gauge is multiplied by ,5, and a couch gauge is multiplied by ,8, now to reduce theſe two factors to one factor, (ſee ex. 290) it will be 0,5 $\div$ 0,8 $=$ 0,625, by which multiply a floor gauge and it will be equal the ſame in a couch gauge, or ,8 $\div$,5 $=$ 1,6, by which multiply a couch gauge, and it will be equal to the ſame in a floor gauge. So 100,8 ×

,625 = 63 bushels the couch gauge to the floor 100,8; but this couch was gauged to 63,6 bushels, so the duty must be upon the couch.

From these things, you may learn that malt is to be gauged several times while it is in making.

How to gauge any tun, tub, back, cooler, &c.

These vessels are of various forms, as prisms, frustums of cones and pyramids, cylinders, cylinderoids, &c. any of which may be gauged by the general rule 43, as applied to the foregoing examples, so in this place I shall only shew the inching of such vessels viz. to find how much liquor they will hold at every, or any inch of depth, which being known and set down in a table, you may by putting a stick into the vessel and seeing the wet inches, know by your table what liquor is in the vessel in gallons, &c.

306. When the bottom of any such vessel is parallel to the horizon, or exactly level, its called an upright vessel, but otherwise an inclined one.

307. What liquor is required to cover every part of the bottom of an inclined tun, is called the drip, or fall of the tun, so G A P is the drip of the tun, A G B C. Fig. 181.

308. That horizontal line D G that touches the highest part G of the bottom A G, is called the horizontal base, and another line C g parallel to D G, touching the lowest point C of the top C B is called the horizontal end.

309. If from C any point in the horizontal end a perpendicular C D be let fall upon the horizontal base G D it is called the height of the tun.

310. The ends of tuns are said to be parallely-posited, when the length of the one end is parallel to either the length or breadth of the other end, and the breadth of one end parallel to either length or breadth of the other end, and if it be an elliptical tun we may for brevity's sake call the transverse diameter the greatest length, and the conjugate the greatest breadth of the bases or ends.

Ex. 311. Let there be an elliptical upright parallel-posited tun, tub, &c. length at top 65, breadth there 60, length at bottom 110, breadth there 100, and the whole depth 12 all in inches, to find the content at every inch deep.

First, to find the diameters at every inch deep, you may measure them in the vessel itself, or find them by this rule (from the nature of the cone viz.) as the height of the tun 12 is to $\left\{\begin{matrix}45\\40\end{matrix}\right\}$ the difference

between the two $\left\{\begin{array}{l}\text{lengths 110 and 65}\\ \text{breadths 100 and 60}\end{array}\right\}$ ſo is any propoſed depth 1 to $\left\{\begin{array}{c}3{,}75\\ 3\frac{1}{3}\end{array}\right\}$ the difference between the $\left\{\begin{array}{l}\text{lengths}\\ \text{breadths}\end{array}\right\}$ at 12 and 11 inches height, now theſe differences being found for 1 inch, you may by continually adding 3,75 the difference of the lengths to 65 the leſſer length, have all the lengths, and by continually adding $3\frac{1}{3}$ to 60, have all the breadths as in the ſecond and third columns in the following table, for the above proportion holds for any depth, and thus having ſet down all the dimenſions you may by rule 44 find all the contents as ſet down in the 4th column which reſpectively taken from 238,9595 the content of the whole tun, leaves the contents in the 5th column, but having by the ſaid rule 44, found the 3 firſt contents 11,4884 and 24,2766 and 38,4340, you may with more eaſe find all the reſt, than by the ſaid rule, thus (by theorem 127) take any 3 contiguous contents and divide each by its dry inches, take the ſecond quotient from the third, add 3 times the remainder to the firſt quotient, multiply the ſum by the dry inches belonging to the required content, the product will be that content. Now the dry inches of the 3 above contents are 1 and 2 and 3, and the dry inches of theſe following will be 4, 5, 6, &c. ſo 11,4884 ÷ 1 (its dry inches) = 11,4884 and 24,2766 ÷ 2 = 12,1383, and 38,4340 ÷ 3 = 12,8113 from which take 12,1383 leaves ,6730 which tripled is 2,019, this added to 11,4884 gives 13,5074, this multiplied by 4 gives 54,0305 fere, for the 4th content. Again 13504 being = the 4th content ÷ 4, from it take 12,8113, the 3d content ÷ its dry inches 3 leaves ,6927 whoſe triple is 2,8781 this added to 12,1383 (the 2d content divided its dry inches 2) gives 14,2164, this multiplied 5 its dry inches gives 71,0820 or 71,1355 by carrying the decimal on further, and thus taking the 3 laſt contents 38,4304 and 54,0305 and 71,1355, dividing each by its dry inches 3, 4, 5, and ſo on as per rule, &c. &c. You may ſoon compleat the table, or by taking any 3 contents together in ſuch a table you may ſoon prove the 4th content, whether right or not, the 3 firſt contents are by rule 44 found,

Thus: $\overline{65 \times 2 + 68\frac{3}{4}} \times 60 + \overline{68\frac{3}{4} \times 2 + 65} \times 63\frac{1}{3} : \times {,}000464 = 11{,}4884$ A. G.

2d. : $\overline{65 \times 2 + 72\frac{1}{2}} \times 60 + \overline{72\frac{1}{2} \times 2 + 65} \times 66\frac{2}{3} : \times {,}000464 = 24{,}2766$ A. G.

3d. : $\overline{65 \times 2 + 76\frac{1}{4}} \times 60 + \overline{76\frac{1}{4} \times 2 + 65} \times 70 : \times {,}000464 = 38{,}4340$ A. G.

In this manner you may find all the contents, but the above method is much easier. The use of this table is easy, thus suppose you come to the above tun and find 7 inches wet, then looking in the table under wet inches and against 7 I find under content from the bottom 167,8240 ale gallons in the tun, or under content from the top, I find 71,1355 ale gallons which it wants of being full.

A TABLE shewing the content in ale gallons to every inch of depth in the upright tun E y n C. Fig. 181.

Dry Inches.	Lengths	Breadths	Content from top C n	Content from bottom E y	Wet Inches
0	65	60	,0	238,9595	12
1	68,75	$63\frac{1}{3}$	11,4884	227,4711	11
2	72,5	$66\frac{2}{3}$	24,2766	214,6829	10
3	76,25	70	38,4340	200,5255	9
4	80	$73\frac{1}{3}$	54,0305	184,9290	8
5	83,75	$76\frac{2}{3}$	71,1355	167,8240	7
6	87,5	80	89,8187	149,1408	6
7	91,25	$83\frac{1}{3}$	110,1497	128,8098	5
8	95	$86\frac{2}{3}$	132,1982	106,7613	4
9	98,75	90	156,0338	82,9257	3
10	102,5	$93\frac{1}{3}$	181,7262	57,2333	2
11	106,25	$96\frac{2}{3}$	209,3446	29,6147	1
12	110	100	238,9595	,0	0

312. But all such tuns are set to lean a little, for the benefit of cleaning them &c. and also have a constant point the easiest to come at, fixt for taking the wet or dry inches, called the dipping place. Now to shew how such inclined tuns may be inched, let us suppose that we have just now inched the upright tun C E y n (fig. 181) whose height C E = n y = 12 inches, and the several contents are as in the above table, and let it be required to inch the inclining tun A C B G or rather q C A G, because the plane C q being parallel to the horizon the vessel can hold no liquor above the point q, and so the hoof q C B need not be regarded, suppose the dipping place to be at n, and let z y, the distance between G D and A F, the horizontal base and bottom of the tun be 2 inches, and let the hoof A P G, or the quantity of liquor just covering all the bottom of the vessel at G, be 15 ale gallons, so when the wet inches are 2, there is in the tun 15 gallons. This quantity of liquor is commonly measured into the tun with a gallon &c. just to cover the bottom and every gallon marked as you pour it in on your rule (as by ex 261,) then the dimensions taken parallel to the surface of the liquor at top C q, and close by it at bottom G P, and

the depth n z perpendicular thereunto, ſo that if a circular veſſel lean to one ſide, the ſurface of the liquor will be an oval and the contrary, now if we ſuppoſe the diameters at C q and G P, and the depth n z to be the ſame as before, we ſhall by the above method have this table, whoſe uſe is the ſame as before. Thns

Ex. 313 If the wet inches be 7 what liquor is then in the tun? Here under content from ale gallons and againſt 7, under wet inches ſtands 143,8098 A G. the anſwer. Alſo, if the dry inches be 7, then the tun wants 89,8187 A G. of being full, &c. for any other

A TABLE, of contents in A G. to every inch of depth in the inclin'd tun A G q C. Fig. 181.

Dry Inches.	Lengths	Breadths	Content from C q	Content from A G.	Wet Inches
1	65	60	,0	253,9595	14
2	68¾	63⅓	11,4884	242,4711	13
3	72½	66⅔	24,2766	229,6829	12
4	76¼	70	38,4340	215,5255	11
5	80	73⅓	54,0305	199,9290	10
6	83¾	76⅔	71,1355	182,8240	9
7	87½	80	89,8187	164,1408	8
8	91¼	83⅓	110,1497	143,8098	7
9	95	86⅔	132,1982	121,7613	6
10	98¾	90	156,0338	97,9215	5
11	102½	93⅓	181,7262	72,2333	4
12	106¼	96⅔	209,3448	44,6147	3
13	110	100	238,9595	15	2

314. Backs or coolers, are often very broad and not deep, that ſo the worts, &c. in them may ſooner cool, they generally are of a priſmatic form, and often have uneven bottoms, in ſuch caſes you muſt take the depths or dips in ſeveral places (as directed in ex. 298) and ſo find a mean depth; then fix a point in the cooler for a dipping place, as that the depth there taken, may be equal to this mean depth: but if this cannot be done you muſt make a juſt allowance for the difference; ſuch large bottomed veſſels are commonly taken at every tenth part of an inch, and if they are priſmatic veſſels, it is very eaſy to do: for (by the foregoing examples) you need but find the content at one inch deep, and one tenth of this content will be that of one tenth of an inch deep, &c. for any number of tenths: in ſuch caſes you need no table of contents &c. for the area at the ſurface being known (and every where equal between top and bottom,) this area multiplied by the depth in wet inches and parts of an inch, will give the content of ſuch a depth.

Ex. 315. Let there be a prifmatic back, length 140 inches, breadth 120, depths at 10 feveral places in inches as here follow.

N°	depth
1	5,1
2	4,8
3	5,0
4	5,2
5	4,6
6	4,9
7	4,5
8	4,7
9	5,2
10	4,9

required the content at 0,7 inch deep

÷N° dep 10)48,9 fum
4,89 mean depth

140 length
120 breadth
282) 16800 area bafe
59,57 A G. at 1 inch deep
0,7 propofed depth
41,999 A G. at 0,7 inch deep.
41,699

And thus you may find the content at any depth you pleafe

Ex. 316. Again, fuppofe you come fometime to this cooler and find the depth 1,2 wet inches, what ale gallons are then in it?

59,57 cooler at one inch deep
1,2
71,484 ale gallons anfwer

317. In fuch veffels, there is always a point (the eafieft to be come at) in the edge of the veffel marked for the conftant dipping place, and if in the laft example you find the depth at the dipping place to be 5 inches then from this 5 you muft take the mean depth 4,89 and there leaves 0,11 inches fo that you muft always take this 0,11 from the wet inches taken at this dipping place, and the aforefaid content 59,57 multiplied by the remainder gives the true content of liquor in the cooler: but if the mean depth exceed that taken at the dipping place, then the difference muft be added to the wet inches taken there, as is eafy to underftand.

318. This will do for backs coolers &c. whofe bafes are equal, and their fides ftreight, but if they be otherwife you muft (by ex. 311 &c.) find the content to every inch deep, if you would be very exact, but the common way is to take or find the dimenfions in the middle of every inch (tho' fome take them in the middle of every 2, 3, 4, &c. inches) and fo get a mean area, for one inch deep, and fo proceeding for every inch of depth you'll have a table as in ex. 311.

319. Thefe veffels are very readily inch'd by the lines on the infide of the two fliders (fee ex. 258) If they are circular and their diameters between 12 and 36 inches. Thus, put the fliding-rule in the middle of every inch of depth to take the greateft breadth or diameter, putting

out the ſlider, or both ſliders, to the oppoſite ſides of the veſſel, then by looking the underſides of theſe ſliders you have the diameter there both in inches and ale gallons, at one inch deep.

320. If theſe veſſels are not quite circular but very near ſo, you may take the greateſt and leaſt diameters (each thro' the middle of the veſſel) and half their ſum may be taken as a mean content. If the diameter is but 12 inches it is had by the ſliding-rule its ſelf becauſe it is 12 inches in length, if the diameter be between 12 and 24 inches, it is had by drawing one ſlider, but if above 24 inches you muſt draw out both ſliders, the gallons on the inſides of theſe ſliders, are found by ex. 277. Thus if the diameter of a circle be 13,4 its content will be 0,5 A G. ſo againſt 13,4 inches you ſet 0,5 on the line of gallons &c. for any other.

321. It is found by tryal that every 10 gallons of hot wort will be but 9 gallons when cold; therefore it will always be as 1 is to 0,9 ſo is any gauge of hot wort to the nett wort, ſo if a hot gauge be 90,3, then, As 1 : 90,3 : : 0,9 : 81,3 the nett content &c. for any other.

To gauge curve-lined tuns, as coppers, ſtills, &c.

322 The common way to gauge theſe veſſels, is thus, take croſs diameters or dimenſions by which you can have the area or content in the middle of every 4, 6, or 8 inches of depth, and this area multiplied by that height gives the content of that part nearly, and thus having found the content to every 6 &c. inches deep, the ſum of all theſe contents muſt be that of the copper, all except the crown whoſe content may be had by ex. 295, or by meaſuring it with water by an ale quart.

323. Otherwiſe by rule 43, take croſs diameters &c. at the top of the copper, top of the crown, and middle between them, if the copper have but one bulge viz. be as one fruſtum, but if it be as ſeveral fruſtums take ſuch diameters for every fruſtum, and height thereof; and ſo find the content by the ſaid univerſal

Rule 43. Then its plain, the ſum of theſe contents will be that required.

Ex. 324. Let d d H H (fig. 129) be a copper with a riſing crown d e H, which is covered up to the diameter d q with 92,8 gallons, ſuppoſe the top diameter H L D (to be the greateſt) = 115 inches, the bottom diameter viz. that over the crown to be degrees, the leaſt = 96 inches, the depth e L = 30 inches, and diameters in the middle of every 6 inches depth to be as in the following table.

To find the content the common way, the copper being circular.

Inches depth	Diameter in the middle of every 6 inches	Content of every 6 inches depth in ale gallons.
6	113,1	213 72
6	109,2	199,26
6	105,28	185,16
6	101,36	171,66
6	97,33	158,22
	Sum of contents =	928,02
	Gallons to cover the crown	92.8
	Coppers content	1020,82

These contents in the 3d column are very readily had by the sliding-rule. Thus, As the gauge point A G on D is to 6 the common depth on C so is each diameter in the second column on D to the content on C as per third column, which third column in one sum is 928,02 A G. the content of the copper above the top of its crown, to which add 92,8 A G. the liquor that covers the crown and you have 1020,82 ale gallons the content of the whole copper. To do this by rule 43 there wants a diameter in the middle between q d and H d, which by the table you see is 105,28 = D B. Then

13225 = sq. H d
44335,41 = sq. 2 D B
9216 = sq. q d
66776,41 sum
30 = depth e L
2003292,3

2003292,3
,000464 = $\frac{1}{6}$ of ,002785
929,5276272 content of d H d g
92,8 crown add
1022,327 + A. G. answer

325. A diameter taken in the middle of every 6 &c. inches cannot be a true mean diameter even if the vessel be conical (see rule 41.) much less so if the vessel is curve-lined as a copper, &c Hence the content thus found must be somewhat too little, and therefore that found by the general rule 43 must be truer, i. e. 1022,327 + is a truer content than 1020,82 and is much easier had.

Ex. 326. Let d d H H (fig. 129) be an elliptical bas'd copper, whose greatest bulge or wideness is at D B out of the middle of the copper and the crown d E H C to bend outward. Such vessels as these whose greatest bulge is not in the middle, nor at top or bottom, must be measured at twice like two different frustums, thus, take two cross diameters 89 and 89,5 at D B the greatest bulge, two at L 80,5 and 80,8

the top two at a 85,5 and 86 in the middle between L and a, by which and the length L b 15, (by rule 43.) find the content of the frustum D B H d. Again, find two cross diameters 83 and 83,5 at E where the crown begins; also two 86,5 and 87 at g in the middle between E and b, which the length 21 E b (by rule 43) find the content of the frustum B D d E H, which two frustums added to the content of the crown d C H, gives the content required.

Suppose that the crown d C H hold 32 ale gallons, Then

diameter at L 80,5 + 80,8	6504,4
diameter at b 89,5 + 89	7965,5
4 times diam. at a 85,5 + 86 + 4 =	29412,0
sum =	43881,9
	15 = L b
	658228,5
inverted factor ,000464 =	464000,0
	26329
	3949
	263
content of B D d L H =	305,41

Again

product of the bottom, diam. 83 + 85,5	6930,5
product of diam. at g, 4 times 86,5 + 87 + 4	30102,0
diameter at b 89,5 + 89 = 7965,5	7965,5
sum =	44998,0
	21 = E b
	944958,00
inverted factor =	464000,0
content of D B E =	438,460
content of B D L =	305,41
content of the crown d C H =	32,00
content of the whole copper =	775,87 sum

327. If you meet with a still or any vessel that hath two bulges, you must consider it as 3 frustums &c. The contents of stills are given in wine gallons, and therefore in such case you must multiply by the factor for wine gallons instead of that for ale i. e. by ,0005666 = $\frac{1}{6}$ of ,003399, instead of ,000464 = $\frac{1}{6}$ of ,002785, all else is the very same. The globical part of a still &c. may be soon had by rule 50. More examples might be added, but I think these sufficient, to any one that is acquainted with rule 43, and what is said about it in this work.

Of cask gauging.

328. Every cask is supposed to be made up of two equal frustums (or one middle zone) join'd together at their greater bases whose common diameter B D (fig. 182,) is called the bung diameter, the diameters d H = d H, at the lesser bases are called head diameters and E L their joint lengths (viz. C L = C E) is called the casks length, so when you come to gauge a cask, take the bung diameter exactly in the middle at C and see if the two head ones be equal, which if they be, it may be gauged as one frustum, by using L E to both frustums instead of C E or C L to one of them. But if the head diameters are unequal you must gauge the cask at twice viz. each side of B D as one single frustum.

329. Several casks may have the same length, bung and head diameters, and yet have different contents on account of their different curvatures as is plain by fig. 182, for the cask G G, will hold more than the cask g g, tho' they both have the same dimensions B D and H d and L E, for this reason casks are divided into 6 forms or varieties as you'll see in what follows, some have but the 4 varieties mentioned in prob. 187, amongst which in cask gauging the 4th form may be left out, for no cask can have its staves streight from head to bung, nevertheless the rules are applicable to conical frustums.

variety	1	two equal frustums of a	spheroid
	2		circular spindle
	3		parabolic spindle
	4		equilateral hyp. spindle
	5		parabolic conoid
	6		cone

These forms are set down according as they curve, the first variety curving most, the second next most, and so on to the last which hath its staves streight from bung to head.

330. Now to find the content of a cask in each form or variety, and see which holds most, let us suppose each to have the same dimensions viz. length L E (fig. 182) = 40 inches, bung diameter B D = 32 inches, head diameter H d = 26 inches.

First, for variety first, being a spheroidical cask, this is done by rule 49.

Thus $32 \times 32 \times 2 = 2048 =$ twice sq. bung diam.
$26 \times 26 = 676 =$ sq. head diameter

$$\begin{array}{rll} \text{sum} = & 2724 & \\ \text{length} = & 40 & \quad \text{Again, for wine} \\ & \overline{108960} & \quad 108960 \\ \text{inverted factor} = & 8429000,0 & \text{Or,} = \quad 3331100,0 \\ \text{ale gallons} = & \overline{101,15} & \text{W G.} = \overline{123,49} \end{array}$$

Here $359,05 \times 3 = 1077,15$ a divisor for ale gallons. Also, $294,12 \times 3 = 882,36$ that for wine, whose square roots are 32,82 and 29,7. so by the

Sliding-rule for ale gallons. As 32,82 on D : 40 on C : : 32 and 26 on D : 38,03 and 25,1 on C, so twice 38,03 = 76,06 added to 25,1 gives 101,16 A. G.

If you would have wine gallons, use the gauge point 29,7, the factors .0009284 for A G. and ,0011333 for W G. are had by ex. 271 or by multiplying 359,05 by 0,2618 and 294,12 by 0,2618. See ex. 184 and 290.

Ex. 331. For the second variety, or that of the circular spindle.

Rule 60. From the sum of the squares of the length and head diameter, take the square of the bung diameter, the remainder multiplied by 14 times the square of the difference of the diameters is a dividend, and 35 times the square of the length, added to 5 times the square of the difference of the diameters, is a divisor, get the quotient and multiply the difference between it, and the sum of twice the square of the bung diameter and square of the head diameter, by the cask's length, and that product multiply by $\left\{\begin{array}{l} ,0009284 \\ ,0011333 \end{array}\right\}$ or divide by $\left\{\begin{array}{l} 1077,15 \\ 882,35 \end{array}\right\}$ for $\left\{\begin{array}{l} \text{ale} \\ \text{wine} \end{array}\right\}$ gallons. or by the

Sliding-rule. As $\left\{\begin{array}{l} 32,82 \text{ for A G.} \\ 29,7 \text{ for W G.} \end{array}\right\}$ on D is to the casks length on C so is the bung and head diameters, and their difference on D to 3 such numbers on C, that if to the second you add twice the first and from that sum take 0,3 of the third, there will leave the content nearly. All from theorem 95 &c. the diameters being 32 and 26, their difference will be 6. So per last rule,

1600 = ſq. 40 E d
676 = ſq. 32 B d
ſum 2276
ſub. 1024 = ſq. 32 BD
1252
504 = ſq. 6 × 14
631008 = the dividend

56000 = 35 times ſq. 40
add 180 = 5 times ſq. 6
56180 = diviſor
ſo 56180)631008
take 11,232 quotient
from 2724 ſq. 26 added to 2 ſq. 32
2712,768
40 = E L

Again for W. G.
108510,720
33331100,0 = ,00113 + invert.
108511
10851
3255
326
33
3
WG. = 122,979

108510,72
482900,0 = ,0009284 invert.
97660
2170
868
43
100,741 A. G.

Sliding-rule, for wine gallons. As 29,7 on D : 40 on C : : 32 and 26 and 6 on D : 46,42 and 30,64 and 1,63 on C, ſo 92,84 (twice 46,42) added to 30,64 gives 123,48, from which take 0,49 (= 0,3 of 1,63) leaves 122,99 wine gallons &c. for ale gallons.

Ex. 332. For variety third viz. that of a parabolic ſpindle, done by rule 56.

1024 } ſq. 32
1024 }
676 ſq. 26
2724 ſum
ſub. 14,4 = 0,4 × 36
2709,6
40 = length
108384,0
4829000,0 inverted factor
97546
2168
867
43
100,624 A. G.

Again, for W G.
108384,000
inv. factor 33331100,0
108384
10838
3252
325
33
3
W. G. = 122,835

Sliding-rule, for wine gallons. As 29,7 on D : 40 on C : : 32 and 26 and 6 (their difference) on D : 46,42 and 30,64 and 1,63 on C, ſo twice 46,42 added to 30,64 and the ſum leſſen'd by 0,4 of 1,63, gives at laſt 122,83 W. G. &c. for A. G.

Ex. 333. For the fourth variety, or that of an equilateral hyperbolic ſpindle.

Rule 61. To the ſq. of the length add the difference of the ſquares of the bung and head diameters; this multiplied by 14 times the ſquare of the difference of the diameters is a dividend, from 35 times the ſquare of the length take 5 times the ſquare of the difference of the diameters, the remainder is a diviſor, take the difference between the quotient of this diviſion and the ſum of twice the ſquare of the bung diameter added to the ſquare of the head diameter, and multiply it by the ſame factors for the ſame meaſure as in the laſt example, or divide by the ſame diviſors 1077,15, or 882,35, for wine. (Theo. 97)

```
      1600 = ſq. 40
      1024 = ſq. 32
      ----
      2624 ſum
ſub.   676 = ſq. 32
      ----
      1948
mul.   504 = 14 × ſq. 6
    ------
    981792 dividend
```

```
         56000 = 35 times ſq. 40
    take   180 = 5 times ſq. 6
         -----
         55820 diviſor
      ſo 55820)981792
    take      17,59 quotient
    from    2724 = twice ſq. 32
          ------     (added ſq. 26
         2706,41
              40 = length
         -------
         108256,4
inv. fact. 33331100,0 for W. G.
         --------
           108256
            10826
             3248
              325
               32
                3
         --------
         122,690 W. G.
         --------
&c. for A. G. 100,505
```

Sliding-rule, for ale. As 32,82 on D : 40 on C : : 32 and 26 and 6 on D : 38,03 and 25,1 and 1,34 on C, ſo twice 38,03 added to 25,1 and leſſened by $\frac{1}{2}$ of 1,34 gives (at laſt 100,49 A. G.

Ex. 334. For the 5th variety, viz. two = fruſtums of a parabolic conoid; here by

Rule 54. the diviſor for $\left\{\begin{array}{l}\text{ale}\\ \text{wine}\end{array}\right\}$ muſt be $\left\{\begin{array}{l}718,1\\ 588,24\end{array}\right\}$ viz. twice $\left\{\begin{array}{l}395,05\\ 294,12\end{array}\right\}$ ſo the factors will be ,0013925 and ,0017 and gauge points 26,8 and 24,25.

1024 ſq. 32
676 ſq. 26
ſum 1700
40
718,1)68000(94,69 A G.
1,22
115,6 W G.

Sliding-rule.

As $\left\{\begin{matrix} 26,8 \\ 24,25 \end{matrix}\right\}$ on D : 40 on C : : 32 and 26 on D : $\left[\begin{matrix} 57,1 \text{ and } 37.6 \\ 69,63 \text{ and } 45,97 \end{matrix}\right]$ on C, whoſe ſum is $\left[\begin{matrix} 94,7 \text{ A. G.} \\ 115,6 \text{ W. G.} \end{matrix}\right.$

Ex. 335. For the 6th variety, viz. two equal fruſtums of a cone. By rule 39.

58 = 32 + 26
58
3364
ſub. 832 = 32 × 26
2532
40 length
101280
4829000,0 inv. factor
91152
2026
810
41
94,029 A G.

A G. = 94,029
1.22
114,71538 W G.

Sliding-rule.

As $\left[\begin{matrix} 32,82 \text{ for A G.} \\ 29,7 \text{ for W G.} \end{matrix}\right]$ on D : the caſks length 40 on C : : 58 the ſum diameters and 6 their difference on D to two ſuch numbers on C, that if to $\frac{3}{4}$ the firſt you add $\frac{1}{4}$ the ſecond, you'll have the content.

336. Theſe are the rules ſet to the ſeveral forms of caſks, except rules 60 and 61, which are approximated as directed after theorem 97, yet very near true, each form is alſo wrought by the ſliding-rule according to the rule by the pen, (except variety 2d and 4th, and thoſe as near as can be done eaſily) a method much ſhorter and alſo truer than that practiſed by moſt authors on gauging, viz. by the mean diameters; but that nothing may be wanting in this ſection, take that method here.

To gauge a caſk by the mean diameter.

337. Firſt to find the mean diameters by the ſliding-rule (ſee ex. 257) on the line of inches, find the difference between the bung and head diameters, and againſt it on the line belonging to its variety, you have a number which added to the head diameter gives the mean diameter, ſo againſt 6 on the inches, you have 4,17 againſt ſpheroid, 3,78 againſt 2d variety and 3,39 againſt 3d variety which reſpectively added to 26 the head diameter gives 30,17 and 29,78 and 29,39, for the mean diameters of the ſpheroid, parabolic ſpindle and parabolic conoid. Otherwiſe, without the rule.

338. Multiply the difference between the head and bung diameters

by	,69	for the	1	variety.
	,68		2	
	,675		3	
	,67		4	
	,525		5	
	,51		6	

This product added to the head diameter gives the mean diameter, i. e. reduces the cask to a cylinder of the same length; hence when you have got this mean diameter work with it and the casks length as if you were gauging a cylinder, and you'll get the answer.

So in the 6 preceeding examples, the difference between the head and bung diameters is 6.

which mul tiplied by	,69	gives	4,14	this added to 26, the head diam. gives the mean diameter,	30,14	of the	1	variety.
	,68		4,08		30,08		2	
	,675		4,05		30,05		3	
	,67		4,02		30,02		4	
	,525		3,15		29,15		5	
	,51		3,06		29,06		6	

How the multipliers are found, see theorems 121, 122, 123, 124, &c. Now any of these mean diameters squared, and multiplied by the casks length, and that product divided by 359,05 gives its content in A G. &c. for W G.

Or by the Sliding-rule.

As the gauge A G. for ale or W G. for wine suppose A G. on D : 40 on C : :	30,14	on D :	101,2	on C nearly for the A G. in the	1	variety.
	30,08		100,7		2	
	30,05		100,6		3	
	30,02		100,5		4	
	29,15		94,7		5	
	29,06		94,0		6	

By comparing these with those 6 foregoing examples done by the true rules it appears, these contents found by the mean diameters are pretty near good, whence the mean diameters in this case, found by the constant factors 0,69 and 0,675, &c. are truer than those found as above by the sliding-rule.

338. Thus you have the art of gauging a full cask, if its form be known, but to do that will require more work than the gauging of it, as is plain by theorem 64, 65, and 66. Also tho' 6 varieties be fixt on, yet there may be more as appears by fig. 182. But the usual way to determine the casks form is by inspection, or as near as you think good by looking at it, and indeed is a better rule than the Cooper re-

gards in making caſks to be in ſuch forms; hence a caſk may come near ſome of theſe varieties by chance, for there is no other rule for it that I know of.

339. Theſe things conſidered, its evident that this method of caſk gauging is at beſt but gueſs work, and therefore to have a more general method, it will be beſt, both in regard of accuracy and readineſs in practice, to work by rule 43 which regards all forms alike, and to take the 4th dimenſion, or diameter, in the middle between head and bung. (See prob. 163.)

How to gauge any caſk without regarding its variety

Ex. 340 Let H B H d D d (fig 128) be ſome caſk whoſe length L E is 40, bung diameter B D 32, head diameter H d 26, and a diameter m G, taken in the middle between the head and bung be 30,4, all in inches, to find its content in ale and wine gallons. By the general rule 43.

1024 = ſq. of B D
676 = ſq. H d
3696,64 = ſq. twice M G

5396,64 ſum
40 = L E

215865,60
2464000,0 ſee ex. 295

86346
12952
863
43

A G. = 100,204

Again for wine gall.
215865,60
7665000,

107933
12951
1295
151

W G. = 122,330

Sliding-rule, Rule 49. As $\left\{ \begin{matrix} 46,4 \text{ for ale} \\ 42 \text{ for wine} \end{matrix} \right\}$ on D : 40 on C : : 32 and 26 and 60 8, (twice 30,4) on D : $\left\{ \begin{matrix} 19 \text{ and } 12,8 \text{ and } 68,4 \\ 23 \text{ and } 16,3 \text{ and } 83 \end{matrix} \right.$ $\left. \begin{matrix} \text{on C} \\ \text{on C} \end{matrix} \right\}$ whoſe ſum is $\left\{ \begin{matrix} 100,2 \text{ ale gallon.} \\ 122,3 \text{ wine gallons.} \end{matrix} \right.$

This is the only method to be followed by any caſk gauger, whether he take it for eaſe, accuracy, or expedition, I have ſo much every where praiſed this rule 43 in this ſection that I think it needleſs to ſay any more about it.

To find the ullage of any ſtanding or lying caſk, &c.

341. If a caſk ſtand upright upon one of its heads, it is called a ſtanding caſk, but if it lye with its axis parallel to the horizon, it is called a lying caſk, now what I ſhall do in this place, will be to find what quantity of liquor is in a caſk part full, in each caſe.

Ex. 342. Let H H d d (fig. 183) be a cask of the 3d variety of the same dimensions with those aforesaid, standing upright and filled with liquor to q z, wet inches O E 28, or dry inches L O = 12, diameter q z at the liquors surface 31,05, diameter m G in the middle between O and L 29,09, to find the ullage.

This is plain by the figure, that it is but to gauge a frustum d t u H E of the cask if it be not half full, viz. only full to t u, or two unequal frustums B D d H and B D q z, if it be above half full, viz. full to O, whence, if the cask is above half full it is easiest to find the content of the empty part q d L H m z, and take it from the casks content so you'll have the content of the full part d q O z H, but if the cask is not half full, it is soonest done by taking the content of that part d t u H E. Therefore, working (as in ex. 340) with q z, H d, m G and L O, as bung head, middle diameters and length you'll find the content of the empty part q d L H z to be 27,949 A G. which taken from 100,6 the cask's content leaves 72,651 A G. in the cask, very near let the variety be what it will.

To work this example the old way by the lines N and S S on the sliding-rule, first find the content 100,6 of the full cask, by its variety and mean diameter, then as 40 the cask's length on N is to the radius of segments 100 on S S, so is 28 the wet inches on N to a reserved number 71,6 on SS. Again, As 100 on B is to the whole content 100,6 on A so is the reserved number 71,6 on B to 72 A G. on A, the liquor in the cask, nearly $\frac{3}{4}$ of a gallon too little as appears by that had by the pen before.

Note. If you use dry inches you'll have the empty content.

Ex. 343. Let H d d H (fig. 184) represent a lying cask full to O, bung diameter B D 32, head H d 26, Length L E 40 as before; dry inches B O = 12 m G a diameter in the middle between the head and bung 30,5, all given in inches to find the ullage or measure of the empty part B L O Q.

Rule 43. To the sum of the areas of the segments at the bung and head, add 4 times the area of the middle diameters segment, multiply that sum by the length L E = L Q and divide by 1692 (viz 6 times 282) or multiply by ,00591 for A G. the same with ex. 291. It is plain by fig. 128 or by fig. 184, that if from the dry inches B O 12, you take H C 3 half the difference between the bung and head diameters there will leave Q H 9 the dry inches at the head, in like manner you get m n 11,25 and Q H 9 being had, annex 3 cyphers to each and divide it by its respective diameter, so you will have 3 versed-sines, with which enter the table under V S, and write out the 3 numbers against

them under ſegments, (ſee rule 19) and each of theſe 3 numbers ſo found, multiply'd by the ſquare of its diameter, you'll have the three aforeſaid areas. Thus

32	12,000	375	againſt which	269013
30,5	11,250	$368\frac{5}{8}$	in the table	263052
26	9,000	$346\frac{1}{8}$	ſtands	241327

Then this mult. by	1024 = ſq. 32	gives the	275.469
the ſq. of its diam.	3721 = ſq. 30,5	area of the	978,816
(viz.) by	676 = ſq. 26	ſeg. =	163,137
		ſum	1417,422
		len.	40
		1692)	56696,880
		quotient	33,5078 A G.

Now this 33,5078 the meaſure of the empty part L B Q, taken from 100,624 the full caſks content leaves 67,116 A G. remaining in the caſk, if the ſame things were given and the wet inches 12, the work would be the very ſame as before, and you would have 33,5078 A G. for the content of the full part.

344. This method of ullaging a ſtanding, or a lying caſk by the middle diameter may always be uſed where great exactneſs is required, but in common practice, as the line S S, on the ſliding-rule ſerved to ullage a ſtanding caſk, So the line S L on that rule will nearly ullage a lying one. Thus, as the bung diameter 32 on N is to 100, on S L ſo is { wet inches 20 / dry inches 12 } on N to { 67,2 / 32,8 } the reſerved number on S L. Then as 100 on B is to 100,6 the caſks content on A ſo is the reſerved number { 67,2 / 32,8 } on B, to the ullage on A viz. { 67,6 A G. in the caſk / 33,0 A G. drawn out of it } erring ſomething above $\frac{1}{2}$ a gallon as per laſt method.

To find the ullage of a lying caſk by its mean diameter.

345. By ex. 338, find the mean diameter ſuitable to the caſks form, then to twice the wet or dry inches, viz. the leaſt of the two, or to either of them when equal (viz. when the caſk is juſt half full) add the mean diameter, and from that ſum take the bung diameter, half the remainder is the height of a mean ſegment, (ſee rule 19) which divide by the mean diameter and look for the quotient under V. S. write out the number againſt it under ſeg. in the table of ſegments, which multiply by the ſquare of the mean diameter, and that product by the caſks length, and then by { ,003546 / ,004329 } or divide by { 282 for ale / 231 for wine } ſo will you have the ullage required, that is, what is drawn out if you wrought with the dry inches (they being leſs) or what remains in the

caſk if the wet inches be uſed (they being leaſt); for if a mean diameter reduces a caſk to a cylinder this method alſo reduces any ſegment thereof to the ſegment of a cylinder, &c.

Ex. 346. Let the dimenſions be the ſame as in ex. 333, and the caſk of the third variety; then by ex 338, its mean diameter is 30,05 which added to 24 (twice 12 the dry inches) gives 54,05 from which taking 32 leaves 22,05 half of which is 11,025, this divided by 30,05 the mean diameter quotes ,367 nearly, againſt ,367 under V S. ſtands ,261284, which multiplyed by 903 the ſq. of 30,05 the mean diameter, gives 235,9894 and this again by the length 40 gives 9439,5760, this divided by 282 quotes 33,47 A G. drawn out, or 33,47 A G. in, if the wet inches be 12. Hence, by the true method, viz. by the middle area, the ullages is 33,5079. By the line S L on the ſliding-rule it is 33; and by the mean diameter 33,47 which is nearer good than that by the ſaid line S L the common way of ullaging, and is alſo eaſier for it does not require the caſks content.

347. This laſt method of ullaging may alſo be done by the new line A L (ſee ex. 256) on the ſliding-rule. Thus, As the mean diameter 30,05 on N is to 18,95 on A L ſo is 11,025 the mean dry inches on N to 33,9 on A L, the reſerved number, Then, as that reſerved number 33,9 on D is to 40 the caſks length on C ſo is 30,05 the mean diameter on D to 33.47 fere, on C the ullage required.

349. Let things be the ſame as in ex. 346, and the caſk of variety 6, the mean diameter of the caſk is (by ex. 338) 29,06, which added to 24 (twice 12) is 53.06, from which take 32 the bung diameter and there leaves 21,06 half of which is 10.53, the mean wet, or dry inches. Then, as 29,06 on N : 18,95 on A L : : 10.53 on N : 33.14 on A L. Again, As 33,14 on D : 40 on C : : 29,06 on D : 30,7 A G on C, the liquor in the caſk if 12 was the wet inches, or what is drawn out if 12 was the dry inches.

350. If you ſuppoſe the dimenſions in ſect. 6 to be taken by a rule divided into (6,558 inches to a part, viz. the cube root of 282) equal parts, then the anſwer to every figure or example in that ſection will be in ale gallons. Thus. if the length of a ſquare priſm be 15 times 6,558 inches, and each ſide of its ſquare baſe 7,2 times 6,558 inches, this veſſel (by ex. 161) will hold 777,6 ale gallons. Alſo, if the length of a cylinder be 8 times 6,558 inches, and its diameter 2,1 times 6,558 inches, this veſſel (by ex. 164) holds 27,708912 A G. and ſo for any other ſolid or veſſel in that ſection. Likewiſe, if you take the cube roots of 231 and of 2150,42, they will be dimenſions for wine gallons and for malt buſhels, it is an eaſy method to gauge by ſuch a rule, which may be divided into gallons and decimal parts of a gallon, as I judge is very eaſy to underſtand, by which you'll in effect, have gauging in this book twice over.

A TABLE of the areas of segments.

V.S.	Seg. Area.	V.S.	Seg. Area.	V.S.	Seg. Area.	V.S.	Seg. Area.
001	000042	045	012554	89	034441	133	062026
2	000119	46	012971	90	035011	134	062707
3	000219	47	013392	91	035585	135	063389
4	000337	48	013818	92	036162	136	064074
5	000470	49	014247	93	036741	137	064760
6	000618	50	014681	94	037323	138	065449
7	000779	51	015119	95	038909	139	066140
8	000951	52	015561	96	038446	140	066833
9	001135	53	016007	97	039087	141	067528
010	001329	54	016457	98	039680	142	068225
11	001533	55	016911	99	040276	143	068924
12	001746	56	017369	100	040875	144	069625
13	001968	57	017831	101	041476	145	070328
14	002199	58	018296	102	042080	146	071033
15	002438	59	018766	103	042687	147	071741
16	002685	60	019239	104	043296	148	072450
17	002940	61	019716	105	043908	149	073161
18	003202	62	020196	106	044522	150	073874
19	003471	63	020689	107	045139	151	074589
20	003748	64	021168	108	045759	152	075306
21	004031	65	021659	109	046381	153	076026
22	004322	66	022154	110	047005	154	076749
23	004618	67	022652	111	047632	155	077469
24	004921	68	023154	112	048262	156	078194
25	005230	69	023659	113	048894	157	078921
26	005546	70	024168	114	049528	158	079649
27	005867	71	024680	115	050165	159	080380
28	006194	72	025195	116	050804	160	081112
29	006527	73	025714	117	051446	161	081846
30	006865	74	026236	118	052092	162	082524
31	007209	75	026761	119	052736	163	083320
32	007558	76	027289	120	053385	164	084059
33	007913	77	027821	121	054036	165	084801
34	008273	78	028356	122	054689	166	085544
35	008638	79	028894	123	055345	167	086289
36	009008	80	029435	124	056003	168	087036
37	009383	81	029979	125	056663	169	087785
38	009763	82	030526	126	057326	170	088535
39	010148	83	031076	127	057991	171	089287
40	010537	84	031629	128	058658	172	090041
41	010931	85	032186	129	059327	173	090797
42	011330	86	032745	130	059999	174	091554
43	011734	87	033307	131	060672	175	092313
44	012142	88	033872	132	061348	176	093074

V. S.	Seg. Area.	V. S.	Seg. Area.	V. S.	Seg. Area.	V. S.	Seg. Area.
177	093836	223	130605	269	170202	315	212011
178	094601	224	131438	270	171089	316	212940
179	095366	225	132272	271	171978	317	213871
180	096134	226	133108	272	172867	318	214802
181	096903	227	133945	273	173758	319	215733
182	097674	228	134784	274	174649	320	216666
183	098447	229	135624	275	175542	321	217599
184	099221	230	136465	276	176435	322	218533
185	099990	231	137307	277	177330	323	219468
186	100774	232	138150	278	178225	324	220404
187	101553	233	138995	279	179122	325	221401
188	102334	234	139841	280	180019	326	222277
189	103116	235	140688	281	180918	327	223215
190	103900	236	141537	282	181817	328	224154
191	104685	237	142387	283	182718	329	225093
192	105472	238	143238	284	183619	330	226033
193	106261	239	144099	285	184521	331	226974
194	107051	240	144944	286	185425	332	227915
195	107842	241	145799	287	186329	333	228858
196	108636	242	146655	288	187234	334	229801
197	109430	243	147512	289	188140	335	230644
198	110226	244	148371	290	189047	336	231389
199	111024	245	149230	291	189955	337	232034
200	111823	246	150091	292	190864	338	232579
201	112624	247	150953	293	191775	339	233026
202	113426	248	151816	294	192684	340	235473
203	114230	249	152680	295	193596	341	236421
204	115035	250	153546	296	194509	342	237369
205	115842	251	154412	297	195422	343	238318
206	116650	252	155280	298	196337	344	239268
207	117460	253	156149	299	197252	345	240218
208	118271	254	157019	300	198168	346	241169
209	119000	255	157890	301	199085	347	242121
210	119897	256	158762	302	200921	348	243074
211	120712	257	159636	303	201232	349	244026
212	121529	258	160510	304	201841	350	244980
213	122347	259	161386	305	202761	351	245934
214	123167	260	162263	306	203681	352	246889
215	123988	261	163140	307	204605	353	247845
216	124810	262	164019	308	205527	354	248801
217	125634	263	164899	309	206451	355	249757
218	126459	264	165780	310	207376	356	250715
219	127285	265	166663	311	208301	357	251673
220	128113	266	167546	312	209227	358	252631
221	128942	267	168430	313	210154	359	253590
222	129773	268	169315	314	211082	360	254550

V. S.	Seg. Area.	V. S.	Seg. Area.	V. S.	Seg. Area.	V. S.	Seg Area.
361	255510	396	289453	431	323918	466	358725
362	256471	397	290432	432	324909	467	359723
363	257433	398	291411	433	325900	468	360721
364	258395	399	292390	434	326892	469	361719
365	259357	400	293369	435	327882	470	362717
366	260320	401	294349	436	328874	471	363715
367	261284	402	295330	437	329866	472	364713
368	262248	403	296311	438	330858	473	365712
369	263213	404	297292	439	331850	474	366710
370	264178	405	298273	440	332843	475	367709
371	265144	406	299255	441	333836	476	368708
372	266111	407	300238	442	334829	477	369707
373	267078	408	301220	443	335822	478	370706
374	268045	409	302203	444	336816	479	371705
375	269013	410	303187	445	337810	480	372794
376	269982	411	304171	446	338804	481	373703
377	270951	412	305155	447	339798	482	374702
378	271920	413	306138	448	340793	483	375702
379	272890	414	307125	449	341787	484	376702
380	273861	415	308110	450	342782	485	377701
381	274832	416	309095	451	343777	486	378701
382	275803	417	310081	452	344772	487	379700
383	276775	418	311068	453	345768	488	380700
384	277748	419	312054	454	346764	489	381699
385	278721	420	313041	455	347759	490	382699
386	279694	421	314029	456	348755	491	383699
387	280668	422	315016	457	349752	492	384699
388	281642	423	316004	458	350748	493	385699
389	282617	424	316992	459	351745	494	386699
390	283592	425	317981	460	352751	495	387699
391	284568	426	318970	461	353768	496	388699
392	285544	427	319959	462	354795	497	389699
393	286521	428	320948	463	355732	498	390699
394	287498	429	321938	464	356730	499	391699
395	288476	430	322928	465	357727	500	392699

Section IX.

Practical Questions.

It is to be observed, that the 40 following questions belong to mensurations, gauging, &c,

Question 1. How many hewn stones of a rectangular form, each 3 foot long and $2\frac{1}{2}$ feet broad., will pave a walk 40 yards long and 3 yards broad?

Divide 1080 feet the area of the walk by 7.5 feet the area of one stone, and the quotient 144 is the number of stones required.

Question 2. How many panes of glass each 7 inches square will suffice 4 windows, each 5 foot high, and 3 feet 7 inches broad?

The area 10320 inches of all the windows, divided by 49 inches, the area of one pane, quotes $210\frac{10}{49}$ panes answer.

Question 3. How many rafts each $2\frac{1}{2}$ inches broad and $1\frac{1}{2}$ inches thick, can be sawn out of a piece of equal squared timber, the length of each end being $17\frac{1}{2}$ inches and breadth 10 inches?

The area of each base of the timber is $17,5 \times 10 = 175$ inches, which divided by $2,5 \times 1,5 = 3.75$ inches the area of one end of a raft, gives 46,666 rafts for the answer.

Question 4. There is a room whose circuit is 20 yards, and height 4 yards to be hung about with tapestry 2 foot wide, all except a door case whose height is 8 foot and breadth 4 feet, what tapestry will do it?

From $60 \times 12 = 720$ feet the area of the room, take $8 \times 4 = 32$, the area of the door case, the remainder 688 feet divided by 2 feet the breadth of the tapestry quotes 344 or $114\frac{2}{3}$ yards, for the answer.

Question 5. How many bricks each 9 inches long $4\frac{1}{2}$ inches broad and 3 inches thick, must be taken to build a wall 100 feet long 20 feet high, and one foot thick?

Here $100 \times 20 \times 1 = 2000$ feet the walls solidity, which multiplyed by 1728 inches in a solid foot, gives 3456000 the walls solidity in inches, which divided by $9 \times 3 \times 4.5 = 121,5$ inches, the solidity of one brick quotes 2844,444 bricks answer.

Question 6. If a piece of round timber be 20 feet solid, how many solid feet will it be when hewn to square timber?

If the diameter of a circle be 1 its area is 0,7854 and 0,5 = half the diameter squared, and then doubled, is the area of the greatest inscrib'd square, therefore, As ,7854 : ,5 : : 20 : 12,732 solid feet answer.

Question 7. If a cellar be 18,75 feet wide, 10 feet long, and 5,2 feet deep, how many floors of earth are therein at 324 solid feet to one floor.

Here $18,75 \times 10 \times 5,2 = 975$ solid feet in the floor, which divided by 324 the solid feet in a floor of earth, or digging, quotes 3 floors 3 feet, the answer.

Question 8. If the solidity of a cylinder be 2150,42, and its height 10, what is the diameter of its base?

The solidity divided by the height quotes 215,042, which again, divided by 0,7854 quotes 273,67, whose square root is 16,5 the diameter required.

Question 9. If the length of a ship's keel be 44 feet, depth of the hold 9 feet, and mid-ship beam 20 feet, what must these dimensions be in another ship of the same mould to carry a double burthen.

By theorem 31. If you cube any of these dimensions, the cube root of the double thereof, will be the like dimension of a ship of a double size, so 44 cubed is 85184, and doubled is 170368 whose cube root is 55,44 feet for the keel sought. Thus having found any one of the dimensions, the rest may be had by the rule of three. Thus, as 44 :

$$55,44 :: \left\{ \begin{array}{l} 20 : 25,22 \text{ mid-ship beam.} \\ 9 : 11,34 \text{ depth of the holds.} \end{array} \right\} \text{ By the Sliding-rule.}$$

Because the burthens are as 1 to 2, it will be as 1 on E is to

$$\left\{ \begin{array}{c} 44 \\ 20 \\ 9 \end{array} \right\} \text{ on D so is 2 on E to } \left\{ \begin{array}{c} 55,44 \\ 25,22 \\ 11,34 \end{array} \right\} \text{ on D, the same as before.}$$

Question 10. If the length of a ship's keel be 80 feet, mid-ship 32 feet, and depth of the hold 14,1 feet, what is her tunnage or burthen in tuns. See Question 169.

The usual way to guage a ship is to divide the product of these 3 dimensions in feet, by 95, or by 100, if there be allowance made for guns &c. Therefore, by the sliding-rule. As 9,78 (the square root of 95) on D is to 14,1 on C so is 50,05 (a mean proportional between 80 and 32) on D to 380 fere, on C the answer. But if the divisor be 100, or guage point 10, the burthen will be 360,96 tuns.

Question 11. If the axis of a globe be 4 inches and it's weight 4 lb, what will be the weight of another globe of the same metal whose axis is 8 inches.

As 64 the cube of the axis is to 4 its weight, so is 512 cube of 8, the cube of the like part of any other like solid to 32 lb. its weight, of the same metal.

Question 12. Whether is half a foot ſquare, or half a ſquare foot greater? Half a ſquare foot is half of 144 ſquare inches = 72, but half a foot ſquare is but 36 inches, the ſquare of 6.

Question 13. What is the difference between half a foot ſolid and half a ſolid foot?

Half a ſolid foot is half of 1728 ſolid inches = 864, but half a foot ſolid is 216 being the cube of 6 inches or half a foot.

Question 14. If one fathom of a cable rope weigh 17 lb. when it is 10 inches about, what would it weigh if it were 18 inches circumference?

As 100 inches the ſquare of the periphery of any cylinder's baſe is to 17 lb. its weight or ſolidity, ſo is 324 the ſquare of any other cylinder's baſe's periphery of the matter or height, to 55,08 lb its weight or ſolidity. See theorem 37.

Question 15. If an oak cheſt be 8 ſolid feet when meaſured on the outſide and but 7 ſolid feet when meaſured in the inſide, what is its weight A ſolid inch of oak weighing 0,537 parts of an ounce.

Since ſolidity is as weight; therefore, As one inch is to its weight ,537 oz. ſo is 1728 inches (the difference between 8 and 7 feet) to 928 oz. ſere, the anſwer.

Question 16. What length of a rope will be fit to tye to a cow's tail, the other end being fixt in the earth, that ſhe may graſs juſt an acre or 4840 ſquare yards, allowing the cow's length 4 yards.

The length of the rope and cow muſt be the radius of a circle whoſe area muſt be 4840. Therefore 4840 divided by 0,7854 is = 6161,09 the ſquare of the whole diameter, whoſe ſquare root is 78,49 half whereof is 39,245 yards the anſwer. See rule 15.

Question 17. What muſt be the dimenſions of a cubical box, to hold 200 oranges of a globular form, each $2\frac{1}{2}$ inches diameter?

If the oranges be laid in rows upon one another, each will take as much room as a cube would do, whoſe ſide is $2\frac{1}{2}$ inches; therefore, the cube of 2,5 = 15,625 multiplied by 200 gives 3125o for the ſolidity of the 2000 cubes, or that of the box. So the cube root of 3125o is = 15,5 ſere, a ſide of the box.

Question 18. If a plank 14 feet long, $1\frac{1}{2}$ thick, and half a foot broad, can be ſold for 8d a foot running meaſure, 7d a foot ſuperficial meaſure, and 10d a foot ſolid meaſure; which of theſe ways muſt it be ſold to make the moſt money?

Its area is $14 \times 1,5 = 21$ feet, its ſolidity is $14 \times 1,5 \times 0,5 = 10,5$ feet, its length 14 feet; ſo, 14 times 8d is = 112d, its price at running meaſure, and 10,5 times 10d is = 105d its price at ſolid meaſure; alſo 21 times 7d is = 147 d value at flat meaſure which is the beſt way to ſell it.

Question 19. If a board be 10 feet long, 8 inches broad at the greater end, and 6 inches broad at the lesser end, how much in length at the lesser end will make one foot.

Because the ends are given in inches, and length in feet, we'll call the foot to be cut off 12 inches (see ex. 83) then to (48) twice the product of 12 the part to be cut off and 2 the difference of the breadths 8 and 6, add (360) the product of the length 10 feet and square of the lesser breadth 6 inches, multiply the sum (408) by the length 10 feet, from the square root of (4080) that product take the product of the lesser breadth 6 inches, and the length 10 feet, (60) and the remainder 3,8 divided by 2 the difference of the two breadths gives 1,9 for the answer. See theorem 125.

Question 20. If a piece of square tapering timber be 10 feet long, 9 inches square at greater base, and 6 inches square at the lesser base, how much in length from the lesser end will make a solid foot. See theorem 126.

First, 9 inches = ,75 F. and 6 inches = ,5 F. and their difference 3 inches = ,25. Then to (1,25 F.) the product of the height 10 F. and the cube of a side (0,5 F.) of the lesser base, add (,75) 3 times the product of 1 F. the part to be cut off and ,25 F. the difference between a side of each base multiply (2,00) the sum by 100 F. the square of the height, and from (5,849) the cube root of that product (200) take the product (5) of the lesser breadth ,5 F. and height 10 F. the remainder 0,849 divid d by ,25 F. the difference of the two bases quotes 3,4 fere for the feet in length require l.

Note. If it be the frustum of a cone you may multiply the part to be cut off by $\frac{1}{3}$ of 0,7854, and then work with that product instead of the said part, and the diameters as with the sides before: Also, if any part was to be cut off from the greater base, take the content of that part from the content of the frustum, and work to cut the remainder from the lesser base as before.

Question 21. If a board be 24 inches broad at the greater end, 8 inches broad at the lesser end, and 20 feet long, where must it be cut o that a foot in length may contain 156 inches area?

The content 156 divided by 12 the length quotes 13 inches for the mean, or middle breadth of the piece to be cut out, (see theorem 26) then (by theorem 9) As 16 (24 — 8) is to 20 the length, so is 5 (13 — 8) to 6. 24 feet the distance between the middle of the piece to be cut out and the lesser end.

Question 22. A hath a piece of square tapering timber 24 inches square at the greater end, 6 inches square at the lesser end, and 60 feet

long; B bids him 12d a foot running meaſure, C offers him 18d a foot ſolid; how muſt he cut it between B and C to make the moſt of it to himſelf?

As the value of a foot in length is $= \frac{2}{3}$ of one ſolid, it is evident, that a ſide of the dividing ſection muſt be = the ſide of a ſquare priſm, wherefore a foot in length will be = to $\frac{2}{3}$ of a foot ſolid: but $\frac{2}{3}$ of 1728 the inches in a ſolid foot is 1152, which divided by 12 inches or a foot in length, quotes 96 inches the area of the dividing ſection, whoſe ſq. root is near = 9,8, then, As 18 inches (24—6) is to 60 feet length, ſo is 3,8 inches (9,8 — 6) to $12\frac{2}{3}$ feet from the leſſer end where the tree is to be cut, and B to have the ſmaller end, C the thicker.

Queſtion 23. Things being the ſame as in the laſt queſtion, ſuppoſe the ſolid foot at 12 d and the foot in length at 18 d, where muſt it be cut to make moſt?

Here, its plain that if ſuch a ſolid foot be cut out of the tree (by queſtion 21) as that its length be $\frac{2}{3}$ of a foot or 8 inches, the value of this foot ſolid will be the ſame in both meaſures, conſequently, if the tree be cut thro' the middle of this piece it will anſwer the queſtion. So $1728 \div 8 = 216$, whoſe ſquare root is 14,7 fere = a ſide of the dividing ſection; then, As 18 (24 — 6) is to 60 feet ſo is 8,7 inches, (14,7 — 6) to 29 feet length running meaſure from the leſſer end to go at 18d a foot.

Queſtion 24. Whether will ſmall, or thick round timber waſte more in ſquaring.

By queſt. 6. Any piece of round timber is to the ſame when ſquared, as ,7854 to ,5, ſo there is no difference, i. e. the waſte is as the thickneſs.

Queſtion 25. If a tree girt 22 inches with a rope of one inch diameter, what is its true girt?

In girding timber &c. the upper ſide of the cord is made to meet, then being ſtretched on a rule, its plain the girt of the tree is taken to be what this upper ſide meaſures too, now if this upper ſide be 22 inches periphery it's diameter will (by rule 17) be 7 inches, but the rope being 1 inch thick, the diameter of the tree or underſide of the rope will be but 5) 7—2) inches; therefore, as 7 : 22 : : 5 : $15\frac{5}{7}$ inches, the true girth of the tree. Hence appears the neceſſity of girding with a ſmall cord.

Queſtion 26. If D L (fig. 185) be the length of an egg 4 inches, d D = 1 inch, the periphery of the greateſt circle G H 10 inches, a periphery taken at e in the middle between d and D = 7 inches, and one taken at a or m m, in the middle between n and L = 6 inches, what is the ſolidity? (See rule 43.)

100 = ſq. H G	100 = ſq. H G	58,1940
196 = 4 □ nn	144 = □ 2 m m	23,5320
296 ſum	244	6)81,7260
1 = length d D	3 = d L	13,621 in. ſolid.
296	732	
,0795 factor	,0795 factor	
23,5320	58,1940	

Queſtion 27. Required the ſolid and ſuperficial content of an elliptical ring, whole diameters taken in the inſide, are 28 and 38 inches, and thickneſs of metal in the ring 2 inches diameter, the ſaid ring being cylindrical.

Here 38 and 28 each added to 2, the thickneſs of metal in the ring, gives 40 and 30 for the two diameters of the oval paſſing thro' the ring's middle, whoſe periphery, (by rule 22) is 110 for the mean length of the ring; then □ 2, viz. $4 \times 110 \times 0{,}7854 = 172{,}788$ the ſolidity in inches, and $3{,}1416 \times 2 \times 1100 = 691{,}152$ inches, the ſuperficial content.

Queſtion 28. If the walls of a building be 20 yards about on the outſide, 16 yards about in the middle, 5 yards high, and 0,5 yards thick, what is the ſolid content of theſe walls? (See theorem 26.)

Here, half the ſum of 20 and 16 is 18 y. the circuit of the wall if taken in the middle; ſo $18 \times 5 \times 0{,}5 = 45$ ſolid yards, the anſwer.

Queſtion 29. Required the axis of the greateſt cylinder, that can be made of a given diameter 20 and diagonal 30; or, of the greateſt cone under a given ſlant length. (See theorem 146.)

The required axis is $= 17{,}31$, the given diameter or ſlant ſide 30, multiplied by 0,577 the ſquare root of $\frac{1}{3}$.

Queſtion 30. Required the axis of the greateſt cone that can be cut out of a globe or ſpheroid, whoſe axis is 30. (See theorem 145.)

This, as in the laſt queſtion, is found $= 17{,}31$. But the axis of the greateſt inſcribed cylinder will be $= 20$, $= \frac{2}{3}$ of 30, viz. 10 on each ſide of the globe or ſpheroid's center.

Queſtion 31. If a rectangular piece of ground is to be £2 for every chain in length, and £3 for every chain in breadth, Quere, the length and breadth, ſo as moſt land poſſible may be had for £40.

To give a general ſolution to all queſtions of this kind, let p = £2, q = £3, s = £40, e = one of the dimenſions, viz. either length or breadth, and a = the other of them; then $a\,e = A$ the area and $p\,e + q\,a = s$ the money to be paid for that area, and by tranſpoſition, &c. $\frac{s - p\,e}{q} = a$; whence, $A = a\,e = \frac{e\,s - e\,e\,p}{q} = $ m a max; ſo, (by

art. 221, prob. 191) $\frac{s-2ep}{q}=0$, hence, $e=\frac{s}{2p}=\frac{40}{4}=10$, and $a=\frac{s-pe}{q}=\frac{40-20}{3}=\frac{20}{3}=6\frac{2}{3}$, then $6\frac{2}{3} \times 10 = 6{,}6\frac{2}{3}$ acres the anſwer.

Queſt. 32. Required the ſolidity of the greateſt cylinder that can be cut out of a ſquare pyramid, whoſe axis is 60, and a ſide of its baſe 30.

Firſt, (by prob. 191) the pyramid muſt be cut at $\frac{1}{3}$ of its axis, ſo the cylinder's height will be $\frac{1}{3}$ of $60=20$; then, as $60 : 30 :: 40$ viz. ($\frac{2}{3}$ of 60) : 20, a ſide of the baſe of the pyramid cut off, which is alſo = the diameter of the cylinder's baſe. So $20 \times 20 \times 0{,}7854 \times 20 = 6283{,}2$ anſwer.

Queſtion 33. A weaver's beam 24 inches in circumference, on which is 95 rounds of cloth $\frac{1}{22}$ of an inch thick, what is the length of the web?

Firſt, As $22 : 7 :: 24 : \frac{84}{11}$ the diameter of the beam and $\frac{1}{22} \times d$ twice 95 is $=\frac{95}{11}$, twice the thickneſs of the ring of cloth, ſo $\frac{84}{11}+\frac{95}{11}=\frac{179}{11}$ the diameter of the beam and web together. Then from $\frac{32041}{121}$ the ſquare ($\frac{179}{11}$) this diameter, take $\frac{7056}{121}$ the ſquare of ($\frac{84}{11}$) the beams diameter, and the remainder $\frac{24985}{121}$ multiplied by 0,7854 gives the area of the end of the ring of cloth, which area divided by $\frac{1}{22}$ (viz. $\times d$ by 22) gives the length of the web in inches, which divided by 36, quotes 99,02 yards the length of the web. See rule 35.

Queſtion 34. What length of wire will come out,
One fourth of an inch about,
From braſs in meaſure juſt a foot;
Pray, ſir, try if you can do't.

That is, out of a ſolid foot of braſs, what length of wire may be drawn that is 0,25 inches circumference? (See rule 36.)

Here, $0{,}25 \times 0{,}25 \times 07958 = {,}004973$, the area of that circle whoſe periphery is $\frac{1}{4}$ of an inch. Then ,004973)1728,000000(347468 inches, length, which divided by 36, and that quotient by 1760 will ſhew it in miles.

Queſtion 35. If the perimeter of a circle, trigon, and ſquare be each equal to unity, which of them is greateſt?

$$\left.\begin{array}{l} 1 \times {,}07958 = {,}07958 \\ \frac{1}{3} \times \frac{1}{3} \times {,}43303 = {,}04811 \\ \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = {,}0625 = {,}0625 \end{array}\right\} \begin{array}{c} \text{area} \\ \text{of} \\ \text{the} \end{array} \left\{\begin{array}{l} \text{circle} \\ \text{trigon} \\ \text{ſquare} \end{array}\right\} \begin{array}{l} \text{by which it appears that} \\ \text{of all ſuperficial figures} \\ \text{contain'd under the ſame} \end{array}$$

perimeter, the circle is the greateſt.

Queſtion 36. A globe, a cube a cylinder,
All three in ſurface, equal are,
In ſolidity, what do they diff'r.

Suppose the surface of each to be 3,1416, then 0,5236 = the solidity, and 3,1416 ÷ 4,7124, and the square root of that quotient taken gives 0,821, the diameter and height of the cylinder which cubed and multiplied by 0,7854 gives 0,42985, the cylinders solidity. Also, 3,1416 ÷ 6 = 0,5236 the square of a side of the cube, whose square root 0,723 being cubed is 0,3786, the cube's solidity. Hence, of all solids under the same superficies, the globe is the greatest.

Question 37. If I take an angle of 50°, with a semicircle, which does not stand level, but makes an angle with the horizon of 33° 45′ what is the true angle of observation.

Let B C (fig. 98) be the edge of the instrument, making an angle ABC = 33° 45′ with the horizon A B, then its plain, while the point of the index moves a slope from B to C, it would move horizontally from B to A, so if B C subtend the observed angle of 50°, B A will subtend the true or required angle, or A C if it be an angle of altitude; so (by axiom 2) As radius : 50° : : co-sine inclination 50° 15 : 41° the true $\angle$ of observation, or : : sine $\angle$ inclination 33° 45′ : 27,6° the true $\angle$, if the observation was for an altitude. Hence appears the necessity of having your instrument level if you are to take $\angle$'s for distances, or $\perp$ if you are to take altitudes.

Question 38. A whin, a thorn, a sheaf of corn,
Each to be measured are;
Come, let us mind, if we can find,
A rule such things to clear.

The usual way to measure irregular solids, such as craggy stone, lumps of metal, bushes of shrubs, &c. is to fill a vessel with water, and then put in the solid, suppose 5 ale gallons of water to run over the top of the vessel, then the content of this immersed solid is 5 times 282 = 1410 solid inches; or, if you take out the solid and measure the empty part of the vessel, it will give the same content.

Question 38. If a tub ten inches deep and no more,
Hold in ale gallons just half a score,
What must the two diameters be,
To be in the ratio five to three?

This vessel being supposed a conical frustum, find (128,284 inches) the content of a conical frustum whose depth (10 inches) and proportion of diameters (3 and 5) are the same with those given in the question, then say, as this solidity (128,284) is to the given solidity (2820 inches) so is (9) the square of (3) the lesser ratio (3) to (190,05) the square of the lesser diameter, or so is (25) the square of the greater diameter, whose square root is the diameter sought, to the square root of 190.05 is 13,9 inches the lesser diameter, and as 3 : 5 : : 13,9 : 23,1 the greater diameter.

Question 40. A cylinder its ſaid, there is to be made,
To take the leaſt wood that can be;
Pray let us ſee, what dimenſions muſt be,
To hold a gallon of brandy.

Divide (462) twice the given ſolidity by ,7854 and (8,38 inches) the cube root of that quotient is the diameter, (ſee theorem 147) by which and queſtion 8 the height is found. (4,2 inches)

The 10 following queſtions ſhew how the contents of bodies may be had by knowing their centers of gravity, and the contrary, ſee theorem 190.

Definition. If in any ſide of any plane or ſurface, we find its center of gravity, and ſuppoſe the plane to turn round, ſo as this ſide deſcribe a circle, as being the radius thereof; then a line perpendicular to the radius on the point on which it turned, is called the axis of rotation, and the diſtance between the ſaid point and the center of gravity, is called the radius of gravity. Then to find the content of ſuch bodies, this is the

Rule 62. Multiply the area of the generating plane, the radius of gravity, and 6,2832 (viz twice 3,1416) into one another, the laſt product is the content. Or the ſaid area multiplied by the periphery of the circle deſcribed by the radius of gravity gives the ſame content.

Queſtion 41. If a circular ſector A Q B S (fig. 153) whoſe radius S Q = 12 chord A B 6, arch A Q B 7 nearly, be turned about the center S, with the radius S Q perpendicular to the axis of rotation, what is the content of the ſolid thus form'd?

By Art. 275
12 = S Q
12 × 2 A B
3×7=21)144
S C = 7 fere for rad. of gra.

7 = A Q B
6 = ½ S Q
42 = area ſector or generating plane
7 = S C radius of gravity
294
6,2832 conſtant factor
1847,2608 content anſwer

Queſtion 42. If the ſector (laſt queſtion) be turn'd about the point Q, ſo as Q S be perpendicular to the axis of rotation, what is the ſolidity of the ſolid, form'd by this revolution?

6,2832 conſtant factor
42 area ſect or gen. plane
263,8944 product

263,8944 product
5 = Q C rad. of gravity
1319,4720 ſolidity required

Queſtion 43. What is the content of a ſolid, formed by the rotation of the ſemicircle A B C (fig. 186) about the tangent A T perpendicular to the diameter C A, = 14 the ſemi-periphery A B C 22.

11 = half of A B C
7 = A O

77 = area semi-circle or generating plane
6,2832 constant factor

483,8064
7 = radius of gravity A O

3386,6448 solidity required

Question 44. But if the semi-circle A B C (fig. 186) revolve round the tangent D T, parallel to the diameter A C, what is the content of the parallel so form'd?

By Art. 275
49 = □ A O
4

22×3=66) 196

nearly 3 =

Note. This quest. is useful in finding the solidities of vaults &c.

7 = O B
3 = O G

4 = B G radius of gravity
6,2832 constant factor

25,1328
77 = area ½ circle

1935,2256 solidity required.

Note. If the axis of rotation do not touch the generating plane, but be at some distance from it, the rule is still the same; also, if you turn the factor 6,2832 into a divisor 0,157, then any of this sort of concise solutions (for so they are when compared with the common methods) may readily be done by a sliding-rule. Thus, As 0,157 on A : the area of the generating plane on B : : the radius of gravity on A, to the content on B.

Question 45. If the rectangle E D C H (fig. 187) be turned about the axis of rotation B A, at the distance A E = 8 from it, what will be the content of the solid viz. of a hollow cylinder, formed by this revolution, D C = E H being = 10 and the breadth H C = E D = 6?

The center of gravity of the generating plane E D C H will (by Art. 274) be at G in the middle of E D, so A E 8 + E G 3 = A G 11 the radius of gravity.

Then 6,2832 constant factor
60 = 10 × 6 = area □ E D C H

376,9920
11 = A G radius of gravity

4146,9120 solidity sought

Sliding-rule. As ,157 on A : 60 (10 × 6) on B : : 11 on A : 4147 on B the content of matter in the hollow cylinder sought.

Question 46. Of half the rectangle (last quest.) viz. H E D be taken instead of the whole one H E D C, the hollow of the figure gene-

rated thereby will be a cylinder, height = H E, diameter = A E, its thickneſs at one end H = 0, and at the other end D = E D = 6, what is the ſolidity of this figure ?

Firſt, the radius of gravity will be (A E 8 + $\frac{1}{3}$ E D 2) = 10, then as 0,157 on A : 30 (area △ H E D) on B : : 10 (radius of gravity) on A : 1885 on B, the content of matter in this hollow cylinder.

Queſtion 47. Given the perpendicular B P 60, (fig. 188.) and the ſegments of the baſe A P = 120, C P = 90 of a plane △ A B C, what will be the ſolidity of a ſolid, formed by the rotation of this △ about the axis C D ?

Here, becauſe the △'s A B P and C B P, have each their centers of gravity G and g, we may reduce theſe two centers to one, (by theorem 185) or find the content at twice, thus, $\frac{2}{3}$ of C P 90 = C g 60, the radius of gravity of the △ C B P, and C G = (C P 90 + P G 40, viz. + $\frac{1}{3}$ A P 120) 130 ; ſo, as 0,157 : area △'s A B P and C B P 3600 and 2700 : : C G and C g 130 and 60 : 2953537,6 and 1017878,4, which added together gives 4071416 the anſwer. In like manner may ſolidities be computed, when their generating planes conſiſt of ſeveral ſorts of figures.

If the contents of any ſuch ſolids as theſe 7 laſt queſtions be known, the center of gravity of the generating plane may, by the laſt rule, be found ; but to find the center of gravity of any ſolid (whoſe greateſt area is at one end, and leaſt area at the other end) without its equation, &c. this is a general

Rule 63. To the greateſt area, viz. the area at the greater baſe, add twice an area taken in the middle between the two baſes, multiply this ſum by the ſquare of the ſolid's axis, and divide the product by 6 times the ſolidity, for the diſtance of the center of gravity from the leſſer baſe. The ſame method anſwers in a plane, by uſing breadths inſtead of areas ; if you divide the ſaid product by 6 times the diſtance of the center of gravity from the leſſer baſe, the quotient is the content. See theorem 190.

Queſtion 48. If B P the ⊥ of a plane △ A B C, (fig. 188) be 60, and the baſe A C 40, what is its center of gravity ?

Firſt, by the property of the △, drawn thro' the middle of B P parallel to A C, will be = 20, ſo by rule 63, twice 20 = 40 + A C 40 gives 80, and 80 × □ B P 60 (3600) is = 288000 the product, which divided by 7200 ſix times the area or content (40 × $\frac{1}{2}$ of 60 × 6) of the figures quotes 40 for the diſtance B d of the center of gravity.

Queſtion 49. If the length of a conical fruſtum be 40 feet, or 480 inches, and diameters at ends 8 inches and 6 inches, where is its center of gravity ?

First, by rule 43, the content of this frustum is found $=111589,632$ when multiplied by 6, viz. 6 times the solidity in inches, then 98 twice square of 7, a diameter in the middle, added to 64, the square of 8, that at the greater base, gives 162, so $162 \times ,7854 = 127,2348$, which multiplied 230400 the square of 480 inches, the axis gives 29314897.92 this $\div$ by 111589,632 quotes 262,7 inches, for the distance of the center of gravity from the lesser end.

Note. In frustums of cones or others of circular bases, you may use the squares of the diameters instead of the areas, and in these of pyramids the squares of like sides. Or, from rules 43 and 63, we may draw a shorter rule for such frustums, the same as by theorem 189,

Rule 64. To half the square of the sum of a side at each base, add the square of a side at the greater base, this sum multiplied by the axis is a dividend, then to the sum of the squares of a side at each base, add the square of their sum, for a divisor, the quote of this division, shews the distance of the center of gravity from the lesser end.

Note. Observe to take similar sides at the bases, and if conical frustums use diameters or peripheries instead of sides, or the ratio's of any similar sides &c. at each base will do.

Question 50. If the greatest and least diameters of a vessel, or $=$ headed cask be 37 and 30,834, what must the length be, when the content is the greatest or convex superficies the least possible. fig. 207.

1. Let c I d B d I c, be such a cask, $2\,a = A\,B = 30,834$, the least $2\,y = d\,c = 37$ the greatest diameter $e = I\,m$ half the cask's length, then (art. 435) $e = 2,3025\,a \times \log. \dfrac{y + \sqrt{:yy - aa:}}{a} = 18,5$ fere, which doubled is $= 37 = I\,m\,I$, the cask's length required, and the curvature is that of the catenary.

The 34 following questions concern more uses of the centers of gravity, the forces of the mechanic powers &c.

Question 51. Two men A and B are to have 5 s for carrying a piece of square timber, length 40 feet, a side at the greater base 8 inches, and a side at the lesser base 6 inches, A bears a foot within the greater end, and B a foot within the lesser end, what money must each man have in proportion to the weight he bears?

The weights (by theorem 184) born are inversely as their distances from the center of gravity; now by the last question the center of gravity from the lesser end is $21\frac{3}{37}$ feet, which taken from 40 feet the whole length leaves $18\frac{4}{37}$ feet its distance from the greater end, and because each man bears one foot within the end, take 1 from each of these and there leaves $20\frac{33}{37}$ and $17\frac{4}{37}$, so as $20\frac{33}{37} : 17\frac{4}{37}$ oz. out of fractions, as $773 : 633 ::$ the weight born by A, to that born by B, consequently as $(773+633)\ 1406 : 5\,s :: \begin{cases} 773 : 2s = 9d \text{ nearly A's sh.} \\ 633 : 2s = 3d \text{ —— B's sh.} \end{cases}$

Question 52. A piece of tapering timber 24 feet long, being laid over a prop, is found to ballance itſelf when the prop is 13 feet from the leſſer end, ſo that point is it's center of gravity, but removing the prop a foot nearer to the ſaid leſſer end, it takes a man's weight of 15 ſtone, ſtanding on the leſſer end to hold it in equilibrium, what is the tree's weight?

By theorem 185. As 1 foot (the diſtance between the prop and center of gravity) is to 13 feet, ſo is 15 ſtone to 195 ſtone the anſwer. By this, if a man know his own weight, he may know the weight of any beam &c.

Question 53. An irregular ſolid 60 inches long, laid over a prop ſet to its middle, and a weight of 25 ſtone, ſet 10 inches from the prop towards the leſſer end, holds the body in equilibrium, but removing the prop 8 inches nearer to the leſſer end, the weight 25 ſtone is removed 15 inches nearer thereunto before the body will be in equilib. upon the prop, what's the bodies weight and ſolidity, $\frac{1}{4}$ of a ſtone of ſuch matter being a ſolid foot? See theorem 187.

Firſt, 8 inches $\times$ 10 inches $=$ 80 and $(10 - 2 + 15)$ $17 - 10 = 7$, then $80 \div 7 = 11\frac{3}{7}$ inches, the bodies center of gravity diſtant from the middle, (ſo by the ſame theorem) 10 inches $\times$ 25 ſtone $=$ 250, which $\div$ by $11\frac{3}{7}$ is 21,875 ſtone, the bodies weight, then, as 1 ſtone is to $\frac{1}{4}$ foot ſo is 21,875 ſtone to 5,468 ſolid feet anſwer.

Question 54. If A B (fig. 189) be a walking ſtick 40 inches long, ſuſpended by a ſtring S D, faſtened to its middle, now if a body be hung at e, 6 inches diſtant from D, and a weight of 2 lb. hung at the ſmaller end A, the ſtick will be in equilibrium, but removing the body to a, one inch nearer to D, the 2 lb weight on the other ſide D is moved to d, within 8 inches of D, before the ſtick will reſt in equilibrio what's the body's weight?

By theorem 185. Multiply (12) the difference between (20 and 8) the diſtances of the 2 lb weight from the point D of ſuſpenſion, by (2) the ſaid weight, that product (24) $\div$ by (1) the difference between (6 and 5) the diſtance of the body from the ſaid point D, quotes 24 lb the weight of the body.

Note. This is a very eaſy and ſimple way to weigh any matter, it requires nothing but any known weight and any walking ſtick &c. divided into inches, or any other $=$ parts.

Question 55. If a two pound weight and a body be in equilibrio, on a ruler, beam, ſtaff, &c. ſuſpended in the middle, or in any other place, and if by moving the body 1, the weight is found to move 12, before they again be in equilibrio, what is the weight of the body? See theorem 183.

This queſtion is the very ſame with the laſt, but the ſolution made more eaſy. Thus, as 1 the diſtance moved by the body, is to 12, that moved by the weight, ſo is 2 lb the weight, to 24 lb the body's weight.

Queſtion 56. If in a pair of ſcales, a body weigh 90 lb in one ſcale, but being put into the other ſcale it only weighs 40 lb, what is its true weight?

By theorem 186, $90 \times 40 = 3600$, whoſe ſquare root is 60, the anſwer. By which you may know if the ſcales be true.

Queſtion 57. If A n and C n be two cords (fig. 190) having each an end faſtened to the nail n, with a ſtaff or ruler A C, faſtened between them, as alſo a plumb line n P, all hanging at liberty on the nail n, now if a weight of 2 lb be hung to the end C of the ſtaff, and a body B to the other thereof, the ſtaff is cut be the plumb in e, ſo, as 1 is to 20 ſo is e A to e C, what is the weight of the body B?

By Art. 281, e is the center of gravity of B and W. therefore

By theorem 183 or 184, As A e 1 : : e C 20 : : W 2 lb : B 40 lb the anſwer, or (by the ſaid art. 281) it is all the ſame whether the ſtaff A C and weights B and W, be ſuſpended by the two cords n B and n C, or at e by the one cord n e, or on a prop ſet to the point e, the equilibrio in any one caſe, holds in all.

Queſtion 58. If two bodies W and b (fig. 191) be in equilib. ($W =$ 110 lb and $b = 2$ lb) on a priſmatic beam A B, ſuſpended on D, what is the diſtance D b, if $DB = 2$ and $DA = 14$?

Firſt, Suppoſe the weight of the beam $AB = 16$, then becauſe it is of = thickneſs the length of any part will be as the weight of that part and alſo the center of gravity of any part D B or D A will be in the middle thereof, ſo (by art. 272) $AD \times \frac{1}{2} AD + Db \times b = DB \times \frac{1}{2} BD + DB \times W$, that is $14 \times 7 + 2Db = 2 \times 1 + 2 \times 110$, or $98 + 2DB = 222$ ſo $Db = \frac{222 - 98}{2} = 62$ anſwer. If you take the weight $W = 1$ lb, 2 lb, 3 lb &c. you may thus find the diſtances D 1, D 2, D 3 &c. which is the conſtruction of the Roman ſteel yard, for weighing bodies at the end B, by moving a known weight on the other arm D A.

Queſtion 59. Two men A and C (fig. 190) bearing a weight of 30 ſtone upon, or hung to a leaver A C at the point e, A bears A e 2 feet from the center of the weight and C bears C e 6 feet therefrom, what weight does each man bear?

This queſtion is the ſame in effect with queſtion 51, i. e. the weights born are inverſely as their diſtances from the center of gravity e, there-

ſore, As 8 $(2+6)$: 30 ſtone : : $\left\{\begin{matrix} 2 : 7{,}5 \text{ ſtone} \\ 6 : 22{,}5 \text{ ſtone} \end{matrix}\right\}$ the weight born by $\left\{\begin{matrix} C \\ A \end{matrix}\right\}$ the anſwer.

Queſtion 60. What is the leaſt number of weights, and the weight of each, that will weigh any number of pounds between 1 lb and 500 tons?

If you take the weights 1 lb and 3 lb, they'll weigh 1 lb, 2 lb, 3 lb, and 4 lb, for 1 weighs 1, 3 weighs 3, 3 and 1 weighs 4, 1 from 3 weighs 2; alſo, if you take the weights 1 lb 3 lb and 9 lb, you'll find that by them any number of pounds may be weighed between 1 lb and 13 lb, &c. &c. Whence it appears that a ſeries of terms in geometrical proportion, whoſe firſt term is 1 and common ratio 3, will weigh at one draught any number of weights between 1 and the ſum of all the terms in the ſeries, therefore the required number of weights muſt be 14, and their weights in pounds 1, 3, 9, 27, 81, 243, 729, 2187, 6561, 19683, and ſo on to 14 terms.

Queſtion 61. How muſt an axle tree be fixt in the ſides of a conical fruſtum or bucket, whoſe ſlant length is 30, diameters 10 and 20, ſo that it may hang with the mouth downwards when empty, but upwards when full?

By comparing the equations in art. 278, it appears that the center of gravity of a ſolid fruſtum is nearer the greater end than that of a hollow fruſtum. Therefore the bottom of the bucket muſt be in the greater end of the fruſtum, and the required point in the common center of gravity of the bottom and hollow fruſtum; ſo by the ſaid art. 278, divide 1500 (twice the greater diameter, 20 + to the leſſer, 10 and × by the length 30) by 90 (3 times the ſum of the diameters) and the quote $16\frac{2}{3}$ is the hollow fruſtum's center of gravity from the leſſer end, or $(30-16\frac{2}{3})$ $13\frac{1}{3}$ from the greater end, now ſuppoſe the middle of the breadth of the bottom, to be $\frac{1}{3}$ within the greater end, and its weight or ſolidity to that of the hollow fruſtum, as 2 to 15, then (by theorem 185) $13 \times 15 \div 15 + 2 = \frac{195}{17} = 11\frac{8}{17}$, the diſtance of the axle-tree from the middle of the bottom's breadth.

Queſtion 62. If A B C (fig. 192) be a rowler (over a mine pit, well &c.) of a A 6 radius with a crook B D E at one end A A, of B D 18 radius, what power muſt be applied to the handle D E to make the rowler A C raiſe a weight W of 200 lb?

It is evident that the radiuſes a A and a D may be taken as two leavers acting againſt each other at the center a, therefore, (by theorem 184) as 18 : 6 : : 200 : $66\frac{2}{3}$ lb anſwer.

Question 63 If the radius of a wheel be 40, and a power of 500 lb applied to its periphery does move a weight at the axle-tree of 40000lb what is the radius of the axle-tree?

The property of the wheel may be eaſily underſtood by the laſt mentioned figure, viz. a D being taken as the radius of the wheel and a A as that of the axle-tree; therefore, As 40000 lb : 500 lb : : 40 : $\frac{1}{2}$ for the anſwer.

Question 64. If there are two blocks or pullies, the one fixt and the other moveable, ſuch as are in ſhips &c. for hoiſting, and to the moveable block there be a weight of 300 lb, what weight pulling at the rope end will raiſe it?

The axis of pullies may be taken as leavers, whoſe fulcrums, are in their middles, or centers of the pullies, whence (by theorem 184) you can by one pulley only raiſe a weight equal to the force that pulls at the other end of the rope; but in any number of pullies any how ordered, or in any machine whatever, (by theorem 183) the diſtances moved over in the ſame time, by the power and weight are equal, ſo in an equilib. (ſee theorem 184) the product of the power and it's diſtance moved, is = to that of the weight and it's diſtance moved in the ſame time, ſo in the above queſtion, $\frac{1}{2}$ of 300 = 150 lb is the anſwer, for you'll (in that caſe) ſee the power deſcend 2, while the weight aſcends one.

Question 65. How muſt three pullies A, C, E, (fig. 193) be combined ſo as to raiſe the moſt weight W, with the leaſt power or force P?

Let the uppermoſt pulley A be fixt to a beam &c. as per fig. then with a little conſideration it will appear, that while the weight W riſes 1 inch, the power P ſettles 7, that is, (by theo. 183) one pound at P will raiſe 7 lb at W, which will anſwer the queſtion. For by the laſt queſtion, a man with two pullies, can raiſe twice his weight, with 3 pullies he'll raiſe thrice his own weight, with 4 pullies, 4 times his weight &c. half the number of pullies being fix'd, and the other half number moveable, and all the parts of the rope parallel to each other, but if each pulley have a fixed rope it muſt be conſidered as a leaver of the ſecond kind, and ſo will double the power of the foregoing laſt pullies, i. e. 4 pullies will increaſe the weight 16 times.

Question 66. If by preſſing with a weight of 30 lb at the end B (fig. 194) of a leaver B A 77 inches long, the end A fix'd at right angles to the head of a ſcrew, which by this preſſure, preſſes or raiſes a weight 1 inch, what is this weight or force of the ſcrew?

Here, in turning the ſcrew once about, the weight W moves but 1 inch; but the power at B moves thro' 484 inches the periphery of a circle whoſe radius is A B 77 inches, therefore (by theo. 183) As 1 inch : 484 inches : : 30 lb : 14520 lb the anſwer.

Note. While the ſcrew turns once round two of its threads will appear, from whence we have this rule, as the diſtance between two contigious threads is to the diſtance moved by the applied power, ſo is the force of the power to the force of the ſcrew in one revolution; hence we ſee the vaſt force of a ſcrew, and that the nearer together the threads are, the greater will the force be.

Queſtion 67. If a weight P of 150lb, can be drawn up the ſide B A (fig. 195) of an upright wall 20 height, what weight W with the ſame eaſe, will be drawn up a plank C A 30 long laid a ſlope from the top of the wall to the bottom, the plank and wallbeing of equal ſmoothneſs?

Here its plain that while the weight W riſes to the ⊥ height B A 20, the power P that draws it (in direction parallel to C A) moves over the diſtance C A 30, therefore (by theo. 183) As A B 20 : A C 30 :: P 150lb : W 225 lb the anſwer, and ſo is B C 22,36 : 167,7 lb the preſſure againſt A C. See queſtion 69.

Queſtion 68. Let things be the ſame as in the laſt queſtion (fig. 196) but inſtead of the weight W, being drawn in direction parallel to the hypothenuſe C A, ſuppoſe it to be drawn up the ſame C A, in direction parallel to the horizon C B?

Here its plain, while the power P moves the weight W from C to A, it moves itſelf from C to B, and the ſaid weight W aſcends from B to C, therefore, (by theo. 183) as A B 20 : $\left\{\begin{matrix}\text{C B } 22{,}36 \\ \text{A C } 30{,}00\end{matrix}\right\}$ :: P 150 : W 167,7 anſwer, and 275 the preſſure againſt A C.

Queſtion 69. Things being the ſame as before, ſuppoſe the weight v to be drawn, or ſuſtained in equilibrio (fig. 195 and 196) by a power q 150lb in an oblique direction q v to the inclined plane C A, ſo as B d = 15, being perpendicular to the line of direction of the power, q v, then in fig. 195, ſuppoſe d C = 18,6?

Firſt, If B e d is ⊥ to the traction q v, then it's plain, B d is ⊥ to q v the direction of the power C B to A B the direction of gravity, and C d to A C the direction of the preſſure againſt the plane; therefore, (by theo. 159) As B d 15 : q 150 lb :: C B 22,36 : v 223,6 lb, and ſo is C d 18,6 : 186 lb the preſſure upon the plane A C, or if we take the ſines of the angles oppoſite theſe ſides, then the ∠ d e v being = 90° the ∠ C d B = comp. e v d, and the ſides being (by theo. 48) as the ſines of their oppoſite angles, it will hold, the power, weight, and preſſure againſt the plane are reſpectively as the ſine of B C d, the plane's elevation, co-ſine of the ∠ of traction e v d (viz. C d B) and C B d co-ſine of the power's direction above the horizon, and when v q inſides with A C, you have the ſame proportion as in queſtion 67, or with B C

as in question 68; from this last question it is plain, that the pressure against the plain is greater when the time of the power's direction v q is below the plane, than when it is above it (all else the same if the ∠s of traction be =) for in fig. 195, d C is greater than it is in fig. 196.

Question 70. What weight will a wedge A B C, (fig. 197) raise by applying a force or power of 150 lb to it's head at A B, the slant length C A = C B = 3 and thickness A B = 4.

Here its plain that while the wood &c. into which the wedge driven, opens its self from H to B, or to A, the slant sides of the wedge moves from B to C or from A to C; therefore, (by theorem 183) as H A = H B = 2 : B C = A C = 3 : : 150 lb : 225 lb the answer, or if the wedge be oblique, As H B + A H = 4 : B C + A C = 6 : : 150 lb : 225 lb the answer, here it appears, the sharper the wedge the less force will drive it.

Question 71. If two bodies W and P be in equilibrio upon two inclined planes A B 40 length, and A C 30 length, their common ⊥ A P 20, what is the ratio of the weights of these two bodies? Fig. 198.

By question 67, As A P 20 : A C 30 : : 1 : $\frac{30}{20}$ = P = 1,5 and as 20 : A B 40 : : 1 : $\frac{40}{20}$ = W = 2, i. e. (because the bodies are in equilibrio) W = 2 and P = 1,5, so, as W : P : : 2 : 1,5 : : 4 : 3 viz. the weight are as the lengths of the planes.

Question 72. If two barrels the one holding 10 gallons and the other 15 gallons, are to be carried hung over a horse's back, how high must the heavier barrel be raised to ballance the lighter?

By the last question, suppose the lighter barrel to hang down the ⊥ A P (fig. 198) and the heavier down A C, then as A P : A C : : 10 : 15, so (by theo. 13) the square root of the difference of the squares of 15 and 10 is 11,3 = P C; therefore, as 10 : 11,3, or as 1 : 1,13, and so must the sides of the horse be raised, that the carriage may ballance, and one side be not heavier nor lighter than the other. But if you suppose the barrels to be suspended at A and B (fig. 149) the ends of a leaver A B, and make it as A G : G B : : 15 : 10, this point G will (by theo. 185) be the common center of gravity of the two barrels, whose weights or contents are 15 and 10, and is the point in the leaver to be laid over the prop &c. that the weights may be in equil.

Question 73. If A C be an inclined plane, on which a cylinder open at top m n, and perpendicular to the said plane is drawn uniformly in direction parallel to A C, required the angle of inclination A C B of the plane with the horizon such that the most water possible may be drawn out of a well at C, by the said cylinder m n b d, in any given time? Fig. 195.

1. It's plain, the greater the angle A C B is, the ſhorter will A C, the way of the water be, and if n r be the ſurface of the water in the cylinder, then the greater the veſſel n r b d, or only it's height r b, is, the more water will be drawn, whence the product of the ſine of the ∠ A C B, and height r b of the veſſel muſt be a maximum, therefore let e = ſine ∠ A C B, then $\sqrt{: 1 - ee :}$ = it's co-ſine (radius = 1) and m b = n d = 1, then the △s A B C and n m r being ſimilar it will be by trigonometry as ($\sqrt{: 1 - ee :}$ ſine ∠ C A B or m r n) : a (m n) : : e (ſine ∠ m n r) : m r = $\frac{ae}{\sqrt{: 1 - ee :}}$, therefore m b — m r = 1 — $\frac{ae}{\sqrt{: 1 - ee :}}$ = r b, ſo r b × e = e — $\frac{aee}{\sqrt{: 1 - ee :}}$ = m, a maximum, and by (art. 421) 1 — $\frac{2ae}{\sqrt{: 1 - ee :}}$ — $\frac{eeea}{\overline{1 - ee}|^{\frac{3}{2}}}$ = 0, ſo we have $\overline{1 - ee}|^{\frac{3}{2}} = 2ae - aeee$, and if we ſuppoſe a = 1, it will be $\overline{1 - ee}|^{\frac{3}{2}}$ = 2 e — e e e, wherein will be found e = 0,4123 the natural ſine of 24° 21′ the anſwer. When the depth and diameter of the cylinder are equal, but if d = the depth and a = diameter, the angle will vary, and our equation become $\overline{1 - ee}|^{\frac{3}{2}}\, d = 2ae - aeee$, in which e may be found, whatever a and d be.

Queſtion 74 If a wall of equal thickneſs a B C D (fig. 199) be built perpendicular to the ſide of a hill, making an angle B a E with the horizon a E of 20°, what weight of the wall will be on each ſide B and a, and whether will it overſet or not, if its height a D = B C be 10 feet, and it's thickneſs D C = a B = 4 feet?

Becauſe the wall is all of the ſame matter and thickneſs, it's center of gravity will be at G in the middle thereof G n (being ⊥ a B) will be = 5 half the height and n a = n B = 2, half the thickneſs, let G e be ⊥ the horizon H e, then will ∠ e G n = ∠ B a E = 20, ſo (by theo. 48) As ſine ∠ n e G is to ſine ∠ e G n 20°, ſo is n G 5 to e n 1,82, therefore, the wall will not overſet, be G e ⊥ horizon, and paſſing thro' the center of gravity G, falls within the breadth a B of the wall, now for the weight, e in this caſe is the center of gravity, ſo (by theo. 185) as e B 3,82 is to e a 0,18, ſo is the weight at a to that at B.

Queſtion 75. A cart load of hay &c. 10 feet high and 4 feet broad is to be drawn on the ſide of a hill elevated 20°, what muſt be the height of the greateſt wheels poſſible, to carry it without overſetting?

Let a B C D (fig. 199) repreſent the load, whoſe center of gravity is at G per laſt queſtion, produce D a and G e till they meet in z, then (by art. 28 :) a z, will be the ½ diameter of the wheel, ſuch that the load will reſt in equilibrio in z, the lower edge thereof, the axle-

tree being at a, so if the wheel be any higher the load will overturn upon z, because then G z ⊥ the horizon, will fall without the side D a, then per similar $\triangle$s z a e and G n e, as e n 1,82 : e G 5,5 : : e a : 0,18 : 0,54 feet = a z the radius of the required wheel.

Question 76. If B a (fig. 156) be a beam, wall &c. whose weight is 500, it's center of gravity at C, (C B being = 22) and leaning, so as to make an $\angle$ A B a with the horizon of 60°, and if a prop A a, be set at A, 40 distant from B the foot of the object, making an $\angle$ B A a, or Q A a with the horizon B A of 55°, what weight doth the prop A a bear?

By theo. 191. Let radius = unity, then the natural co-sine of 60° is = 0,5, and the natural sine of 55° is = 0,8191, so $\frac{22 \times 500 \times 0,5}{40 \times 0,8191}$ $= \frac{5500}{32,764} = 167,8$, the weight born by the prop A a, which taken from 500 the whole weight leaves 332,2, for the weight press'd at B, and (by art 28 :) is in direction of a line drawn from B to the meeting of A a (produced) and a line drawn thro G, perpendicular to the horizon.

Question 77. Things being as in the last question, and the prop A a at the same place, what weight doth it bear, when the object B a presses with the greatest force, and the prop supports with most ease?

By theo. 192. When the object makes an $\angle$ with the horizon of 51° 50′ then it presses the prop A a with the greatest force possible, which prop at the same time must make equal $\angle$s with the object and horizon, therefore the comp. of 51° 50′ is 38° 10′ whose natural sine is 0,6180, and 180 — 51° 50′ = 128° 10 half whereof is 64° 5′ the angle of the prop's inclination, whose natural sine is 0,8995, so $\frac{22 \times 500 \times 0,618}{40 \times 0,8995} = 188,9$ the answer.

Question 78. A beam B C (fig. 200.) whose center of gravity is at G, or a leaver B C bearing a weight G parallel to the horizon A D, upon two loose props C D and B A, making on angle B A D with the horizon of 50°, what angle must the prop C D make therewith, to support the beam in equilibrio, B C being = 8 and C G = 12?

1. Thro' G draw F G ⊥ the horizon, and produce A B to meet in F, thro' F and B draw F B A so is B A (by article 281) the other prop, then, (by theorem 47) as B G 8 : G C 12 : : tangent $\angle$ B F C 40° (comp. $\angle$ F B G 50°) : tangent $\angle$ G F C 51,32 whose compliment is 38,28 = $\angle$ F C G or A D C the answer.

Question 79. If the weight G (last question) be 500, how much of it doth each prop bear; or, if it be hung by the two ropes F B and F C, what is the tension of these ropes, or the weights they bear?

Draw I H parallel to F B, then, (by article 281.) as sine ∠ F I H 91° 32′ (∠ A 50° + ∠ D 38° 28′ and taken from 180°) : 500 (the weight denoted by F H) : : sine ∠ H F I 51° 32′ : 391,7 fere, the weight upon A B, and so is sine ∠ F H I 40 : 321,5 the weight upon C D, that is, the weight borne by A B is to that borne by C D as 391,7 : 321,5 or as 39 : 32; therefore, as (39 + 32) 7 is to 500 so is $\left\{\begin{array}{l}32 : 225,3\\39 : 274,6\end{array}\right\}$ the just weight borne by $\left\{\begin{array}{l}\text{C D or F C,}\\\text{A B or F B,}\end{array}\right\}$ in these directions. But, if you would know the pressure in any other direction, as that ⊥ to the horizon: then, (by theorem 157) as radius (∠ B E A) : 274,6 (force in direction B A) : : sine ∠ E A B 50° to 210,3 the pressure in direction B E ⊥ to the horizon, and : : sine ∠ E B A 40° : 176,5 the push outward in direction E B parallel to the horizon. The line may be done for the prop C D.

Question 80. Fig 157. If D C E be an upright wall height C D 20, built at the foot of a slope wall A B C, of the same height A B, and slope side B C 30, and the vacant triangle B C D, between the two walls be filled with earth, sand, &c. with what force doth it press each wall?

By art. 288, As 30 : 20 : : 1 : $\frac{2}{3}$, the weight sustained by the wall C B, so $(1 - \frac{2}{3})\ \frac{1}{3} =$ that sustained by C D E, in direction D C, and as B C 30 : A C 22,36 : : $\frac{2}{3}$: $\frac{44.72}{90} = 4959$, the pressure in direction A C perpendicular to the wall D C E, and acts at L in direction G L (C L being $= \frac{2}{3}$ of $20 = 13\frac{1}{3}$.)

Question 81. If a solid inch of the wall D D E (last question) be to a solid inch of the △ B C D as 4 to 5, what must be the thickness of the said wall, that it may not be overset by the pressure of the △?

1 B D 22,36 × $\frac{1}{2}$ D C 10 = 223,6 the content of the △ B C D, then (by theo. 193) $E\,e\,x : y + P : = \dfrac{AC \times DC \times W}{\square\, BC}$, also, D C × E C = the wall's (C D E) content. Likewise, if e be the center of gravity of the two weights P (pressing upon C) add y (upon R) then, by theorem 185) $\dfrac{\frac{1}{2}\,CE \times y}{P + y}$ (R being in the middle of C E, because C D E is equal thickness) so $E\,e = C\,e + \frac{1}{2}\,CE = \dfrac{\frac{1}{2}\,CE \times y}{P + y} + \frac{1}{2}\,CE$; there-

fore, $Ee \times : y + P : = : y + \frac{1}{2} P : \times CE = \frac{AC \times DC \times w}{\square BC}$, or $=$ $\frac{AC \times \frac{2}{3} \square DC \times w}{\square BC}$, because the force in direction A C acts at L and $LC = \frac{2}{3} DC$. Then, (by art. 288) $P = w \frac{DC \times w}{BC}$, which put in the last equation for P, we get $: y + w - \frac{DC \times w}{BC} : \times CE = \frac{AC \times \frac{2}{3} \square DC \times w}{\square BC}$, but by the question, $y = DC \times CE \times n$ and $w = \frac{1}{2} AC \times DC \times m$ (n being taken $= 4$ and $m = 5$) which substituted in the last equation for y and w we'll have, after reduction compleating the square &c. $CE = \sqrt{\frac{\frac{1}{3} m \times \square AC \times \square DC}{\square BC} + : 1 - \overline{\frac{DC}{BC} \times \frac{\frac{1}{2} m \times AC}{2n}}\Big|^2} : - : 1 + \frac{DC}{BC} : \times \frac{\frac{1}{2} m \times AC}{2n}$, but to have the equation shorter, let $1 - \frac{DC}{BC} = z$, then $EC = \sqrt{: \frac{m \times \square AC \times \square DC}{3 \square BC \times n} + \frac{zzmm \times \square AC}{16nn}} : - \frac{zm \times AC}{4n}$, in numbers it is, $CE = \overline{\frac{5 \times 500 \times 400}{3 \times 900 \times 4} + \frac{4 \times 25 \times 500}{9 \times 16 \times 16}}\Big|^{\frac{1}{2}} - \frac{2 \times 5 \times 22,36}{3 \times 4 \times 4} = 6,1$ ans.

Question 82. What must be the thickness of the stone wall C D E (fig. 157) to resist the pressure of the Δ of earth B C D, the weight of stone being to that of earth as 3 to 2, and allowing a heavy body to loose $\frac{1}{3}$ of its weight in sliding down an inclined plane B C, on the account of friction?

Here $n = 3$ and $m = 2$, but m must abate $\frac{1}{3}$ of it's weight because of friction as is found to answer to experience in planes middling smooth so $m = \frac{4}{3}$ ($2 - \frac{1}{3}$ of 2, or $\frac{2}{3}$ of 2), so to have m and n out of frictions it will be as $3 : \frac{4}{3} :: 9 : 4$, that is, $n = 9$ and $m = 4$, which put in the last general equation, gives $\sqrt{\frac{4, 500, 400}{3, 900, 9} + \frac{4, 16, 500}{9, 16, 81}} : - \frac{2, 4, 22, 36}{3, 4, 9} = CE \sqrt{\frac{800000}{2700,9} + \frac{32000}{11664}} : - \frac{178,88}{106} 5,3$ the answer.

But if the pressure of the Δ B C D in direction DC downwards be

taken away, or which is all the ſame, the two laſt terms in the equation being but ſmall may be neglected, and then the ſolution will be very eaſy; thus, $CE = \overline{\frac{m \times \square AC \times \square DC}{3n \times \square BC}}\Big|^{\frac{1}{2}} = \sqrt{\frac{5 \times 500 \times 400}{3 \times 4 \times 900}}$ $= \sqrt{\frac{1000000}{10800}} = \sqrt{92,6} = 9,6$, for the anſwer to queſtion 81, and $CE = \sqrt{\frac{800000}{24300}} = \sqrt{32,9} = 5,7$, which is but a ſmall matter more than 5,3 the laſt anſwer; and therefore is to be uſed in ſuch caſes before 5,3, becauſe this 5,3 is the breadth on the equilib. ſo 5,7 muſt do more than ballance.

Queſtion 83. If the wall C D E (fig. 157) have no breadth at top D and a double breadth 2 C E = 11,4 at bottom, how much more preſſure will it then reſiſt, the ſide C D next the preſſure being upright and the other ſide D E ſlant?

Firſt, half of 5,7 is = 2,85 = C R, R being the center of gravity when the wall is of = thickneſs, but $\frac{2}{3}$ of 11,4 = 7,46 +, when no breadth at top; ſo, as 2,85 : 7,46 : : the force of the former : that of the latter. That is, the wall C D E, will endure a preſſure againſt the ſide D C before it turn over upon E, which is as 2,85, and if it become the wall A B C it muſt be preſſed againſt B A with a force as ($\frac{1}{3}$ of 11,4) 3,8 before it overturn upon C, but a force to overſet it upon A by preſſing at B C is as 7,46 +, each of theſe preſſures being in the ſame direction and at the ſame height above the bottom, and thus appears the reaſon why tapering walls &c. are ſtrongeſt.

The walls here mentioned are ſuppoſed to be equally ſtrong, or ſtick well together. (See queſtion 108 and 109.)

Queſtion 84. How many ſolid inches are in a pair of bellows A B C when the ſides A B, A C, are each a circle of 10 inches diameter, and makes an angle B A C with each other at the pipe A of 30°, and what is the greateſt quantity of air they can poſſibly hold? (Fig. 201)

1. Its plain by the figure, that if another pair of = bellows A C D, be laid upon A C, with their pipe over C, theſe two pair of bellows ſo joined will form an oblique cylinder A B C D, half of whoſe content will be the anſwer; ſo by trigonometry, as radius : A C 10 : : ſine $\angle$ B A C 30° : $\perp$ C P 5 = height of the cylinder A B C D therefore $0,7854 \square AB \times \frac{1}{2} CP = 0,7854 \times 100 \times 2,5 = 196,35$ ſolid inches, and the content muſt be the greateſt poſſible, when the ſides C A and C B are $\perp$ to each other, and then C P = C A = A B = 10, whence $100 \times 0,7854 \times 5 = 392,7$ ſolid inches, for the greateſt content poſſible.

Question 85. This and the 29 following questions are about wheel carriages, the strength and stress of timber, walls, &c. A carriage of 8 hundred weight is to be drawn up the side of a hill making an ∠ with the horizon of 30°, what force will be sufficient to draw it when the radius of each wheel is 24 inches or 2 feet?

(By prob. 196, art. 265). The carriage goes with most ease when the traces &c. are parallel to the hills side &c. Therefore, (see fig. 147) as sine radius : CE 2 feet : : sine ∠ C E H 60° (comp. hill's elevation) : C H 1,75 which squared and taken from 4 the square of CE leaves 0,9375 whose square root is 96. Then, as C E 2 : 8 hundred weight : : 0,96 : 3,84 hundred weight, the force fit to hold it on the hill's side.

Question 86. Things being as in the last question, but suppose the traces, or direction, to make an ∠ with the hill's side of 20°, what force will the carriage require in that direction to hold it on the side of the hill?

By theo. 180. As sine 70° : sine 90° : : 3,84 last weight found : 4,1 hundred weight, the answer.

Question 87. Things being the same as in question 85, what force will be sufficient to hold the carriage on the top of the hill or obstacle, whose ⊥ height is 0,25 feet (2 feet — 1,75 feet) in a direction parallel to the horizon? See theorem 182.

1. Multiply 3,75 the difference between 4 the wheel's diameter and 0,25 the object's height by 0,25 the said height, the square root of 0,9375 that product is 0,96. Then, as 1,75 feet (radius 2 feet — object's height 0,25 feet) is to 8 hundred weight, so is 0,96 to 4,3 hundred weight the answer; thus you see that in any position of the traces, that parallel to the object is the best.

Question 88. If a wheel carriage is to be drawn on rough uneven ground, how much easier will it be drawn on wheels of 3 feet radius than on those of 2 feet radius? See theorem 181.

As $\frac{1}{3}$: $\frac{1}{2}$: : 2 : 3, so is the force required to draw the greater wheels to that required to draw the lesser ones.

Question 89. There are two wheels, the radius of one is 2, and of the other 5, which will have the most advantage to drive an obstacle before it? See art. 267.

As $\sqrt{2-1}$: $\sqrt{5-1}$: : 1 : 2, so is the force required by the lesser wheel to that required by the greater one. Hence, in this case which seldom happens, small wheels may have the advantage; but in the foregoing questions great ones have it.

Question 90. Whether is the friction on great or small axle-trees, the most?

1. The friction or rubbing parts may be taken as a force acting against the force to draw the carriage, and therefore (theo. 183) the greater the velocity of these parts, the greater will the friction be; so if the wheels bear the same weight, and turn about in the same time, their velocities (viz.) that of their axle-trees, will be as the diameters of the said axle-trees. Hence, small axle-trees have less friction than great ones, and in any machine, the smaller the parts that rub against one another, the less is the friction.

Question 91. If a board be 2 inches thick and 10 inches broad, how much more weight will it bear edge way, than broad way? See art. 301.

1. In all the like sections, the square of the depth, or side of the section $\perp$ the horizon, multiplied by the breadth, or side parallel thereunto, is as the strength. Therefore, $2 \times 2 \times 10 = 40$, and $10 \times 10 \times 2 = 200$, so, as $40 : 200$, or as $1 : 5 ::$ the strength broadway : the strength edge way.

Question 92. If any beam &c. making an angle with the horizon of 60° can bear 200 C weight, what weight will it bear in the same place, when parallel thereunto. (By art. 300) This 200 C. weight multiplied by 0,5 the natural co-sine of 60° the inclination, gives 100 C. weight the answer.

Question 93. An iron bar bearing 500 lb bends thro' 1,8 feet, (b) then is just a breaking, a steel bar bears the same weight (c) but deflects only 0,1 foot (B) then is just a breaking, with what velocity (v) must a ball 5 lb weight (w) be thrown against each to break it, see the latter part of art. 311?

1. $d = bc \div 2w = 1{,}8 \times 500 \div 10 = 90$, then, (theo. 166) as $\sqrt{16} : 32 :: \sqrt{d}\ (\sqrt{90}) : 76$ nearly feet per second $= v$, for the iron bar; and taking 0,1 for 1,8 we get $v = 9$ nearly, for the steel. Hence, force to break the iron : force to break the steel $:: 76 \times 500 : 9 \times 500 :: 76 : 9$ the answer.

Question 94. How much weight will an oak beam 10 feet long and 1 foot square bear, before it break, when a like piece of oak a foot long, and $\frac{1}{10}$ of a foot square beares 320 lb to break it? See art. 308.

Here $e = 1$, $d = 0{,}1$, $g = 0{,}5$, $v = 320$ lb, $w = 0{,}44$ lb, the weight of a peice of oak 1 foot long, and 0,1 of a foot square, $a = dd = {,}01$, $E = 10$, $D = 1$, $A = DD = 1$. Whence $y =$

$$\frac{DDDee \times : \frac{1}{2}w + v : - \frac{1}{2}dEEDDw}{dddEe} = \frac{318{,}02}{{,}01} = 31802 \text{ ans.}$$

Question 95. How long must the last mentioned beam be, to break with its own weight? See art. 308.

1. From $dEEw = Deew + 2Deev$, we get $E = \overline{\frac{Deew+2Deev}{dw}}\Big|^{\frac{1}{2}} = \sqrt{\frac{640,44}{,044}} = 120,6$ feet, the anſwer.

Queſtion 96. If a ſquare priſm of oak 1 foot long and 0,1 of a foot ſquare, bear 320 to break it, what are the dimenſions of a ſimilar oak priſm, that breaks with its own weight? See art. 308.

1. $E = : ew + 2ev : \div w = 640,44 \div 0,44 = 1455,54$ feet the length, and as $1 : 0,1 :: 1455,54 : 145,554$ feet, a ſide of it's baſe.

Queſtion 97. If a ſquare priſm of oak ſupport 320 lb weight, before it break, required the dimenſions of the ſtrongeſt ſquare beam of oak poſſible, ſimilar to the ſaid priſm, being a foot long, and 0,1 of a foot ſquare? See art. 308.

Here $e = 1$, $v = 320$ lb, $w = 0,44$ lb the weight of the ſaid priſm then $E = \frac{2e \times : w + 2v :}{3w} = \frac{2 \times : 0,44 + 640 :}{1,32} = 970,37$ feet the length, and as $1 : : 0,1 : : 970,37 : 97,037$ feet a ſide of it's baſe, and the weight it will bear is $y = \frac{: w + 2v : \times EEe - EEEw}{2eee} =$ 132300000 lb, nearly.

Queſtion 98. If a ſquare oak beam 10 feet long and 1 foot ſquare, 31802 lb, what will each priſm bear when it is ſlit into two triangular ones, ſupported the ſame way as the beam was, with a ſide parallel to the horizon?

1. Here (ſee theo. 195) we have $P = \frac{1}{4}$, $p = \frac{1}{3}$, $D = d$, $E = e$, $G = g = \frac{1}{2}E = \frac{1}{2}e$, $c = \frac{1}{2}C = 1$, $w = 440$ lb the weight of 10 ſolid feet of oak, the ſquare beam, $v = 31802$ lb, which duly ſubſtituted in the ſaid theorem, we'll get $y = : w + 6v : \div 8 = 23907$ lb, the weight borne by each priſm, and is much above half of 31802 lb, the weight borne by the whole beam.

Queſtion 99. Whether is a ſquare beam ſtronger, when it lies corner uppermoſt, or a ſide uppermoſt? See art. 302.

Let a = area of the ſquare, d = it's ſide, D = its diagonal, then $\frac{1}{3}da$ is the ſtrength, when the weight preſſes one of its ſides, and $\frac{5}{18}Da = \frac{5}{18}da\sqrt{2}$ (for $DD = 2dd$) $= \frac{7}{18}da$ nearly, the ſtrength in direction of its diagonal, i. e. as $\frac{1}{3} : \frac{7}{18} :: 6 : 7 ::$ the weight it will bear upon a ſide to that it will bear upon a corner, for in both caſes, the length and weight, of the beam is the ſame.

$*_*{*}$ U

Question 100. Whether are round, or ſquare beams ſtronger? See art. 302.

1. Suppoſe two beams, the one round and the other ſquare, of the ſame matter, length, content, and conſequently weight, then if d = the diameter of the round beam or cylinder, it's ſtrength will be ,2451 ddd, and D the ſide of a ſquare = the circle will be $\sqrt{,7854\,dd} = ,887\,d = D$, but $\frac{1}{3} D^3 = ,236\,ddd$, is the ſtrength of the ſquare in direction of it's ſides, and by the laſt queſtion $\frac{7}{6}$ of ,236 ddd = ,275 ddd, the ſtrength of the ſquare, in direction of it's diagonals. Therefore, theſe three ſtrengths are as the numbers 245 for the round 236 for the ſquare on one of it's ſides and 275 if on one of its corners.

Question 101. If a round beam 1 foot diameter, 10 foot long 320 lb weight, bear 30000 lb, what weight will it bear if hollowed, diameter of the hollow 0,5 foot? See art. 302, and 308.

Let D = 1, the diameter of the cylinder d = that of its bore, a = the area of the ring e = 10, the length w = 320, v = 30000, then $G = g = \frac{1}{2} e = \frac{1}{2} E$, in the plane of a circle we have $\frac{5}{16}$ d, in the periphery thereof $\frac{3}{8}$ d, in each caſe = $\frac{1}{3}$ d nearly, ſo $\frac{1}{3}$ D a is nearly as the ſtrength of the hollow cylinder, and 0,245 D D D, as the ſtrength of the round beam, but a = D D — d d : × ,7854 = ,75 × ,7854 = ,589, alſo as ,7854 D D × 10 : 320 : : ,589 × 10 : 230 = z, then as, 245 D D D : $\frac{1}{2}$ e w + e v : : $\frac{1}{3}$ D a (,196 D) : $\frac{1}{2}$ e z + e y,

$$\text{whence } y = \frac{,196 \times : w + 2v : - ,245\,z}{,245 \times 2} = 24013 \text{ lb anſwer.}$$

Note. If the area a, be very ſmall, $\frac{3}{8}$ D, ſhould be uſed, but if a be pretty great, and d but ſmall, it is better to take $\frac{5}{16}$ D. This laſt anſwer will be truer if we take $\frac{11}{32}$ D, half the ſum of theſe two, tho' the difference is very little.

Question 102. If a ſquare beam 1 foot ſquare (D) 10 feet long (e) weight 440 lb (w) bear 31802 lb (v), what weight (y) will it bear when cut into a ſquare pyramid of the ſame length and baſe? See theo. 195.

1. Becauſe a pyramid is $\frac{1}{3}$ of it's circumſcribing priſm, therefore $z = \frac{1}{3}$ w, alſo $G = \frac{1}{4}$ e, $g = \frac{1}{2}$ e, D = d, P = p, C = c, by which the ſaid theorem, becomes $\frac{1}{2}$ w + v = $\frac{1}{4}$ z + y, whence y = v + $\frac{5}{12}$ w = 31802 + 183,33 = 31985,33 lb. Whence, the part will bear more than the whole.

Note. Theſe beams ſince queſt. 90, may either be fixt at one end or ſuported at both ends, either parallel to the horizon or make any angle therewith provided, any two beams in the ſame queſtion have the ſame poſition. In this queſtion the beam is fixt at one end, if it be ſupported at both ends it muſt be two = pyramids.

Question 103. If a cylinder of any kind of metal 1 foot diameter (d) 10 feet (e) length, weight (w) 900 lb, fixt at one end, and at the other bear a weight of 5000 lb before it break, (v) what weight, or force (y) will be fit to twine it about? See art. 302, and 304.

1. 245 d d d, or $\frac{5}{16}$ of ,7854 d d d, is as the lateral strength, and ($\frac{1}{4}$ b d d) $\frac{1}{16}$ b d d $= \frac{1}{8}$ of 3,1416 d d d (because 2 d $\times$ 3,1416 $=$ b) is as the twisting strength, but as $\frac{5}{16}$ of ,7854 : $\frac{1}{8}$ of 3,1416 : : 5 : 8 : : lateral strength : twisting strength. Therefore, as 5 : $\frac{1}{2}$e w $+$ e v : : 8 : $\frac{4\,we + 8\,ve}{5} = y = 87200$ lb answer.

Note. Because $\frac{1}{8}$ of 3,1416, is always the same, let d be what it will; therefore, the strength of cylindrical pieces, or of any similar pieces of timber, being twisted, will be as d d d, the cubes of their diameters. Therefore, if a force of 5 lb weight twine a hazel rod of half an inch diameter, a force of 40 lb, will twine one of 1 inch diameter, for as, 125 (cube of $\frac{1}{2}$) : 1 : : 5 lb : 40 lb.

Question 104. A cylinder one foot diameter (d), 10 feet long (e) 900 lb weight (w) fixt at one end parallel to the horizon, (as in the last question) and at the other end bears (v) 5000 lb at it's outmost strength, if it be laid on a plane parallel to the horizon, what weight or force (y) will pull it asunder? See art. 303.

1. Here G $= \frac{1}{2}$e, g $= \frac{1}{2}$d, s $= \frac{5}{8}$ d, so : $\frac{1}{2}$ e w $+$ e v : $\times$ d $=$ g s y, whence, $y = \frac{d\,e \times : \frac{1}{2}w + v :}{g\,s} = \frac{10 \times : 450 + 5000 : \times 16}{5} =$ 174400 lb the answer.

Question 105. If the last mentioned cylinder, be suspended by one end, what weight at the other end will pull it asunder? See art. 303.

1. Because in this case its own weight 900 lb, is also employed in pulling it asunder, therefore (see the last question) 174400 — 900 $=$ 173500 lb to part it at top.

Question 106. A beam length e, weight w, fixt at one end, bearing a weight v, at the other end, what weight will it bear in the middle, when supported at each end, and also, when fixt, or nailed down at each end? See art. 311.

1. When fixt at both ends, then $\frac{1}{4}$ e $\times \frac{1}{2}$ w $+ \frac{1}{2}$ e $\times \frac{1}{2}$ v, is as the stress, if only one end be fixt, but the other fixt end bears $\frac{1}{2}$ weight, therefore, half of $\frac{1}{8}$ e w $+ \frac{1}{4}$ e v $= \frac{1}{16}$ w $+ \frac{1}{8}$ e v, is as the stress when nailed down at both ends; but $\frac{1}{2}$ e w $+$ e v, $\propto$ stress when fixt at one end, and $\frac{1}{8}$ e w $+ \frac{1}{4}$ e v, when supported at both ends, hence these stresses are as 1,8 and 2 so the weights they'll bear, are as 8,1 and 4.

Question 107. Two similar rectangular beams, or plates, of the same depth d, lengths e and E, breadths b and B, weights w and z, weights borne v and y, whether is the stronger? See art. 299, and 307. The plates &c. being of the same matter.

1. B d d $\times$: $\frac{1}{2}$ e w $+$ e v : $=$ b d d $\times$: $\frac{1}{2}$ E z $+$ E y : and taking d d out of each side, and dividing by e and E, it will be e B $\times$: $\frac{1}{2}$ w $+$ v : $=$ E b $\times$: $\frac{1}{2}$ z $+$ y : but because the solids in respect of length and breadth are similar, e B $=$ E b, for as B : E : : b : e, therefore, w $+$ 2 v $=$ z $+$ 2 y. Hence, if w and z, the weights of the solids, be neglected, as they may in small ones, we'll have v $=$ y, that is, they will bear equal weights, as per art. 299.

From these questions appear the impossibility of things being infinitely great, for the strength being as the cube of the depth, and the stress as the product of the matter and length, it's plain the strength increases in a less ratio than the stress, and therefore, any wall, tree, beam, mechanic engine, and even man or beast, may be taken so large, as to break, or be crush'd by it's own weight, whence it is, that little animals, machines &c. (similar and of the same matter) are more active, will carry more weight, leap further &c. for their size, than great ones, because the strength increases in a less proportion than the size, or weight, and so cannot resist any violence in the same ratio of size.

Question 108. A wall is to be built with a plane rectangular face towards the wind, what must be the form of its backside, that it may be equally strong to resist the wind? See art 305.

1. If the parts of the wall stick well together, the back-side thereof must be a streight slope, or the thickness of the wall a plane triangle whose hypothenuse is the said slope. But if the wall is built of loose materials, the said slope should be a common parabolic curve.

Question 109. In what form must a tower &c. be built to be equally strong throughout, in resisting the wind? See art. 305.

1. As in the last question, when the weight presses uniformly, or the parts stick well together, then a cone standing on its base is best. Otherwise, parabolic conoids are equally strong throughout; for a cone or pyramid fixt at base, or a wedge so fixt at it's thicker end, and its two parallel sides, parallel to the horizon, a weight, or power pulling at the other end, will no sooner break it in one place than in another. Also, if a beam &c. has each side cut in the form of a parabola, fixt at base, pull'd at any where, it is equally strong between the base, and place so pull'd at, and therefore equally fit throughout to support itself. Also, if a spring be made by the first part of the said art. 305, the stress will in every part be proportional to the strength.

Question 110. A veſſel, or wall, to hold a bank of earth, or any fluid body, in what form muſt its out-ſide be, that it may be equally ſtrong throughout, when the in-ſide is ſtreight, and upright? See art. 305. and the laſt queſtion.

A wall to anſwer this end, if the parts ſtick well together, ſhould be concave in form of a ſemi-cubical parabola. But if of looſe materials then a right line or ſloping plane ought to be it's figure.

Question 111. A beam, or foot-bridge, horizontally ſupported at each end, into what form muſt its under-ſide be cut that it may be equally ſtrong throughout to ſupport a variable weight? (Fig. 202) See art. 306.

1. The form of the under-ſide muſt be A Q B, a half ellipſis, but if the beam ſhould be equally ſtrong throughout to ſupport its own weight then Q A and Q B ſhould be two common parabola's, in which caſe $\frac{1}{3}$ of the beam may be cut away without any loſs of ſtrength.

Question 112. A bow is to be made out of a cylinder 10 feet long, and 1 foot diameter, into what form muſt it be cut to be equally ſtrong throughout in bending, and what is its ſolidity when ſo cut (fig. 203) See art. 305.

1. This is the ſame thing as to make a ſpring equally ſtrong &c. If the bow is to be made tapering from the middle then its form A D E B, muſt be two conoids A D E and B D E of two equal cubic parabola's, joining in the middle on one common baſe at E, diameter there equal 1 foot, and therefore, it's ſolidity will be to that of the whole cylinder as $\frac{3}{5}$ to 1, or as 3 to 5, and therefore $\frac{2}{5}$ of ſuch a cylinder may be cut away without loſs of ſtrength, but if the bow is to be made of half a cylinder, and the inſide A D B, to be every where of the ſame breadth, then the out-ſide A E B, muſt be two common parabolic curves, A E = B E, then it's ſolidity will be to that of the half cylinder as $\frac{2}{3}$ to 1, or as 2 to 3, ſo $\frac{1}{3}$ of the half cylinder may be cut away.

2. Otherwiſe, If we ſuppoſe A B perpendicular to the horizon, and a weight w laid upon A to bend the bow, then ſince this weight w acts in direction A B, it may be ſuppoſed to act at C, in any part of A B, as at the end of a lever C D, whence (by theo. 190) the ſtreſs at D, will be as C D, or as A C, or as A D, becauſe, C D, A C, and A D, are all nearly right lines. Therefore, if d = e E, the depth of the body A E B e D be = it's breadth, then if the ſtrength which is as b d d, be every where as the ſtreſs, that is, A C be as b d d, it will be the very ſame with the firſt part of art. 305, and ſo the anſwer as before. In like manner, if E A e be a prop ſet oblique to the horizon, to ſupport a weight w, it's evident, this prop to be equally ſtrong throughout, muſt be cut into the form of half the bow, with it's greater baſe ſet on the horizon F e E. Alſo, if a piece of timber &c. be bended in

ſeveral places, the force to break it in any point D, will be as a perpendicular D C, let fall from that point upon A B, the line of direction of the force, let the bending be what it will, and in any direction.

Queſtion 113. If a beam A B (fig. 161) 10 feet long, ſupported at both ends, bend thro' a diſtance C D 6 inches from it's middle, what will this deflection be if the ſaid beam be 20 feet long? See art. 310, and 311.

1. Let $e = 10$, $E = 20$, $a = CD = 6$ inches $= 0,5$ feet, then b and d being = in both, it will be as $e\,e\,e\,e : (0,5)\,a :: E|E\,E\,E\ \frac{a\,E\,E\,E\,E}{e\,e\,e\,e}$ $= 8$ feet, the anſwer; for as $e : w :: E$: it's weight, the weights being as the lengths, but if the beam be a breaking with 6 inches deflection, then as $e\,e : a :: E\,E : E\,E\,a \div e\,e = 2$ feet, the deflection of the longer beam, when it breaks.

If two props E and F (fig. 204) are to be ſet to the ſlender cylinder D e H e D, ſo that the bending may be the leaſt poſſible, then D e = D e muſt be $= \frac{1}{7}$ of D H D, the whole length of the cylinder?

Theſe queſtions are of great uſe in the conſtruction of machines, teaching how to make their parts equally ſtrong, and of the leaſt weight &c.

Alſo, If the ſtrength of any ſmall piece of any ſort of timber &c. be known, the ſtrength, or weight, that any beam of the ſame matter will bear may by theſe queſtions ſoon be found. Thus, if a ſquare priſm of oak, a foot long, and an inch ſquare, be horizontally ſupported at each end, it will bear in the middle 320 lb weight before it break, and ſo may any other body be tried; and thus, the proportion of ſtrength of the following bodies is found, viz. oak, box, yew, plumb-tree 11, elm, aſh, $8\frac{1}{2}$, walnut, thorn $7\frac{1}{2}$, red fir, hollin, elder, plane, crab-tree, apple-tree 7, beech, cherry-tree, hazel, $6\frac{1}{2}$, alder, aſp, birch, white-fir, willow 6. Iron 107, braſs 50, bone 22, lead $6\frac{1}{2}$, fine free ſtone 1. A good hempen rope of an inch circumference drawn in length with 1000 lb will break. See Emerſon's mechanics, for theſe experiments.

The 26 following queſtions are about pendulums, muſical cords, ſounds, bells &c.

Queſtion 114. A pendulum length 11, = C S B (fig. 158) ſuſpended at S, at 1 within the end, what weight B muſt be at the lower end, that the pendulum may ſwing the faſteſt poſſible, when a weight C, of 100 lb is fixt at top.

1. Let $S\,C = c = 1$, $S\,B = b = 10$, and the points O and G, the centers of oſcillation and gravity, then (art. 294) $\frac{B\,b\,b + C\,c\,c}{b\,B - c\,C} =$

$SO = 2SG = \frac{2Bb - 2Cc}{B + C}$, which multiplied crofs-wife and reduced, is $BB - : \frac{4Cbc - Cbb - Ccc}{bb} : \times B = - \frac{CCcc}{bb}$ put $-2z = \frac{4Cbc + Cbb + Ccc}{bb}$, then $BB - 2z = -\frac{CCcc}{bb}$ complete the fquare &c. then you'll find $B = z + \surd : zz - \frac{CCcc}{bb} :$ $70,5 + \surd : 4970,25 - 100 : = 70,5 + 69,6 = 140,1$ lb anfwer.

Queftion 115. A pendulum 11 long, fufpended 1 within the upper end, what weight B muft be at the lower end, that the pendulum in vibrating may have the greateft momentum poffible, with the moft eafe when a weight C is fixt at it's upper end, of 100 lb? See queft. 253.

1. Let $c = SC = 1$ (fig. 158) $b = SB = 10$, then (art. 292.) $\surd : \frac{Bbb + Ccc}{Bb - Cc}$: is as the time t of vibrating, which time is as v the velocity, and v B, is as the momentum, therefore $B \surd : \frac{Bbb + Ccc}{Bb + Cc}$ $\propto vB \propto$ the forces of the body, B, or C, therefore by making this expreffion, or $\surd : \frac{BBBbb + BBCcc}{Bb - Cc}$: it's equal a maximum, we'll (by art. 223, or 421,) B alone variable $\frac{3BBbb + 2BCcc}{\surd : B^3b^2 + BBCcc : \times \surd : Bb - Cc :}$ $- \frac{b\surd : BBBbb + BBCcc :}{\overline{Bb - Cc}|^{\frac{3}{2}}} = 0$, which reduced is $BB + : \frac{Cbcc - 3bbCc}{2bbb} : \times B = \frac{CCccc}{bbb}$, put $2z = \frac{Cbcc - 3bbCc}{2bbb}$, then it will be $BB + 2z = \frac{CCccc}{bbb}$, which by completing the fquare &c. gives $B = z + \surd : zz + \frac{CCccc}{bbb} : = 7,25 + \surd : 52,5625 + \frac{10000}{1000} = 15,16$ lb the anfwer.

Hence, any machine working beams, as levers, &c. with an accelerated motion, is in its beft perfection, when if the power B be in equilibrio with the weight C, and there be taken 1,516 B inftead of B, or nearly 1,5 B, viz. the faid power B increafed one half; for it is plain, if a pendulum fwing flower it lofes time, and if fafter it will be too much fhaken or ftrefs'd.

Note. In the following queſtions about pendulums, any body will anſwer the ſame end, whoſe length between the point of ſuſpenſion and center of oſcillation is = the length of the pendulum. (Queſt. 116)

If a cylinder, or priſm, of equal matter every where, and 58,8 inches length, be ſuſpended cloſe by one end, it will be a ſecond pendulum, for $\frac{2}{3}$ of 58,8 = 39,2 &c.

Queſtion 116. If in any latitude, a heavy body by the force of it's own gravity fall thro' a diſtance of 139$\frac{1}{4}$ inches in the firſt ſecond of time, what muſt be the length of a pendulum in that latitude to vibrate ſeconds?

By theo. 175. As the ſquare of 3,14159 &c. is to 1 ſo is 139$\frac{1}{4}$ inches to 19,6 inches, which doubled gives 39.2 inches the length of a ſecond pendulum viz. of one that vibrates 60 times in a minute.

Queſtion 117. If the length of a ſecond pendulum be 39,2 inches, what is the length of a half ſecond pendulum? See theo. 174.

As ſquare 2 ſeconds (4) is to ſq. 1 ſecond (1) ſo is 39,2 inches to 9,8 inches the anſwer.

Queſtion 118. Pray what length muſt a pendulum be,
To vibrate once in ſeconds three;
Where ſecond pendulums have their lengths,
Thirty-nine inches and two tenths.

By queſt. 117. It will be as ſq. 1 (1) is to ſq. 3 (9) ſo is 39,2 to 352,8 inches anſwer. So that if a rope or cord have one end faſtened to the top of a houſe &c. and at the other end there be a weight, and if it vibrate once in 3 ſeconds, the length of the cord or height of the houſe is 352,8 inches.

Queſtion 119. If a pendulum meaſures mean or equal time when it vibrates in an arch of 5°, what ſeconds will it loſe per day when it is made to vibrate in an arch of 7°? See theo. 177.

From 49 = ſq. 7, take 25 = ſq. 5, there leaves 24, and $\frac{3}{7}$ of 24 is 10$\frac{2}{7}$, the required number of ſeconds loſt per day.

Queſtion 120. If a pendulum beats ſeconds when the height of the barometer is 30,25 inches, what time will it beat when the ſaid height is 36 inches? (Art. 443.)

1. Here m and $\frac{c}{d}$ are each very ſmall, ſo $2mb + \frac{4cc}{3d} - \frac{16cc}{9dd}$ &c, = I D, will be $2mb + \frac{4aa}{3d}$ = I D nearly, the difference between the arch's of deſcent and aſcent B A and A I. Now if m be negative or — m, or ſuppoſed to accelerate the motion of the pendulum, ſo as juſt to overcome the reſiſtance of the medium, the pendu-

lum will then vibrate ſtill in the ſame arch, and then $ID = 0 = 2mb + \frac{4aa}{3d}$ whence $3mbd = 2aa$; therefore as m, the force acting uniformly on the body to counterballance the reſiſtance of the medium, is to 1, the weight of the pendulum, ſo is $2aa$ to $3bd$. Hence, if the force m, which keeps the pendulum in motion, be conſtant, a, the arch deſcribed will be as $\sqrt{bd}$, or if m be variable and bd conſtant, then a is as $\sqrt{m}$.

2. Again, when $m = 0$, c will be $= a$, and then $T = 3{,}1416\sqrt{b} \times : 1 + \frac{aa}{6dd} - \frac{2aaa}{9ddd}$ &c. which ſuppoſe $= 1$ ſecond, the time of deſcribing the arch a, when the height of the barometer is $30{,}25 = K$, let $h = 36$ the other height, and $c =$ arch deſcribed at that height in the time t; then becauſe $\sqrt{bd} \propto a$, or (becauſe b is conſtant) $\sqrt{d} \propto a$, and (it's plain) the denſity of the medium or height of the barometer is inverſely as d, it will be as $\sqrt{h} : \sqrt{H} :: a : a\sqrt{\frac{H}{h}} = c$ and as $h : H :: d : \frac{Hd}{h} = D$, whence, if $3{,}1416\sqrt{b} \times : 1 = \frac{aa}{6dd}$ &c. $= T = 1$, then $3{,}1416\sqrt{b} \times : 1 + \frac{cc}{6DD}$ &c. $: = t$, therefore as $T : t :: 1 + \frac{aa}{6dd} : 1 + \frac{cc}{6DD}$, where if a and d be known, the true anſwer may be had; but the following way may give a gueſs and is much eaſier, thus, ſince $h \propto d$ and $H \propto D$, let a be ſuppoſed equal 6, then as $T : t :: 1 + \frac{aa}{6hh} : 1 + \frac{cc}{6HH} :: 1 + \frac{aa}{6hh} : 1 + \frac{aa}{6Hh} :: 1 + \frac{1}{216} : 1 + \frac{1}{181{,}5} ::$ 1 ſecond : 1,0008 ſeconds the anſwer, which (tho' ſo great an alteration as $5\frac{1}{2}$ inches in the barometer can rarely happen) is ſo ſmall as ſcarce to be regarded, the alteration which happens by heat lengthening the rod of the pendulum is much more, tho' in theſe parts it is not ſo much as in hotter climates, but becauſe heat raiſes mercury and expands metals, a cylinderic tube filled with mercury, might be ſo taken for the rod of a pendulum, as nearly would correct this error.

3. If h and H be inverſely as the denſities or any other two perfect fluids, the above work holds true, as well in the ſmall arch of a circle as in any arch of a cycloid for which this queſtion is intended.

4. In the above value of T, when d is infinite, or the resistance of the medium $= 0$, then $T = 3,1416 \sqrt{b}$, the same with art. 424. Also, $T = 3,1416 \sqrt{b}$ the same thing whether m be $= 0$, or $=$ any finite number; hence, if the resistance m be uniform, the vibrations will be isochronal, and performed in the same time as if there were no resistance, if the bobs of pendulums be pretty dense, as of lead, iron, &c. the resistance they'll meet with in air, is so little as not to be observed.

Question 121. If there be 12 threads in an inch length on the screw of a pendulum, and the clock gain 3 minutes per day, or 24 hours, how many threads must the bob be let down to measure mean time? See theorem 177.

Here $\frac{2}{37}$ of 3×12 is $= 2$ threads nearly, for the answer; but if the clock lose three minutes per day, the bob must be raised near two threads. By these two last questions any pendulum clock may be regulated, the last method being easiest is most practised.

Question 122. Required the length of a second pendulums rod, it's bob being a globe (as commonly they are) of 2 inches radius. See art. 292.

Let $2l = 29,2$ in. the length of a 2d pendulum, $r = 2$ the radius of the bob, to find d, the distance between the centers of the bob and pin on which the pendulum hangs, then $2l = d + \frac{2rr}{5d} = d + \frac{0,4rr}{d}$ whence $dd + ,4rr = 2dl$, and by compleating the square, $dd - 2dl + ll = ll - ,4rr$ which solved gives $d = l + \sqrt{:ll - ,4rr:} = 39,16$ inches, answer. Or $(39,16 - 2)$ 37,16 inches, the length of the rod between the pin, and periphery of the bob.

Question 123. If s a (fig. 206) be 30 inches the length of a pendulum's rod between the pins, on which it hangs, and the bob a, and its weight to that of the bob as 1 to 2, what is it's true length, the radius d a of the bob being $1\frac{1}{2}$ inches?

The rods of pendulums are commonly light in respect of the bobs, but if their weights be considerable, then the common center of oscillation of the rod and bob, points out the true length of the pendulum, let O be this center, B the center of oscillation of the bob and A, that of the rod, then comparing this question with theo. 194, we'll have $a = SA = \frac{2}{3}$ of $Sa = 20$, $A = 1$, $Sd = (30 + 1,5) = 31,5$, $B = 2$, and b (by art. 292) $= 31,5 + \frac{2\,\square\,da}{5\,da\,S} = 31,5 + \frac{0,9}{31,5} = 31,53 = SB$ whence $\frac{Aaa + Bbb}{Aa + Bb} = SO = 29$ inches fere, answer, then

(by queſt. 117) As 29 : 39,2 : : □ 1 ſecond (1) : 1,35 whoſe ſquare root is 1,19 ſeconds the vibrations this compound pendulum will make in 1 ſecond.

Queſtion 124. If a pendulum 39,2 inches long, vibrate ſeconds at London (latitude 51° 32′) what time will it gain or loſe, and how much per day, when removed to Whitehaven, (latitude 54° 56′). See queſtion 271.

Since the earth turns round its axis in 24 hours it's evident that every particle of matter on it's ſurface will in that time deſcribe a circle the greateſt of which will be that deſcribed by a particle under the equinoctial, viz. on the middle of the earth and conſequently will have the greateſt velocity, which velocity manifeſtly endeavours to throw away every thing from the center, ſo that where this velocity is greateſt, the force of gravity (which impels) towards the center muſt be leaſt, from which conſiderations, Sir Iſaac Newton has proved, that the gravity at the poles is to the gravity at the equator, as 692 to 689, therefore the decreaſe of gravity at the equator is $\frac{3}{692}$ parts of the whole, but this decreaſe at the equator is to that in any other latidude, as the ſq. of radius is to the ſquare of the ſine of that latitude, therefore, as 1 (□ radius 1) is to ,6131 (□ ſine 51° 32′) ſo is 3 (decreaſe of gravity at the equator) to 1,8393 (decreaſe of gravity at London) which added to 689 the gravity at the equator gives 690,8393 the gravity at London, in like manner you'll find 691,009 for the gravity at Whitehaven. Then as 690,8393 is to 691,009, ſo is 39,2 to 39,21 inches the length of a ſecond pend. at Whiteh. by queſt. 117, the clock will be found to gain 4,176 ſeconds per day. From this ſolution it appears that 689 lb at the equator will weigh 691,009 lb at Whitehaven, i. e. if a perſon can ſuſtain 691,009 lb at the equator, he will be equally ſtrained with 689 lb at Whitehaven, which is the reaſon that the clock gains time there, for where gravity is quickeſt, the vibrations of pendulums muſt be ſo too, gravity being the cauſe of that motion. By queſtion 116.

Queſtion 125. If a muſical ſtring weighing 8,64 grains or ,0015 lb length $2\frac{1}{2}$ feet or 30 inches, be ſtretched with a weight at one end of 10 lb, how many times will it vibrate in one ſecond, allowing a heavy body to deſcend 16 feet in the firſt ſecond of time from a ſtate of reſt? See theorem 179.

Here $\sqrt{:\dfrac{2 \times 16 \times 10}{2{,}5 \times {,}0015}:} = \sqrt{\dfrac{320}{{,}0037}} = \sqrt{85333} = 292{,}1$ times anſwer.

Queſtion 126. If a ſeconds pendulum, be made with a thread and leaden bullet, and put to ſwinging in an arch of 12 inches, and the bul-

let be obſerved to fall $\frac{1}{4}$ of an inch ſhort of 12 inches the firſt vibration how many ſeconds will it ſwing before it be at reſt? (Queſtion 120, art. 443.)

1. Since $I D = 2 m b + \frac{4 c c}{3 d}$ &c. if d be infinite, or the reſiſtance of the medium $= 0$, then, $I D = 2 m b$, i. e when the reſiſtance m is uniform, I D the difference between the arches of deſcent and aſcent, B A and A I is ſtill the ſame, let the arch deſcribed be what it will, and (ſtep 4th queſtion 120) all ſuch arches are deſcribed in the ſame time, and the reſiſtance this pendulum meets with from the air, being ſo ſmall as to be neglected, it will ſwing ſo many times before it be at reſt, as there are quarters of an inch in 12 inches, viz. 48 times or ſeconds.

Queſtion 127. With what weight muſt a cord 6 feet long, be ſtretched parallel to the horizon, that it may vibrate half ſeconds allowing 16 feet for the deſcent of gravity in the firſt ſecond? See theorem 179.

Here, $\sqrt{\frac{w l}{2 s a}} = t$, viz. $\sqrt{\frac{6 w}{32 f}} = 0{,}5$, or $0{,}25 \times 32 f = 6 w$ i. e. $8 f = 6 w$, or $f = \frac{3}{4} w$, hence, the tenſion muſt be 3 fourths of the cord's weight.

Queſtion 128. There are two muſical cords of equal lengths and diameters, with what force or tenſion, muſt each be ſtretched, to ſound a fifth? See theorem 178.

Here, the vibrations being as 3 to 2, the tenſions muſt be inverſely as the ſquares of theſe numbers viz. as 9 to 4.

Definition, If two muſical ſtrings vibrate in the ſame time the concord is moſt perfect, and more agreeable to the ear than any other, and is called uniſon, and ſo on as here ſet down.

If the times of the vibration of two muſical ſtrings be,

As	1	to	1	it is called	uniſon, which is moſt pleaſing to the ear,
	1		2		diapaſon or octave, leſs pleaſing,
	2		3		diapente or fifth, leſs pleaſing,
	3		4		diateſaron or fourth &c. of which this laſt

and the next that follows it in order are not ſo pleaſing to the ear, and are therefore called imperfect concords, nor are there above ſeven notes (beſides the half notes called flats and ſharps, by which the natural notes made half a note higher, or lower, as the nature of the muſic requires) in all the infinite variety of tones fit to merit a place in muſical compoſition.

Queſtion 129. If two muſical ſtrings of the ſame kind A and B have their tenſions equal, and their diameters as 5 to 6, what muſt be their lengths to ſound a diapente or 5th? See theo. 178.

In this caſe the vibrations being as 2 to 3, or as 3 to 2 it will be, as $3 : 2 :: 5\,l : 6\,L$, whence, $10\,l = 18\,L$, or $l = 1{,}8\,L$, ſo as $1 : 1{,}8 :: L : l$, that is the length muſt be as $1 : 1{,}8$.

Queſtion 130. There are two muſical ſtrings of equal length whoſe weights are as 5 to 6, what muſt be their tenſions to ſound an octave?

(By theorem 178.) As $1 : 2 :: \surd\frac{5}{f} : \surd\frac{6}{F}$, ergo $2\surd\frac{5}{f} = \surd\frac{6}{F}$, or $20\,F = 6\,f$, whence, as $20 : 6$, or as $10 : 3 :: f : F$, i. e. the tenſion of the lighter ſtring muſt be 10, to make it vibrate twice while the heavier ſtring whoſe tenſion is 3, vibrates once.

Queſtion 131. There are two muſical ſtrings of equal tenſions whoſe lengths are as 3 to 1, and their vibrations are as 5 to 6, in the ſame time, viz. in ſeſquiditonus, what is the ratio of their diameters? See theorem 178.

As $5 : 6 :: 3\,d : D$, or as $5 : 18 :: d : D$, that is, the diameter of the longer ſtring is to that of the ſhorter as 5 to 18, and vibrations as 5 to 6. From theſe queſtions may muſical inſtruments be ſtrung to the beſt advantage.

Queſtion 132. If in a minutes time you eſpy,
The moving wings of a ſmall fly,
Nine thouſand times to ſhift;
What number then muſt be the ſwings,
In the ſame time of a bee's wings,
To ſound a perfect fifth?

As the fly is leſs than the bee, let her ſwings in the ſame time be more than thoſe of the bee's, then to make a fifth, the vibrations in the ſame time being as 3 to 2, we'll have as $3 : 2 :: 9000 : 6000$ anſ.

Queſtion 133. If a flute 10 inches long be uniſon with a muſical ſtring, what muſt be the length of another flute of the ſame bore and force of wind, to ſound an octave with the ſaid ſtring?

Definition, In wind inſtruments of muſic, the ſound being made by the vibration of a column of elaſtic air contain'd in the tube, the time of vibration or tone of the inſtrument, muſt vary with the length and diameter of the ſaid column of air which compreſſes it, in the ſame manner as the tones of muſical ſtrings vary with their lengths, diameters and tenſions, from whence (by queſtion 128) the length of the required tube will be 5 inches.

Queſtion 134. If the axis of a ſpeaking trumpet be 10 feet, and the area of the orifice at the out end 0,5 foot, how many times will it magnify the voice of the ſpeaker? See queſtion 137.

If we ſuppoſe, a perſon plac'd in the center of a ſphere, whoſe radius is equal the length of the tube 10 feet, and there ſpeak without

the tube, it's then plain his voice will be equally diffused thro' the whole surface of the sphere, but speaking thro' the tube with the same strength of voice it will be only diffused thro' that part of the sphere's surface which subtends the out end of the tube, and because this part of the said surface is but small, it may be taken for the area of the said end of the tube, and then it will be, as 0,5 the area of the said orifice is to 1256.64 the surface of a sphere whose radius is 10, so is unity (1) to 2513,28 times answer.

Question 135. If the axis of a speaking trumpet (otherwise called the stentorophonic tube) be 10 feet, the diameter at the lesser end 1 inch, and the diameter at the greater end 8 inches, what must be the diameter at every foot in length when the trumpet is made to the best advantage?

Since the force of the voice is propagated thro' a series of elastic bodies of air in the tube, it is evident (from art. 258) that if these portions of air be in geometrical progression, the voice will receive the greatest augmentation possible. Whence, if the required diameters divide the axis into 10 equal parts, these parts being small may be taken as cylinderical bodies of air in geometrical progression,, but cylinders of equal altitudes are as their bases, and these cylinders being by supposition indefinitely short, will be nearly as their diameters, whence the diameters of the tube, must be in geometrical progression, and therefore if they be 11 in number, the least being 1 (Q) and the greatest 8 (Qe^{n-1}) see art 258, it is required to find e the common ratio of the diameters n being equal 11, so $n-1=10$, therefore, Q being equal 1, we have, $e^{10}=8$, or $e=\sqrt[10]{8}=1,232$, so the least diameter viz. 1 being multiplied 1,232 gives 1,232, the diameter at a foot length from the lesser end, and $1 \times \square\, 1,232$ gives 1,515 inches the diameter at two feet from it, and so on. From this it appears that a speaking trumpet formed by the revolution of the logarithmetic curve about its axis, will augment the sound, more than one of equal length of any other shape.

Question 136. What must be the weight of a bell to sound the lower octave, with a smaller bell of the same kind that weighs 25 pounds?

The sound of a bell consists of a vibratory motion of its parts much like that of a musical string, now by the foregoing questions, the length of a string gives the lower, or graver octave to a like string half that length, or half the length gives the acute or sharp octave to the whole length, so as 1 (the cube of 1) is to 8 (the cube of 2) so is 25 lb to 200 lb answer.

It is proved by experiment that if a bell be rung in the exhausted receiver of an air pump, it will give no sound, whence it follows that

ſound is nothing but the tremors and vibrations of the particles of a ſonorous body impreſſed on thoſe of the air, and as a pool of ſtagnated water, being any how ſtruck on its ſurface, will by that ſtroke be put into waves, which are greateſt where the ſtroke is made, and decline gradually in a ſpherical form around the ſaid ſtroke till at laſt they have no motion, ſo the air being ſtruck, the waves, or pulſes of it give the greateſt ſound neareſt the ſtroke and as theſe pulſes weaken the ſound is leſſened, till at laſt it cannot be heard. Hence, ſounds are ſtrong or weak, according as we are nearer to, or farther from the ſounding body, and ſince (by theo. 178) the vibrations of a ſounding body are each made in the ſame time and theſe vibrations being the cauſe of ſound, it's evident that the intervals of aerial pulſes will be ſo too, and that all ſounds whether loud or low, which are excited by the vibrations of the ſame body, are of one tone ; alſo, thoſe bodies which vibrate ſloweſt have the graveſt or deepeſt tone, and thoſe bodies which vibrate quickeſt, have the ſharpeſt or ſhrilleſt tone, ſounds are loudeſt when the preſſure of the atmoſphere is greateſt viz. in dry weather for then the particles of moiſture in the air are feweſt, ſo it's elaſticy is greateſt, Sir Iſaac Newton, (by comparing the vibrations of theſe aerial waves, or pulſes with thoſe of a pendulum, whoſe length is equal to the height of a homogenial atmoſphere every where of the ſame denſity with the air at the earth's ſurface, the bulk of a particle of air equal to one of water or ſalt, the air to the vapours in it as 10 to 1 &c.) proves the motion of ſound to be uniform and at the rate of 1142 feet per ſecond, but in every 10 miles we muſt allow about half a mile when the wind blows ſtrongly againſt the ſound, and deduct the ſame when it blows with it, but when it blows croſs the ſound it makes little alteration in its velocity, all which agrees with experiments of firing guns &c.

Queſtion 137. If a body A, be placed 3 feet diſtant from a lighted candle and a body B, 10 feet therefrom, what proportion of light hath each ?

If the ſame quantiy of light be placed in the centers of two different ſpheres its evident the ſurface of each ſphere will be illuminated by this equal quantity of light inverſely as the ſaid ſurfaces, but the ſurfaces of ſpheres are as the ſquares of their diameters or radiuſes therefore, as 9 (ſq. 3) is to 100 (ſq. 10) ſo is the light received by B to that received by A, the ſame is to be obſerved in heat, cold ſound, &c.

Queſtion 138. If it be 10 ſeconds of time between dropping a ſtone 6 lb weight into a pit, and hearing it ſtrike the bottom, how deep is the pit, and with what weight doth the ſtone fall at bottom, allowing ſound at the rate of 1142 feet per ſecond, and the deſcent of heavy bodies 16 feet in the firſt ſecond of time ?

1. Let d = the pit's depth, then as $\sqrt{16}$ feet : 1 ſecond : : $\sqrt{d}$: $\frac{1}{4}\sqrt{d}$ = time of the ſtone's deſcent, and as 1142 feet : 1 ſecond : : d : $\frac{d}{1142}$ = time of the aſcent of ſound, now by the queſtion the ſum of theſe two times is = 10 ſeconds, that is, $\frac{1}{4}\sqrt{d}+\frac{d}{1142}=10=t$, ſo $d+285{,}5\sqrt{d}=1142\,t$, and by compleating the ſquare, $d+285{,}5\sqrt{d}+\square\frac{285{,}5}{2}=1142\,t+\square\frac{285{,}5}{2}$, whence $\sqrt{d}=\sqrt{:}\ 1142\,t+$ ſquare of $\frac{285{,}5}{2}:-\frac{285{,}5}{2}=35{,}6$ nearly, which ſquared is d = 1267,36 feet the depth ſought.

2. Now the body falling 16 feet = S, the firſt ſecond of time, it then has 16 feet velocity, which in the ſame time will carry it uniformly over 2 S, twice that ſpace, therofore, as $\sqrt{S}$: 2 S : : $\sqrt{d}$: $2\sqrt{dS}$, the uniform velocity acquired by falling thro' d ſo 6 lb $\times\, 2\sqrt{dS}=6\times 2\times 35{,}6\times 4=1708{,}8$ lb, the required force with which the ſtone ſtrikes the bottom.

3. If $d=\frac{1}{64}$, then $\sqrt{dS}=0{,}5$ and $6\times 2\sqrt{dS}=6$ lb, or the force of the ſtroke = body's own weight, but if d = 0, then this force is = 0, now if the velocity be the ſame, this force or momentum is the ſame, let the body move in what direction it will, but if it fall directly downwards, it's evident it preſſes with it's own weight more than the momentum, whence 1708,8 + 6 = 1714,8 lb (in this queſtion) is the force wherewith the body preſſes at the bottom of the pit the moment it comes there. This is called preſſure, and ought in all ſuch caſes to be conſidered from momentum, if the body move down, an inclined plane, it's weight thereon may be found by what goes before.

Queſtion 139. If a man ſtanding at the ſide of a river oppoſite to a bank or ſome houſes &c. on the other ſide, hear his voice reflected back in 3 ſeconds of time (called echo) what is the river's breadth.

When ſound ſtrikes any large obſtacle (ſee art. 259) it is reflected back with the ſame velocity, ſo in this caſe, the ſound moves twice over the river in 3 ſeconds; therefore, half of 1142 × 3 = 1713 feet anſwer.

Note. To increaſe the hearing with a tube, it ſhould be made as directed in queſtion 135, the longer the better, and ſet the leſſer end to the ear, and the wider end gathering in more pulſes of air will mightily increaſe the hearing, &c.

Here follow 26 queſtions concerning the maxima of bodies moving in fluids, how to conſtruct mill's engines &c. to the beſt perfection the force of moving bodies &c.

Question 140. If the length of a ſolid be 50, and the radius of the greater baſe 20, required the radius of the leſſer baſe, and form of the ſolid, that moving in a perfect fluid in direction of its axis with the leſſer baſe foremoſt, it may be the leaſt reſiſted poſſible, or leſs, than any other ſolid of the ſame length, baſe and ſurface. Fig 210.

1. In art. 435, we have $ayuvvv = zzzz$ for the equation of the curve generating this ſolid, which (art. 418) is found $e = \frac{1}{a} \times : \frac{3}{4}$ $nnnn + nn - 2{,}3025 \log. n :$ and $y = \frac{\overline{nn+1}^2}{na} = \frac{nnn+2n}{a} + \frac{1}{na}$ which made a minimum (n variable) we get $\frac{1}{a} \times : 3nn - \frac{1}{nn} + 2 :$ $= 0$, which ſolved gives $n = \frac{1}{\sqrt{3}}$, ſo $y = \frac{\overline{nn+1}^2}{na} = \frac{3{,}08}{a}$ fere for the leaſt ſemi-ordinate, alſo, if in the value of e, we write $\frac{1}{\sqrt{3}}$ for n, its equal, we'll have $e = \frac{5}{12a} - \frac{2{,}3025 \log \frac{1}{\sqrt{3}}}{a}$, which taken from the aforeſaid value of $e = \frac{1}{a} \times : \frac{3}{4} nnnn + nn - 2{,}3025 \log. n :$ leaves $e = \frac{1}{a} \times : \frac{3}{4} nnnn + nn - \frac{5}{12} - 2{,}3025 \log. n\sqrt{3} :$ by which values of e and y (e, by the queſtion being $= 50$ and $y = 20$) and taking e to begin at the middle of the leaſt ordinate, we get $50 \times : n^3 + 2n + \frac{1}{n} : = 20 \times : \frac{3}{4} n^4 + nn - \frac{5}{12} - 2{,}3025 \log. n\sqrt{3} :$ from which n is found near $= 3{,}5$, then $y = 20 = \frac{\overline{nn+1}^2}{na}$ we find $a =$ $2{,}5$, whence $y = \frac{3{,}08}{a} = 1{,}2$ the required radius of the leſſer baſe; now if the ſolid be form'd by the rotation of the ſpace EBRQ about the axis EQ, then $RQ = 20$, $EQ = 50$, $EB = 1{,}2$, and to find as many ſemi-ordinates $Fz = y$, with their reſpective abſciſſes $EF = e$, as you pleaſe, you may take $n =$ any numbers between $\frac{1}{\sqrt{3}}$ and $3{,}5$ ſuppoſe $= 3, 2, 1,$ &c. and you'll have $y = \frac{\overline{nn+1}^2}{na} = 13{,}33, 5,$

1,6, &c. and $e = \frac{1}{a} \times : \frac{3}{4} n^4 + nn - \frac{5}{12} - 2{,}3025$ log. $n \surd 3$: $= 30$, 5,8, 0,34, &c. by which the figure is easily constructed. It appears that the least ordinate E B cannot be $= 0$. yet if E Q and Q R be unlimited, the constant quantity a may be taken as great as you please, and then $y = \frac{3.08}{a} =$ E B, will be very small.

Question 141. If A B (fig 208) be the breadth of a river or canal, required the position of the flood-gates D A $=$ D B, resisting the water with the greatest ease?

First, Upon B D produc'd let fall the perpendicular A E, now the longer either gate is, suppose the gate B D, the greater will be the pressure of the water against it. Also, the longer timber of the same diameter is, the weaker it is; hence, in each of these cases the resistance is inversely as the length of the gate, and therefore, in both cases together it is as $\frac{1}{\square BD}$, but per similar Δs, as $\square BD : \square BC :: \square BA : \square BE$, or because B C $= \frac{1}{2}$ A C, and $\frac{1}{4}$ is constant, it will be as $\square BD : 1 :: 1 : \square BE \propto \frac{1}{\square BD}$, which is as the resistance of the gate B D; again, the force with which the gate A D resists the pressure of the gate B D at D, is $=$ the two forces E D parallel to B D and so avails nothing, and A E perpendicular, which so, is the only active force (the sum of these two forces being $=$ the one force A D,) hence $\square BE \times AE =$ a maximum, i. e. putting e $=$ A E and a $=$ B E, then : $aa - ee : \times e = aae - eee$, so (by art. 221) $aa = 3ee$, then $e = a \surd \frac{1}{3} = ,5773$ the sine of $35^\circ\ 16' = \angle$s D B A and D A B as required.

Question 142. If the length of a conical frustum be 50, and the radius of its greater base 20, required the radius of the lesser base, that moving in a perfect fluid, in direction of its axis with the lesser base foremost, may meet with less resistance than any other such frustum of the same axis and base? Fig. 216.

Let P n F, be a right angled triangle by whose rotation about F P, a cone is form'd in which E B n F, forms the required frustum, moving in direction F E, put r n $=$ e, r B $=$ F E $=$ b $=$ 50, n F $=$ a $=$ 20, then $\square nF - \square rF = aa - \overline{a-e}|^2 = 2ae - ee$, and $\square nB = bb + ee$, also $\square nr = ee$ this compared with $1 : \frac{rr}{ss}$ (art. 423) gives

$\frac{aa}{ee} : \frac{2ae - ee}{bb + ee}$, or ſuppoſing the reſiſtance of the baſe (a a) to be $=$ 1, we have $\frac{ee \times : 2ae - ee :}{bb + ee}$, for the reſiſtance of the ſurface deſcribed by n B, to which $\overline{a - e}|^2$, that of the circle deſcribed by B E, and we'll have $\frac{aabb - 2aebb + eebb + aaee}{bb + ee}$, for the whole reſiſtance of the fruſtum deſcribed by E B n F, which by the queſtion muſt be a minimum, and therefore making e variable we'll (art. 421) have $\frac{2abbee + 2ebbbb - 2abbb}{\overline{bb + ee}|^2} = 0$, whence $ee + \frac{bb}{a}e = bb$, and by compleating the ſquare &c. $e = \frac{b}{2a} \surd : 4aa + bb : - \frac{bb}{2a}$ $= 8$ nearly, ſo n F 20 — n r 8 = r F = B E = 12, anſwer.

Queſtion 143. If the axis of a ſolid be 50, and its ſolidity 3612264 (S) required its greateſt diameter, ſo that moving in a perfect fluid in direction of its axis with the leſſer baſe foremoſt may meet with leſs reſiſtance than any other ſolid of the ſame length and ſolidity? Fig. 210.

1. The equation of this ſolid (art. 435) is $auvvv = yzzzz$, which being ſolved gives (art. 418) $e = \frac{3ann + a}{2 \times : \overline{nn + 1}|^2}$, and $y = \frac{an}{\overline{nn + 1}|^2}$, where if $n = 0$, then $y = 0$, ſo in this caſe the curve will meet the axis, or the leaſt ordinate $= 0$, but when $n = 0$, $e = \frac{1}{2}a$, therefore, that the abſciſſa's and ordinates may begin together, or be $= 0$, at the ſame time this $\frac{1}{2}a$, muſt be taken from the above value of e, and then $e = \frac{3ann + a}{2 \times : \overline{nn + 1}|^2} - \frac{a}{2} = \frac{ann - annnn}{2 \times : \overline{nn + 1}|^2}$, where it appears that n muſt be leſs than unity, otherwiſe the numerator will be negative, then from $y = \frac{an}{\overline{nn + 1}|^2}$ and $e = \frac{ann - annnn}{2 \times : \overline{nn + 1}|^2}$, we get $y = \frac{2e}{n - nnn}$, which made a ſolid (C = 3,1416) we get $c\dot{u}yy =$ $4ee\dot{u}c \div : \overline{n - nnn}|^2$, whoſe fluent (theo. 78) is $\frac{4ceee}{3 \times : \overline{n - nnn}|^2} =$ S, ſo $\frac{3S \times : \overline{n - nnn}|^2}{4c} = eee = 125000$, therefore, $n - nnn =$

$\sqrt{:125000\frac{4c}{3S}:} = \frac{1}{8}$ nearly, whence $n = \frac{1}{2}$, and therefore, $a =$

$\frac{2e \times : \overline{nn+1}|^2}{nn - nnnn} = 833\frac{1}{3}$, so $y = \frac{an}{\overline{nn+1}|^{\frac{1}{2}}} = 240$, which doubled is 480 the required diameter, i. e. if this solid be formed by the rotation of the curve A B R, about A Q, $= e = 50$, and Q R $= y = 240$, the content of this solid $= s = 3612264$, and by taking $n =$ many fractions between 0 and $\frac{1}{2}$ ($a = 833\frac{1}{3}$ always) you may have as many abscissas and their respective semi-ordinates as you please, and so draw the curve.

Question 144. If a perfect fluid in direction C B, striking against a plane n B P, make an ∠ C B n therewith of 30°, Required the ratio of the forces of this fluid to push the plane forward, and to turn it about. Fig. 210.

1. Draw C n ⊥ B n and n r ⊥ C B, let $e =$ sine ∠ n C B = co-sine ∠ C B n, radius $= 1$, then $1 - ee =$ □ sine ∠ C B n; then if C B, which suppose $= 1$, express the whole force of the fluid, C n will express that part of it which tends to move the plane forward in direction C B and n r being ⊥ that direction, must therefore express the part of that force which tends to turn the plane about; but the whole force to move the plane forward being as □ sine incident ∠ C B n, viz. as 1 (□ radius) : 1 (C B) : : $1 - ee$ (□ sine ∠ C B n) : $1 - ee =$ force C n, and by trigonometry, as radius (1) : C n ($1 - ee$) : : $e : e - eee =$ force n r; whence, as force C n to push the plane forward : force n r to turn it : : $1 - ee : e - eee$: : $1 : e$: : (in this quest.) $1 : \sqrt{0{,}75}$, i. e. as radius : the co-sine of the ∠ of incidence, answer.

2. If $e - eee = m$ a maximum, then $1 - 3ee = 0$, so $e = \sqrt{\frac{1}{3}}$ $= 0{,}57733$ the natural sine of 35°16′ whose comp. is 54′44′; hence the water has the greatest force against the rudder of a ship, &c. to turn the ship, when it makes an ∠ with the keel of 54°44′. Also the wind blowing parallel to the axis of a wind mill, gives the sails the greatest force to turn, when the plane of the sail makes an ∠ with the said axis of 54°44′.

Question 145. If the velocity of the wind be at the rate of 3 feet per second, with what force will it strike a plane of 10 feet area when it blows perpendicularly against it? See question 178, and art. 323.

Wind being a stream of air, any thing struck thereby may be looked upon as in that fluid, and a cylinder of air, a foot base, and a foot height (viz. a foot solid of air) weighing ,07268 lb, we'll have $a =$,07268 lb, $v = 3$ the given velocity $S = 16$, so (from art. 323) $R =$ $\frac{vva}{4S} = \frac{,07268\,vv}{64} = ,00113\,vv = ,07268\,h$, and taking $A =$

the area of any plane in feet, and F = force in pounds weight, we have this general expression ,00113 v v A = ,07268 h A = F, = ,10170 lb, in this case, because A = 10 and v = 3, or v v = 9.

The velocity of the wind may be very easily computed thus, take a feather or some light body, and letting it go in an open plane where the wind is not molested, observe the time of its flight by a half second watch or pendulum, then measure the distance it has flown, so you'll have it's velocity, by this method the Rev. Dr. Derham found that the velocity of a very great wind was at the rate of 66 feet per second and that at a medium its velocity is at the rate of 17,6 or 22 feet per second.

Question 146. If the area of a wind-mill's sail be 50 feet, and the velocity of the wind 20 feet per second, with what force will it strike a sail, when the sails are set to turn to the best advantage?

By question 144, the force of the fluid in direction A B, is to its force in direction C D, as a a : $\frac{aae - eee}{a}$, or as a a : $\frac{aaa\sqrt{\frac{1}{3}} - aaa\sqrt{\frac{1}{27}}}{a}$ (because e = a $\sqrt{\frac{1}{3}}$) which by reduction and taking a = 1, will be as 1 : $\sqrt{\frac{4}{27}}$, or as 1 : $\frac{2}{5,19}$, or as 5,19 : 2, which is nearly as 13 : 5 : : the whole force of the wind, to that part of its force which blows against the sails when they are posited in the best manner for turning, therefore, per last question, $\frac{5}{13}$ × ,00113 v v A = ,000435 v v A = F the required force, and because v = 20, and A = 50, we'll have ,000435 v v A = 8,7 lb, the force constantly press'd against the given sail, which multiply'd by 4 the number of sails (if they are all equal in area) gives 34,8 lb, the force against them all.

Question 147. Things being as in the last question, suppose the 4 sails to be equal and alike, and the distance between the center of gravity of each sail and the center of the axle-tree to be 5 feet, required the velocity of the sails, when the mill is charged with such a weight as to perform the greatest effect?

By this question, if the mill be charged with a weight of 174 lb, it will be in equilib. with the force of the wind, so (by question 150) $\frac{4}{9}$ of 174 lb is = 77 $\frac{1}{3}$ lb with which the mill being charged it will perform the greatest effect, and (by question 150) $\frac{1}{3}$ of 20 the winds velocity is = 6 $\frac{2}{3}$ feet per second for the sails velocity, also, 5 × 34,8 = 174 lb, the force with which the axle-tree is press'd to turn, and if d = the distance between the center of gravity of any sail and that of its axle-tree, then ,00113 v v A d = F, the force to turn the machine.

Question 148. If the four rectangular sails of a wind-mill be each 8 feet long, 2 feet between the lower end of each sail and center of the axle-tree, and velocity of the wind 30 feet per second, what must be (b) the breadth of each sail, when set in the best manner for turning, that the axle-tree may be in equilib. with 174 lb weight?

Because the force of the pressure acts at the center of gravity of each sail, we may take $A = 8 \times b \times (2 + \frac{1}{2} \text{ of } 8)\ 6 = 48\ b$. Then (by question 146) $,000435\ v^2 \times 48\ b = F = 174$ lb, or $18,792\ b = 174$, so $b = \frac{174}{18,792} = 9,3$ the breadth of all the 4 sails, therefore $9,3 \div 4 = 2,4$ feet, breadth of each sail, and the mill will then work best if charged with $77\frac{1}{3}$ lb weight $= \frac{4}{9}$ of 174 lb.

Question 149. If the wind blow at the rate of 30 feet per second, perpendicularly against a wall 200 feet long, and 20 feet high, with what force doth it endeavour to overset the wall?

The wall being a rectangle of equal thickness, its center of gravity will be in its middle, viz. at 10 feet height, then $A = 200 \times 20 \times 10 = 40000$, so (by question 146) $,00113\ v\,v\,A = 4,52\ v\,v = 4068$ lb, answer.

Question 150. If a stream of water constantly pressing or striking the pallets of a wheel or engine, mill &c. with (w) 810 weight of water, be just fit to give the machine motion with what weight (p) of water must the pallets be struck when the effect of the machine is the greatest possible, allowing the whole weight on the machine to loose $\frac{1}{5}$ by friction when the velocities of the pallets and stream are equal?

1. Let $v =$ the velocity of the stream, $e =$ the required velocity of the pallets, then $v - e =$ the difference of these velocities, or that with which the pallets are struck, now the force (art. 319) being as the square of the velocity, and p and w the weights that ballances the force of the stream when its velocity is v and $v - e$, we have as $v\,v : p :: \overline{v-e}|^2 : \frac{p \times \overline{v-e}|^2}{v\,v} = w$, if there were no friction, but its manifest $\frac{p \times \overline{v-e}|^2}{v\,v}$ must ballance both w and the friction, now let $a = 0,1$ ($= \frac{1}{2}$ of $\frac{1}{5}$) then $a\,w + a\,p =$ the friction when the velocity of the pallets is v, therefore as $v : a\,w + a\,p :: e : \frac{e\,a\,w + e\,a\,p}{v} =$ the friction when the velocity of the pallets is e, but to be more universal, let this friction be expres'd by $\frac{e\,n\,w + e\,m\,p}{v}$, then $\frac{p \times \overline{v-e}|^2}{v\,v} = w$

$+\frac{enw+emp}{v}$, or $\frac{p}{v}\times\overline{v-e}|^{2}=wv+enw+emp$, whence

$w=\frac{p}{v}\times\frac{\overline{v-e}|^{2}}{v+en}-\frac{emp}{v+en}$, which multiplied by e, gives $ew=$

$\frac{p}{v}\times\frac{\overline{v-e}|^{2}e}{v+en}-\frac{eemp}{v+en}=$ the momentum, which by the question must be a maximum, so making e variable (prob. 201) and taking the fluxion $=0$, we'll get $vvv-4vve+3vee-2veen+2ceen-2mvve-mnvee=0$, but if we divide this equation by $v-e$, it will be abridged to $vv-3ev-2nee\frac{-2mvve-mnvve}{v-e}=0$,

now taking m and n each $=0,1$, we'll find $e=0,3\,v$ nearly, whence $w=\frac{p}{v}\times\frac{:\overline{v-e}|^{2}:-emp}{v+en}=\frac{39\,p}{103}$, whence $p=\frac{103\,w}{39}=$ 2139,2, or the velocity of the stream e to v, as 3 to 10, also, $ew=\frac{39\,p\times 0,3v}{103}=\frac{117\,pv}{1030}=$ the momentum.

2. If we suppose the friction to arise only from the motion of the weight w, then $m=0$, and so $vv-3ev-2nee=0$ (from the last general equation) whence $e=\frac{2v}{3+\sqrt{:9+8n:}}=$ (if $n=0,1$) $\frac{2v}{6,13}$ $=0,32\,v$, so $w=\frac{p}{v}\times\frac{\overline{v-e}|^{2}}{v+en}=\frac{p}{v}\times\frac{\overline{0,67\,v}|^{2}}{1,03\,v}=\frac{0,4489}{1,03}p=\frac{44,89}{103}p$, so $\frac{103\,w}{44,89}=p=1858,5$ or the velocity of the stream to that of the pallets v to e, as 100 to 32, also $ew=\frac{pe}{v}\times\frac{\overline{v-e}|^{2}}{v+en}=\frac{44,89\,p\times 0,32\,v}{103}$ $=\frac{143,648\,pv}{1030}=$ the momentum.

3. If the machine have friction arising from the weight of its parts, it may be considered in the weight w, thus, suppose it were known that 10 weight, applied to the pallets (the engine having no charge) would give them a velocity v, then w being put $=820$ instead of 810, the power p will be found so much greater as to answer this weight.

4. If there were no friction at all, then m and n, are each $=0$, and our general equation becomes $vv-3ve=0$, so $e=\frac{1}{3}v$, whence

$w = \frac{p \times \overline{v-e}|^2}{vv} = \frac{4}{9}p$, so $p = \frac{9}{4}w = 1822,5$, and $e\,w = \frac{4v}{27}p$, the momentum.

Question 151. A rectangular sluice constantly 25 inches (a) deep of water, by drawing up this sluice-gate 9 inches, the water striking the pallets of a mill's wheel &c. is just fit to put the machine in motion, how far must it be drawn up, when the engine performs the greatest effect possible, allowing no friction?

1. If an engine be in equilib. with a weight w, and that weight be made $\frac{9}{4}$ w, the engine (question 150) will perform the greatest effect; and if the orifice be rectangular, (art. 331) the weight of water discharged will be as the area of a parabola, of the same base and height with the water in the sluice, so let $b = 25 - 9 = 16$, $e = a -$ required height, then $\frac{2a}{3}\sqrt{a} - \frac{2}{3}b\sqrt{b} \propto w$, and $\frac{9}{4} \times \frac{2}{3}a\sqrt{a} - \frac{2}{3}b\sqrt{b}$: $\propto \frac{9}{4}w, = \frac{2}{3}a\sqrt{a} - \frac{2}{3}e\sqrt{e}$, that is, $\frac{9}{4}a\sqrt{a} - \frac{9}{4}b\sqrt{b} = a\sqrt{a} - e\sqrt{e}$, whence $\frac{5}{4}a\sqrt{a} - \frac{9}{4}b\sqrt{b} = -e\sqrt{e}$, suppose $= z$, then $eee = zz$, so $e = z^{\frac{2}{3}} = 12,116$, and $25 - 12,884$ inches answer. See question 155.

Question 152. A water mill is to be built where there is a fall of water of 24 feet, whether will a wheel of 18 feet radius with 6 feet fall, or one of 16 feet radius with 8 feet fall, grind most corn with least water.

The velocities of falling bodies being as the square roots of the heights fallen thro' we'll have (per question 150) $18 \times \sqrt{6} =$ momentum of the greater wheel, and $16 \times \sqrt{8} =$ that of the lesser wheel therefore, as $18\sqrt{6} : 16\sqrt{8}$, or as $\sqrt{243} : \sqrt{256}$: : the quan. of corn grinded by the greater wheel to that by the lesser wheel in the same time, and with the same quantity of water, so the lesser wheel will perform better,

Question 153. If a current of water have 8 feet perpendicular descent, what must be the diameter of a wheel to receive the greatest force possible from the said current striking A the middle of the wheel.

Let $a = 8$ feet $=$ B E (fig. 212) the height of the fall, and $e =$ G E $=$ A D the radius of the wheel, then per last question $\sqrt{a-e}$ is as the velocity of the water at G, and (by question 150) G A (e) $\times \sqrt{:a-e:} = e\sqrt{:a-e:}$ is as the force at G, which by the question must be a maximum, viz. $e\sqrt{:a-e:} = m$, or which is the same, $aee - eee = m$, so (by art. 221) $e = \frac{2}{3}a = 5\frac{1}{3}$ feet, the radius sought. The

ſtream ſhould always ſtrike the wheel directly, for if it ſtrike it obliquely it preſſes the wheel againſt the other ſide, and ſo increaſes the friction, beſides, the advantage is leſs, for if q m be the direction of the fluid, it ſtrikes the wheel at G, with a velocity $\sqrt{:BG:}$ draw A m $\perp$ q m, then A m $\times \sqrt{BG} =$ the momentum in this oblique ſtroke, but if B G, be the direction of the fluid, then A G $\times \sqrt{BG} =$ the momentum, &c.

Queſtion 154. If the ſtream (laſt queſtion) have the ſame fall B E, but run in direction q m making an angle B q G with the horizon of 30° what then muſt be A m the diameter of the moſt advantageous wheel?

Having found A G $= \frac{2}{3}$ a $= 5\frac{1}{3}$ by the laſt queſtion, then by trigonometry, as radius 1 : G A $5\frac{1}{3}$: : s $\angle$ A G m 30 : A m $2\frac{2}{3}$ feet, anſwer. If it be required to find what angle B q G, the ſtream muſt make with the horizon, ſo as to ſtrike the wheel with the greateſt force poſſible, let s = its ſine, then $\sqrt{:1-ss:} =$ its co-ſine, and by trigonometry, $\frac{2}{3}sa =$ A m, $\frac{2}{3}a\sqrt{:1-ss:} =$ G m, and $\frac{2}{3}sa\sqrt{:1-ss:} =$ G n; then, G n + G B = B n $= \frac{1}{3}a + \frac{2}{3}sa\sqrt{:1-ss:}$, and by the laſt queſtion, $\sqrt{:Bn:} \times \frac{2}{3}sa = \frac{2}{3}sa\sqrt{\frac{1}{3}a + \frac{2}{3}sa\sqrt{1-ss}}$ = a maximum by the queſtion, or, $\overline{\frac{2}{3}sa}|^2 \times \frac{1}{3}a + \frac{2}{3}sa\sqrt{:1-ss:}$ i. e. $\frac{4}{27}aaa \times :ss + 2sss\sqrt{:1-ss:} =$ m a maximum; ſo (by theo. 149) $2s + 6ss\sqrt{:1-ss:} - \frac{2s^4}{\sqrt{1-ss}} = 0$, (s being variable) which by reduction is $\sqrt{:1-ss:} = 4s^3 - 3s$, and by involution and tranſpoſition $1 = 16s^6 - 24s^4 + 10ss$, which equation ſolved gives s = 0,7214 the ſine of 46°30′ Thus, if the water run down a trunk, &c. making an angle with the horizon of 46°30′ and the radius of the wheel be found to anſwer this angle, &c. and the water ſtrike its pallets perpendicularly, then the advantage is greateſt.

Queſtion 155. If a rectangular flood-gate before a mill, engine, &c. be 2 feet broad, and 6 feet deep of water, and it being drawn up half a foot, the water running againſt the pallets be fit to give the wheel motion, with what force doth it ſtrike the pallets, and how much farther muſt it be drawn, that the machine may perform the greateſt effect poſſible, ſuppoſing the ſtream to ſtrike the pallets perpendicularly, at the bottom of the orifice, the friction being nothing.

1. Here $2 \times 0{,}5 = 1$ foot = a, the area of the orifice, and h = 5,75 feet the depth of water above the middle of the orifice, ſo h a = 5,75 ſolid feet of water, but a cubic foot of water is = 6,25 lb, and 112 lb

= 1 C weight, so $\frac{62,5\,ha}{112}$ = 3,208 C. or more accurately, (question 151) thus, as 6 is to 4 (square of 2) so is 5,5 (6 — 0,5) to 3,6666, whose square root is 1,914, then $\frac{2}{3}$ of 6 × 2 — $\frac{2}{3}$ of 5,5 × 1,914 = 0.982 feet = a, and h = 6 feet, the depth of the sluice, so h a = 5,892 solid feet of water, which by the question ballances the engine, therefore (question 150) $\frac{9}{4}$ of 5,982 = 13,257 feet of water, with which the pallets being press'd, the engine will perform the greatest effect, so 13,257 ÷ 6 (h) = 2,209, the area of the lower part of a parabola, whose height (a) is required, the axis of the whole parabola being = 6, and greatest ordinate = 2, so as 6 : 4 : : 6 — a : 4 — $\frac{2}{3}$ a, whose sq. root is $\surd$: 4 — $\frac{2}{3}$ a : then $\frac{2}{3}$ of 6 × 2 — $\frac{2}{3}$ of $\overline{6 - a}$ × $\surd$: 4 — $\frac{2}{3}$ a : = 2,209, that is, 12 — 6 $\surd$: 4 — $\frac{2}{3}$ a : + a $\surd$: 4 — $\frac{2}{3}$ a : = 3,3135, or $\overline{6 - a}$ × $\surd$: 4 — $\frac{2}{3}$ a : = 8,4865, which by involution and multiplication is 144 — 72 a + 12 a a — $\frac{2}{3}$ a a a = sq. 8,4865 = 72 almost, which reduced gives a a a — 18 a a + 108 a = 108, this cubic equation, solved gives a = 1,25 feet nearly, answer.

But because this method is tedious in practice, the first method may serve for a guess, thus $\frac{9}{4}$ of 1, the first mentioned area is 2,25, which divided by 2, the breadth of the sluice, gives 1,125 feet (less then 1,25 by 0,125, the true height) for the height, the sluice-gate is to be drawn up from the bottom, when the engine works to the best advantage.

Note. the pallets or paddles of a wheel, should be just so many, and so set as that the water may strike them, perpendicularly one after another; and if the water is little, it is better to have boxes than paddles, because the pressure is increased by the weight of the water in the boxes &c.

Question 156. If the breadth of a stream of water be 20 feet, depth 4 feet and velocity 5 feet per second, with what force doth it press a plane set perpendicularly to it? See art. 321.

Here v = 5, and a = 20 × 4 = 80, the area of the plane, but because the plane is press'd to be over-set, or driven directly forward, by the force acting against its center of gravity, we must take a = 20 × 4 × 2 (half of 4) = 160, then s being = 16, we'll have h a = $\frac{vva}{2s}$ = $\frac{5 \times 5}{32}$ × 160 = 125 solid feet of water = 7812,5 lb (62,5 × 125) answer.

The velocity of any stream may be found by dropping in some light matter that will swim, and measuring the distance it moves in one se-

cond, by this means you'll find that the velocity of a ſtream is much greater near the middle, than at the ſides, owing to the friction &c. againſt the ſides and bottom, where the water is ſhallow, now if you take half the ſum of theſe two velocities, it may ſerve for the mean velocity of the water.

Queſtion 157. If the depth of a ſluice be 10 feet, and there be a hole in it's bottom 2 feet area, what quantity of water will run out in 1 minute, or 60 ſeconds, and with what velocity, the ſluice being conſtantly kept full? See theo. 196.

Here $t = 60$, $a = 2$, $h = 10$, then $6{,}128\,t\,a\sqrt{2hs} = 6{,}128 \times 120 \times \sqrt{:320:} = 13089{,}4$ ale gallons, that will run out in a minute, and with a uniform velocity $v = \sqrt{2sh} = \sqrt{320} = 17{,}8$ feet per ſecond.

Queſtion 158. If there be a rectangular ſlit 2 inches wide and 6 inches deep, at the top of a ſluice kept always full of water, what quantity of water will be diſcharged thro' this orifice in a minute's time? See art. 330.

Here $h = 0{,}5$ feet $a = 2$ inches $\times$ 6 inches $= 1$ foot, $t = 60$ ſeconds, and s always $= 16$, ſo $\frac{2}{3}$ of $6{,}128\,t\,a\sqrt{2hs} = 245{,}12\sqrt{16} = 980{,}48$ ale gallons anſwer.

Queſtion 159. Two equal conical fruſtums depth $e = 30$, diameters of the greater and leſſer baſes 20 and 10, each filled with water &c. each ſtanding upright, the one upon the greater and the other upon the leſſer baſes, required the ratio of the time in which each will be emptied by an equal orifice in the bottom of each veſſel? See art. 426. Here $C = 60$ and $c = 30$.

1. As $10\ (20 - 10) : 30 :: 20 : 60$ the axis of the whole cone, and ſo is 10 to 30, the axis of the cone cut off. Then as $2cc + \frac{4}{3}ce + \frac{2}{5}ee$ is to $2CC - \frac{4}{3}Ce + \frac{2}{5}ee :: 3360 : 5160 :: 28 : 43 ::$ velocity at the leſſer baſe : velocity at the greater baſe, and in this caſe time being inverſely as velocity, it will be, as $43 : 28 ::$ time of running out at the leſſer baſe : time of running at the greater baſe.

Queſtion 160. If a column of water a B (fig. 213) 20 feet high, be kept conſtantly full of water, what angle muſt a ſpout a, at it's bottom make with the horizon a D, to throw the water to D, five feet diſtance from the ſaid ſpout a? See art. 366 and 333.

If a hole at a, be ſmall in reſpect of the cylinder's &c. baſe, then the height fallen thro' to acquire the velocity of the ſpouting water at a, muſt be $= 10$, half of the columns height, then this height doubled gives 20 the amphitude of the ſpouting water under an angle of 45°

and for this angle to any other amphitude it will be, as this greatest amphitude 20 is to the radius 1, so is any other amphitude, 5 to 0.25 whose sine belonging, is $14^{\circ}\ 45'$, half of which is $7^{\circ}\ 22'$ for the required angle, and then the path of the water is the parabolic curve a e D, and taken from 90° leaves $82^{\circ}\ 38'$, for the direction of the spout, and then the path is a u D. And thus, any such cases may be solv'd as in the questions of gunnery.

Note. If the area of the spout be equal to that of the column's base, then (by art. 333) the greatest amphitude is double to what is here taken.

Question 161. If each side of a cube full of water be 2 feet, what pressure doth the whole cube sustain? See art. 334.

A cubic foot of water being $62\frac{1}{2}$ lb, and each side of the cube 2 feet it will be $2 \times 2 \times 2 \times 62,5 = 500$ lb the pressure against the bottom, half whereof is 250 lb the pressure against each side, and because there are 4 sides, therefore, $250 \times 4 = 1000$ lb the pressure against all the sides, so $500 + 1000 = 1500$ lb, the whole pressure sustain'd by the cube, i. e. it is press'd with three times the weight of the water that fills it.

Question 162. If the breadth of a sluice be 20 feet, and its depth 4 feet, with what force is the bank or wall press'd that holds it in.

Because the depth is 4, the center of gravity will be 2 from the bottom so $20 \times 4 \times 2 \times 62,5 = 10000$ lb answer.

This compared with question 156 may seem strange, that the force of standing water should be greater than that of running water, the reason is; If the stream were at rest then the plane in it would have no pressure for then it is equally supported on either side by the stagnant water, but in this question there is no water on one side of the plane and so the other side is press'd by the water against it, &c.

Question 163. If a heavy body falling from a height of 15 feet into clay, snow, soft earth &c. do make an hole 5 inches deep, how deep would the hole be if it fell from a height of 20 feet.

If a body fell upon soft or yielding matter, its evident it must take up some time in making the dent, or cavity, and therefore, in that time may be said to move in a resisting matter or medium, but by art. 317) the resistance is as the square of the velocity, and (by theo. 166) the distances fallen thro' are as the squares of the velocities, therefore, it will be as $15 : 5 :: 20 : 6\frac{2}{3}$ inches deep answer. But if the body fall upon a hard unyeilding substance, its then plain there will be no time spent in destroying the generated velocity by the fall, and so by (theo. 155) the effect will be as the velocity, and thus the effect of a

ſtroke may be as the ſquare of the velocity, or as the velocity, or any way between two, according as the matter ſtruck, yields, or not yields to the ſtroke, &c.

Queſtion 164. In what time (T) will a conical fruſtum ſtanding upright on its leſſer baſe, be emptied thro' a hole in it's bottom of one inch diameter, the depth of the fruſtum being e = 30 inches and diameters 20 and 10 inches?

1. As 10 (20 — 10) : 30 : : 10 : 30 = c, the axis of the cone cut off, now becauſe the diameters and orifice are circles, we may leave out 0,7854, the circular factor, and take A = ſquare 20 = 400, and a = ſquare 1 = 1 (theo. 197) then becauſe the veſſel is ſuppoſed to be full of liquor and none to run in while it empties, n will be = 3, alſo s = 16 feet = 192 inches, we'll from $2\,T\,a\sqrt{}:\dfrac{A\,e\,s}{n\,A\,A - a\,a}$: = q for the quantity run out in T ſeconds, of a cylinder, height = e and area baſe = A = upper baſe of the fruſtum, get $T = \dfrac{q}{2\,a}\sqrt{}:$ $\dfrac{n\,A\,A - a\,a}{e\,A\,A\,s}$: = 144 ſeconds (q being 30 × ſq. 20) that is, an upright cylinder full of water depth = 30, and diameter = 20 inches will be emptied by a hole in its bottom of one inch diameter in 144 ſeconds of time, then (art. 426) as $2\sqrt{}\,e : p\sqrt{}\,e \times : 2\,c\,c + \frac{4}{3}\,c\,e + : \frac{2}{5}\,e\,e$, ſo is 144 : 168,6 ſeconds the anſwer.

Note. here $\sqrt{}\,p$, the perameter of the cone is $= \dfrac{\text{diam.}}{\text{axis}} = \dfrac{20}{60} = \frac{1}{3}$, ſo $p = \frac{1}{9}$.

2. If the fruſtum ſtand on the greater baſe, with the ſame orifice there, then (queſtion 159) as 43 : 28 : : 186,6 : 121,5 ſeconds, the time of evacuation.

Here follow 26 questions concerning the specific gravities of bodies, the properties of the air, the construction of weather glasses, pumps, &c.

A TABLE of specific gravities of Bodies.

Fine gold — —	19,640	coal — —	1,520
standard gold —	18,888	brazil wood —	1,031
lead — —	11,340	box wood — —	1,030
fine silver —	11,092	bee's wax —	,955
standard silver —	10,536	oak — —	,920
copper — — —	9,000	logwood — —	,913
copper half-pence —	8,915	ice — —	,908
fine brass —	8,350	beech — —	,854
cast brass — —	8,100	ash — —	,820
steel — —	7,850	yew — —	,760
iron — — —	7,644	elm — — —	,750
pewter — —	7,471	crab-tree — —	,700
tin — —	7,320	cedar — — —	,613
cast iron — —	7,000	fir — — —	,580
lead oar —	6,200	cork — —	,238
pebble stone — —	2,700		
glass — —	2,600	FLUIDS.	
flint — —	2,570	quicksilver —	14,000
common stone —	2,500	urine — —	1,032
brick — —	2,000	milk — —	1,031
earth — —	1,984	sea-water — —	1,030
chalk — —	1,793	ale — —	1,028
clay — —	1,712	common water —	1,000
sand — —	1,520	common air — —	,001,2

These are mean specific gravities of these solids and fluids, in respect of their goodness, fineness, dryness, texture, &c. also heat and cold will make some difference; this table not only shews the ratio's of the specific gravities, but also the weight of a cubic foot of each in averdupoise ounces, for a cubic foot of common water is found to weigh very nicely 1000 averd. ounces, so a cubic foot of lead, will be 11340 oz. one of copper 9000 oz. one of mercury or quicksilver 14000 oz. one of air 1,2 oz. &c. for any other, and may, if needful, be reduced (by ex. 262, sect. 8) to troy weight.

These specific gravities are found (by art. 341, 342, &c.) thus let the specific gravity of water be $= 1000 = c$, then if the body be heavier than water first weigh it in air, and suppose its weight there $= 9000$ oz. $= A$, then having a cord fastened to it let it go into water and observe nicely what weight will hold it in equilib. when its

covered with water suppose 8000oz. $= B$, then $\frac{cA}{A-B} = a = 9000$ oz the specific gravity of the body sought which 9000 looked for in the table is found to be copper, and so as 1000 : 9000 : : 1 : 9 : : the specific gravity of water to that of copper; also, if the specific gravity of the body be 9 you'll have $c = \frac{aA - aB}{A} = 1$ for that of the fluid, but if the body is lighter then water, then tie a piece of metal to it to make it sink in water, and then taking $e =$ the weight of the compound in water, and a, A, B, c, as before, we have $\frac{cA}{A+e-f} = a$ $=$ the specific gravity of the light body. Thus, if I take a piece of metal whose weight in water is 31 oz. and tie it to a bag of beans and the weight of the whole in air be 101 oz. and in water 7 oz. then, $\frac{cA}{A+e-f} = \frac{1000 \times 101}{101+31-7} = \frac{101000}{125} = 808$ oz. that is the specific gravity of water is to that of beans as 1 to 0,808, hence a cubic foot of beans weight 808, oz. or $50\frac{1}{2}$ lb averdupoise, and thus may you find the specific gravity of any thing what ever, but for fragments, dust, powders, &c. it will be best to weigh them in close boxes, or metal buckets, observing to ballance the weights of such boxes &c. both in air and water.

Question 166. If a crown &c. made of gold and silver together, weight 128, specific gravitiy 16, how much of each of these metals is in the mass? See art. 344.

From $d = \frac{abc}{bz - az + ac}$ we'll have $z = \frac{:b-d:\times ac}{:b-a:\times d} =$

$\frac{:19-16:\times 11 \times 128}{:19-11:\times 16} = \frac{4224}{128} = 33$ the weight of gold in the the mixture, so $128 - 33 = 95$, the weight of silver therein.

Note. I have here taken the specific gravity of gold to that of silver as 19 to 11, because that of the crown is 16 to such parts.

Question 167. If the specific gravity of a man's body, water and cork be as 10, 9, 2,25, how much cork will make him swim?

If he be made with cork tied to him of the same specific gravity with water, $(9 = d)$ its plain if a little more cork were added he could not sink, so from (art. 344) $d = \frac{abc}{:b-a:\times:z+ac}$ we'll have

(ſuppoſing the man's weight 150lb $= z$) $c = \frac{zd \times : b - a:}{a \times : b - d:} =$

$\frac{150 \times 9 \times : - : 10 + 2\frac{1}{4}:}{10 \times : 2\frac{1}{4} - 9:} = \frac{-150 \times 9 \times 7,75}{-10 \times 6,75} = 155$lb, the weight of the man and cork together, ſo $155 - 150 = 5$lb of cork, the anſwer.

Queſtion 168. If a ſolid inch of ſome matter weigh 8 ounces, avoird. what is its ſpecific gravity, weight of water being 0,578 oz. per inch ?

By art. 346, it will be as the ſolidity in inches 1 : the weight in ounces : : 1 to weight, or $\frac{8}{1 \times 0,578} = 13,8408$ (0,578 being the ounces weight of a ſolid inch of water) i. e. as 1 : 13,0408 : : the ſpecific gravity of water : that of the matter.

Queſtion 169. If in water, a cube of fir, ſink 3 inches downright, and if each ſide be 12 wide, what ounces is its weight.

Since a ſolid inch of any fluid bears a ſolid inch of any matter of the ſame ſpecific gravity. it will be, as 1 ſolid inch of water is to 0,578697 ounces its weight, ſo is $(12 \times 12 \times 3)$ 432 ſolid inches the wet part of the cube to 249,997104 oz. its weight anſwer.

By this method it will be eaſy to find the burden and weight of any ſhip &c. for, if you find the ſolidity of the wet part of a ſhip when ſhe ſwims empty (in inches) and multiply it by the weight of a cubic inch of ſuch water as ſhe ſwims in, the product is the ſhip's weight in ounces. And if in like manner, you find her weight when loaden, its plain the difference of theſe two weights will ſhew the weight of her burden, which may be reduc'd into tuns ; thus, a cubic foot of ſea-water weighing 1030 oz. or 64,375 lb. and 2240 lb being = 1 tun, ſo as 64,375 lb : 1 foot : : 2240 lb : 34,78 feet, the weight of a tun of ſea-water, ſo if you find the ſolidity of the ſpace in the inſide of the ſhip in feet, between the light, and laden marks, and divide that ſolidity by 34,78, you'll have the true weight of the burden in tuns, which is a much truer method than the common one in queſt. 10.

Queſtion 170. What muſt be the thickneſs of metal in a hollow cube, made of 4 ſolid inches of copper, that it may ſwim in one inch deep of water ?

Let $s = 4$ the ſolidity of metal, $a =$ a ſide of the required cube, and $e = 1$ the inches, $D =$ the ſpecific gravity or denſity of copper, and $d =$ that of water, which are as 9 to 1, now $aae =$ the bulk of the wet part of the cube, and $s =$ bulk of the metal, ſo (by art 338)

$daae = DS$ whence $a = \sqrt{\frac{DS}{de}} = 6$ inches, therefore $6 \times 6 \times 6 = 216$ inches the ſolidity of the required cube, from which taking 4 the quantity of metal leaves 212, the ſolidity of the cavity whoſe cube root is 5,963 inches, the length of the inſide of the hollow cube, ſo $6 - 5{,}963 = 0{,}037$, half of which is ,0185 inches, the required thickneſs of the hollow cube.

Queſtion 171. If the axis of a copper ſphere be 10 inches, what muſt be the thickneſs of the ſhell, that it may ſwim in the air?

To give a general ſolution to queſtions of this nature, Let $D = 10$ inches the axis of the ſphere, $d =$ the axis of the cavity or ſhell, $e =$ the inches of depth that the ſphere is to be immerſed in the fluid, $p = 0{,}7854$, and m to n as the denſity of the fluid to the denſity of the metal, then $\frac{2}{3} pDDD =$ the ſphere's ſolidity, and $\frac{2}{3} pddd =$ the cavity's ſolidity, their difference is $\frac{2}{3} p \times : DDD - ddd$ the content of metal; alſo, $pDee - \frac{1}{3} peee =$ the ſolidity of a ſegment of the fluid in which the ſphere is to ſwim, or be in equilib. with, ſo (by art. 338) $\frac{2}{3} mp \times : DDD - ddd : = np \times : Dee - \frac{1}{3} eee :$ ſo, $ddd = DDD - : 3Dee + eee : \frac{n}{2m}$, but in this queſtion the ſphere is to be wholly immerſed in the fluid, ſo $e = D$, and then, this laſt general theorem becomes $ddd = DDD - \frac{n}{m} DDD = DDD \times : 1 - \frac{n}{m} :$ ſo $d = D \sqrt[3]{1 - \frac{n}{m}}$, now by the foregoing table the denſity of copper is to that of air as 90000 (n) to 12 (m), or lower, as 7500 to 1, whence $d = \sqrt[3]{1 - \frac{n}{m}} \times D = \sqrt[3]{1 - \frac{1}{7500}} \times 10 = \sqrt[3]{\frac{7499}{7500}} \times 10 = 0{,}99978$, whence $\frac{D - d}{2} = {,}00011$ parts of an inch, the thickneſs of metal required, but

Note. There muſt be no air in this ſphere, for if it be filled with air, it will ballance the ſame bulk of air without, and ſo cannot be in equilib. therewith.

Queſtion 172. If two ſolid feet of ſome light matter as wool, feathers &c. weigh 4 lb, how much will it weigh when preſs'd into half a foot bulk?

(By art. 316,) the weights loſt by theſe two bulk are, as $\frac{1}{2}$ to 2, or as 1 to 4, now a ſolid foot of air, being $= 1,2$ oz. its plain the matter will loſe $1\frac{1}{2}$ times more in 2 feet bulk than in $\frac{1}{2}$ a foot bulk, ſo $1\frac{1}{2} \times 1,2 = 1,8$ oz. hence 4 lb 1,8 oz. is the anſwer; hence, the cloſer light ſubſtances are tied together, the more they will weigh.

Queſtion 173. The denſity of gold being to that of braſs as about 2 to 1, if 2 lb of gold be in equilib, with 2lb of braſs when the air is in a mean gravity, or the mercury in the barometer ſtands at the height of 28 inches, what will they differ in weight when the mercury ſtands at 31 inches?

1. A cubic inch of gold weighs 10,36 ounces troy and a cubic inch of air $\frac{2}{7}$ of a grain, when the air is in a mean ſtate, ſo as 10,36 oz, : $\frac{2}{7}$ grains : : 12 oz. : $\frac{24}{72,52}$ grains, which 12 oz or 1 lb of gold will loſe when the air is in a mean ſtate, (2) as 28 : $\frac{2}{7}$: : 30 : $\frac{60}{196} = \frac{15}{49}$ gr. the weght of the air (1 inch) when the mercury ſtands at 30 inches height, then as 10,36 : $\frac{15}{49}$: : 12 : $\frac{180}{507,64} = \frac{45}{126,91}$ grain more loſt by a pound of gold in that ſtate of the air, now $\frac{24}{72,5} = ,37$ and $\frac{45}{126,91} = ,3546$ and $,37 - ,3546 = ,0154$ grains, which doubled is ,0308 gr. more loſs in 2 lb when the air is heavieſt, now if the bulks of braſs and gold were =, their loſſes would be = and ſo the equilibrium ſtill kept, but the bulk of braſs being double to that of gold it muſt loſe double $,0308 = ,0616$ gr. and therefore $,0616 - ,0308 = ,0308$ gr. that muſt be put to the braſs to keep the equilib. the anſwer.

Queſtion 174. If the ſolidity of the cavity of a pair of ſmith's bellows filled with air be 495 inches, and the bellows be preſſed together in a ſecond of time, required the velocity of the air thro' the noſe of 0,6 inches area?

Here 495 ſolid inches, the quantity of air voided in a ſecond, being divided by 0,6 inches the area of the orifice thro' which it paſſes gives 825 inches = 68,75 feet for the length of a column of air that would be generated in one ſecond of time, and is therefore the required velocity of the air per ſecond thro' the noſe of the bellows pipe.

Queſtion 175. If the capacity of the receiver of an air pump be to that of the barrel as 8 to 1, and the denſity of the air in the receiver = 1, what will be the denſity of the air there after 3 ſtrokes, or turns of the piſton?

Here d = 1, n = 1, m = 8 and s = 3, so (by theo. 198) $\frac{1}{729}$, or as 729 : 1 : : the density before the first stroke to that after the third stroke, or because the rarity is inversely as the density it will be as 1 : 729 : : the rarity before the first stroke to that after the third stroke.

Question 176. The density being = 1 as in the last question, and the capacities of the barrel and receiver equal, suppose each = 1, how many turns must be made to rarify the air in the receiver 100 times?

From theo. 198, we have $\overline{2|}^{s} = 100$, so by the logarithms $s \times \log. 2 = \log. 100$, whence $s = \frac{\log. 100}{\log. 2}$ 6,6 the turns required.

Question 177. If any quantity of air is reduc'd into a fourth part of it's first bulk, what more force is required to keep it there? See art. 351.

If the same body of air is confin'd in $\frac{1}{4}$ of the space, it must require 4 times more force, for it there acts with 4 times more force.

Note. Gunpowder fired is nothing but an elastic fluid, and from hence it appears that the greater quantity of this fluid is contained in the same space, the more violent will the explosion be.

Question 178, It is proved by experiments that mercury in the barometer settles $\frac{1}{10}$ of an inch, when the barometer is removed 85 feet directly upwards, now if the mercury at that time on the earth's surface, stand in the barometer at the height of 30 inches, and the density of the air be every where the same, what must be the height of this body of air called the atmosphere?

If a tube be filled with mercury and hung perpendicular to the horizon, the upper end being close stopped and the lower end open, the mercury in that tube will stand at the height of 30 inches, the cause of which can be nothing but the pressure of the air acting on the bottom of the tube, for if ever so little air get in at top, the fluid falls to the ground, whence a column of mercury 30 inches height is in equilibrium with a column of air, of the same base, or, one of mercury $\frac{1}{10}$ of an inch height in equilib. with one of air of the same base and 85 feet height, so it must be, as 0,1 inch : 85 feet : : 30 inches : 25500 feet = 5 miles nearly for the answer.

2 Hence 25500 × 12 = 306000 inches, the height of a column of air in equilib. with one of mercury of the same base and 30 inches inches height, therefore, as 30 : 306000 : : 1 : 10200 : : the density of air to that of mercury.

3. The particles of air near the earth's ſurface muſt bear the preſſure of thoſe above them, whence, the air cannot be every where of the ſame denſity, but muſt be lighter the higher we go, and ſo its height will be indefinite, but if we fix the boundary of the atmoſphere, at that height where it has power to reflect a ray of light, which is the utmoſt limit of twilight, then this height by trigonometry will be found $= 44\frac{1}{2}$ miles.

4. If the baſe of the aforeſaid column of mercury be one foot, then 1 foot $\times$ 30 inches ($= 2,5$ feet) it's height gives 2,5 feet ſolidity whoſe weight is $=$ about 2187,5 lb, which muſt alſo be the weight of a column of air of the ſame baſe, whence it appears that every foot area on any ſurface, as man's body &c. is preſſed with 2187,5 lb weight, which vaſt preſſure he could not ſuſtain were it not that the air acted equally in all directions and ſuffers him to feel this weight no where.

5. But this preſſure acting on the ſurface of any fluid, cauſes it to riſe into any pipe or tube where the air is by any means taken out of ſuch a tube, and becauſe mercury is the heavieſt of all fluids, it riſes to the leaſt height of any, hence we ſee the reaſon of ſucking, and that it is but taking away the air out of the pipe &c. and the fluid follows, to the height of 30 inches if it be mercury, but if water, to the height of 35 feet, (the denſity of mercury being to that of water as 14 to 1) and no higher.

6. The mercury is by obſervations ſeen at all heights between 28 and 31 inches, which ſhews the ſtate of the air to be variable this 3 inches difference is called the ſcale of variation, being that graduated part at the top of the barometer between the greateſt and leaſt heights of the mercury; there has been ſeveral inventions to enlarge this ſcale, ſome by bending the top of the barometer, becauſe fluids riſing to the ſame height muſt run further in an oblique direction than in a $\perp$ one, others have had the tube conical becauſe as the baſe decreaſes the length muſt increaſe to contain the ſame ſpace, but on the account of friction &c. the upright barometer is ſtill the beſt: you may make a weather glaſs with any fluid; thus, take a tube, or long vial bottle, and let it be ſomewhat more than half fill'd with the fluid, then ſtop the mouth of it wirh your finger &c. and turn it down into a veſſel of the ſame fluid, then take away your finger &c. and ſet the veſſel and tube in it, againſt a wall or in a frame for that purpoſe (out of the ſunſhine for heat as well as preſſure affects fluids) then as the preſſure of the air increaſes or decreaſes, upon the ſurface of the fluid in the veſſel it will cauſe that in the tube to riſe or ſettle, for the preſſure of air above the fluid in the tube cannot change becauſe the top of the tube

is close stopped, it is best to make these kind of barometers when the air is in a mean state viz. when the mercury in the common barometer stands at the height 29,5 inches, this being the mean between the two extreams 28 and 31.

7. Any fluid may be exhausted by heat, so if a tube is heated by the fire till the air is gone out of it, and then its open end (the top end hermetically sealed up) be immersed into a vessel of mercury, the mercury will rise in the tube to the common height, and if the tube be not above 28 &c. inches long the fluid will run over at top, if it be open there. See art 349.

Question 179. If mercury in the barometer stand at the height of 30 inches, how high would it rise in the same tube if there were 20 inches height of water in it. See art. 351.

Its plain that the water and mercury together must be in equilib. with a column of mercury (b) 30 inches height, therefore the density of mercury to that of water being as (m to n) 14 to 1, it will be $e = \frac{mb - na}{m} = b - \frac{na}{m} = 30 - \frac{20}{14} = 28\frac{4}{7}$, for the height of mercury in the tube so $20 + 28\frac{4}{7} = 48\frac{4}{7}$ inches, the height of the mercury and water together answer.

Question 180. When mercury in the common barometer stands at the height of 30 inches, what must be the ratio of the diameters of a compound barometer of mercury and water that each of these fluids may have 20 inches height in the compound tube. See art. 351, and the last question?

$$dd = \frac{:b - e: \times m}{na} = \frac{:30 - 20: \times 14}{20} = \frac{140}{20} = 7, \text{ so } d =$$

$\sqrt{7}$, that is, as $1 : \sqrt{7}$: : the diameter of the mercury to that of the water.

Question 181. If a tube 35 (h) inches long 30 (b) inches of it filled with mercury, and then the orifice stopp'd with the finger and turn'd down into a vessel of the same fluid, the finger then taken away, it is plain there will be 5 (c) inches of air in the tube, quere (e) the height of the mercury in this tube, that in the common one at this time being (b) 30 inches? See art. 352.

Here $e = \mp\sqrt{:bc + \overline{\frac{h-b}{2}}\vert^2 :} + \frac{b+h}{2} = 45$ if $+$ be taken, but $= 20$ if $-$ be taken before the surd sign, but 45 is more than 35 the whole length of the tube, so 20 inches is the answer.

Again, ſuppoſe the tube to be but 23 inches long, all elſe the ſame then $= \overline{+}\sqrt{} : cb + \overline{\frac{h-b}{2}}\Big|^2 : + \frac{b+h}{2} = \overline{+}\sqrt{}: 150 + \overline{\frac{30-23}{2}}\Big|^2$ $: + \frac{30+23}{2} = 39{,}24$ inches for the greater root, and $= 13{,}76$ inches for the leſſer root, which is the anſwer.

Queſtion 182. How much air muſt be in a tube 10 inches long, that the mean height of the mercury in it may be 8 inches? See art. 352.

Here is given $b = 29\frac{1}{2}$, the mean height in the common barometer $h = 10$ the propoſed tube's length and $e = 8$ the mean height in it, to find c, ſo it will be, $c = h - e - \frac{he+ee}{b} = 10 - 8 - \frac{80+64}{29{,}5}$ $= 2 - \frac{16}{29{,}5} = 1\frac{13{,}5}{29{,}5}$ inches of air, anſwer.

Queſtion 183. What muſt be the diameters of the two tubes (fig. 166) that the variation in this compound barometers leſſer tube may be to that in the common barometer as 10 to 1? See art. 353.

Here it is, as $e : v :: ddm : 2m - dd - 1$, or ſo is $14dd : 28 - dd - 1$, but by the queſtion, as $e : v :: 10 : 1$, therefore, as $10 : 1 :: 14dd : 28 - dd - 1$, whence $d = 3{,}3541$ viz. the diameters muſt be as 3,3541 to 1, (the denſity of mercury being to that of water as 14 to 1) in like manner, you may find a ſcale of variation as you pleaſe, (and uſe any other fluid with mercury as well as water) for if $2m - dd - 1$ be taken $= 0$, or $dd = 2m - 1$, then $d = \sqrt{}: 2m - 1 := \sqrt{27} = 5{,}2$, that is, if the diameter of A I F be to that of C K F as 5,2 to unity, the ſcale of variation here will be infinite in reſpect of that in the common barometer, but for all this, the common barometer is better, for this compound one is difficult, both to make and keep in order.

Queſtion 184. If F B (fig. 165) be a barometer whoſe ſcale of variation is D G, how much muſt this ſcale be bent out that it may be 3 times as long?

Let G L be ⊥ the tube F B, and with three times D G in your compaſſes and one foot in D, interſect G L in L, ſo is G L the required ſcale, for its plain while the fluid would riſe perpendiculary from D to G, it would fill the ſpace G F. See queſtion 178.

Queſtion 185. If mercury in the common barometer at the bottom of a hill ſtand at the height of 30 inches, and at the top thereof, at the height of $29\frac{1}{2}$ inches, what is the height of that hill? See art. 349

The height of mercury in the barometer being as the denfity of the air and this denfity decreafing in a geometrical ratio as the altitudes increafe in arith. progreffion, its evident, the latter will be inverfely as the logarithm of the former, that is (fee queft. 178) as log. of $\frac{1}{29,9}$: 85 feet : : log. of $\frac{1}{29,4}$: 86,45 feet, that is, if the barometer be carried fo high till the mercury fall $\frac{1}{2}$ an inch, it muft be raifed 86,45 feet higher to make it fettle $\frac{1}{10}$ of an inch more, and if in this manner the heights be found for every $\frac{1}{10}$ of an inch fall of the mercury its plain the fum of all thefe heights will be the height required. Thus

As the log. of $\frac{1}{29,9}$: 85,29, and : : log. of $\frac{1}{29,7}$: 85,58, and fo is log. of $\frac{1}{29,6}$: 85,86 and : : log. of $\frac{1}{29,5}$: 86,16, then $85 + 85,29 + 85,58 + 85,86 + 86,16 = 427,89$ feet the anfwer.

Again, fuppofe that by carrying the barometer to the bottom of a pit &c. the mercury in it rife 5 tenths of an inch viz. there ftand at 30,5, then it will be as log. of $\frac{1}{29,9}$: 85 : : the logs. of $\frac{1}{30}$, $\frac{1}{30,1}$ $\frac{1}{30,2}$, $\frac{1}{30,3}$, $\frac{1}{30,4}$, and $\frac{1}{30,5}$ to 84,72, 84,44, 84,16, 83,89, and 83,61 refpectively, the height which caufes the mercury to rife $\frac{1}{10}$ of an inch as you are a going down the pit; then, $85 + 84,72 + 84,44 + 84.16 + 83,89 + 83,61 = 420,82$ feet the pit's depth. In this manner you may conftruct a tube to fhew the heights either afcended or defcended, or both, and fo by moving with a barometer either upwards or downwards, you'll fee the diftance to every tenth of an inch &c. fall or rife of the mercury, but if the height is but fmall, this way may ferve viz. as 0,1 inch : 85 feet : : 0,5 inch : 425 feet anfwer, which 2,89 feet in the firft cafe and $(425 - 420,82)$ 4,18 feet in the fecond cafe from truth.

Queftion 186. If A B (fig 214) be a tube open at the end B, with a ring of lead &c. laid about that end to make it fink that end foremoft in deep water, when fufpended by the other end A, which is clofe ftopp'd, how far in the tube will water rife, when it is 300 feet under water, the faid tube being 10 inches long?

By art. 351. As A D: A B : : the force of the air in the whole tube A B, to its force when preffed by the water &c. into the fpace A D, now (by queftion. 178) the preffure of the air in a mean ftate

ſuſtains a column of water ($14 \times 29{,}5$ inches height of mercury) 413 inches high, but mercury is not quite 14 times heavier than river water, ſo we'll take this height = 400 inches, ſo if the tube be ſunk 400 inches it will there be half full, becauſe then the tube full of air, will by the water at that depth, be preſſed into half its firſt ſpace, and for any other depth it will be (by the above proportion) 3600 inches = 300 feet is to 400 inches the ſtandard height as ($\frac{400}{3600} = \frac{1}{9}$) 9 to 1, that is, the tube at 300 feet under water will be 9 parts full and 1 part empty, be its capacity more or leſs, whence if the tube were much wider at bottom B, than at top A, the water would not riſe ſo high in it, becauſe the greater the diameter, the leſs the height to contain the ſame ſpace, and thus we have the nature of the diving bell, with which you may be let down into the water of a good depth &c. hence, alſo is the nature of an inſtrument called Sea-gauge, which being ſunk to the bottom of deep water, there ſtrikes, and loſſes a weight which carried it down, and then riſes to the top again and ſhews how far the air in it has been compreſſed, by which the depth is known as above, See queſtion 190.

Note. No other fluid like air will endure compreſſion, for by experiments they either preſs thro' the pores of the veſſel or break it.

The thermometer, a tube commonly ſet with the barometer, to ſhew the degrees of heat and cold, is made of ſpirits of wine ting'd with cochineal.

Queſtion 187. If the ſucking pipe of a water pump be 28 feet long, (b) and the piſton can be moved 8 feet every ſtroke (a) how many ſtrokes muſt be made before the water come to the top of the pipe? See art. 354.

Suppoſing a column of water 31 feet high (c) at that time to be in equilib. with the atmoſphere, and the diameter of the barrel = that of the pipe.

Here, $2z = a + b + c = 8 + 28 + 31 = 67$, ſo $z = 33{,}5$ whence $e = z - \sqrt{:zz - ac:} = 4$ feet nearly, for the height to which the water riſes the firſt ſtroke, now this 4 feet taken from the pipe's height 28 feet leaves 24 for a ſecond (b) the reſt of the letters being as before, we'll have $e = z - \sqrt{:zz - ac:} = 4{,}2$ feet, the height the water riſes the ſecond ſtroke, and this taken from the laſt b 24 leaves 19,8 for a third b, and ſo on as before, you'll find 4,6, 5,1, 8,7 feet, for the heights of water raiſed by the 3d, 4th, 5th, ſtrokes, whoſe total is = 28 feet, all but 1,4 feet, hence the water at the 6th ſtroke has only 1,4 feet to riſe in the pipe, after which it muſt run into the barrel.

Question 188. If a pipe be (b) 20 feet long, and a column of water 35 feet (c) in equilib. with the atmoſphere, what length (a) muſt a ſtroke be made with the piſton to raiſe the water in the pipe (e) 10 feet? See laſt queſtion.

Here $a = \frac{be + ce - ee}{c - e} = \frac{be}{c - e} + e = \frac{200}{25} + 10 = 18$ feet anſ.

Note. In theſe queſtions the piſton is ſuppoſed to deſcend every ſtroke, cloſe to the head of the pipe and then aſcend to the higheſt elevation making every ſtroke equal, for if it do not deſcend from the higheſt to the loweſt elevation, there may no water riſe in the pump, as is thus proved from the laſt queſtion, its evident when the piſton is put cloſe down to the pipe, and then raiſed 8 feet above it, the air in the 28 feet of pipe would be expanded into (28 + 8) 36 feet, did not the water riſe (ſee queſtion 187) 4 feet, whence the ſaid 28 feet of air is in this caſe expanded into (36 — 4) 32 feet; then as the piſton deſcends, the water by its tendency downwards ſhuts the valve v (fig. 167) ſo that neither air nor water can paſs into the well, now its plain when the piſton has deſcended ſo low as to drive this 32 feet of expanded air into its firſt ſpace of 28 feet, it will then be in equilib. with the external air, ſo if the piſton fall any lower, the air below it will become denſer than that above it ſo forces open the valve D to make its eſcape, therefore 32 — 28 = 4 feet, the ſpace that the piſton muſt fall from the higheſt elevation to thus compreſs the internal air, or make it of equal denſity with the external air, but if no air eſcape thro' the valve D, no water can riſe in the pump, whence, if the firſt ſtroke be 8 feet the ſecond muſt be more (in this caſe) than 4 feet, or no water can riſe, and ſo on for any other.

Question 189. If the diameter of a pump's barrel be 6 inches, the diameter of the pipe 2,1 inches, its height above the water 16 feet, the height of a column of water in equilib. with the atmoſphere 31 feet each ſtroke of the piſton 2 feet, and the deſcent of gravity 16,1 feet, what muſt be the velocity of the piſton per ſecond that the water may follow it, or the pump work to the beſt advantage? See theo. 199.

Here, $D = 6$, $d = 2,1$, $b = 16 + 2 = 18$, $c = 31$, and $s = 16,1$ whence, $v = \frac{2dd}{DD} \times : \surd : sc - \surd sb : = \frac{8,82}{36} \times : \surd\ 499,1 - \surd\ 289,8 : = \frac{8,82}{36} \times : 22,34 - 17,02 : = \frac{8.82}{36} \times 5,32 = 1,3034$ feet per ſecond, or (1,3034 × 60) 78,204 feet per minute, ſo 78,204

÷ 4 (because in one stroke are 4 feet, viz. 2 up and 2 down) = nearly 20 strokes in a minute, answer.

Question 190. If by any means water can be made to ascend in a tube filled with air, and none of the air go out, its manifest that the air in the tube will by the rising water be driven into a less space, and so have a greater spring or elastic force; as for example, if the air be compress'd into half the first space, or the vessel $\frac{1}{2}$ fill'd with water, then it's spring will be double, or have the force of twice the atmosphere. Now because one atmosphere can raise water to the height of 34 feet in vacuo, it follows that if there be a spout made a little below the surface of this rais'd water in the tube, that it will spout to the height 34 feet directly upwards provided the tube be constantly kept full of water to the same place, for the force of the air in the tube being double to that of the external air, one half thereof will be spent in overcoming the force of the said external pressure, so the other half must raise water to the same height that the pressure doth in vacuo, in like manner if the air be compressed into $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. of its first space the water will spout to the heights of 68, 102, 136 &c. feet, from whence we have the construction of the engine for quenching fire in cities &c. and may be done by a common pump, by fastening leather &c. about the piston at the head of the pump so that no air can pass, and then having made a few strokes to raise the water above the spout, let the spout be opened and the water will flow in a continual stream in any direction so long as the workman pleases to keep the water at the same height. But if to the barrel of the pump (fig. 167) you fix an air vessel or hollow cylinder d q, all close stopped except at the bottom d, there be a hole to communicate with the pump. Now its plain when the water in the pump rises to d, it will run into the tube d q and drive the air into a space T q, whence, the water in the pump will be press'd with both this air T q, and that above it at A, in the cistern or barrel, and then turning a stop-cock at A, it will run out in a continual stream, the aforesaid engine consists of one large air vessel, and two pumps.

Quest. 191. The engines for raising water by fire, called fire-engines are thus, A B C (fig. 215) is a copper vessel partly filled with water to D E, which being set over a fire and made to boil will fill the upper-part D B E, with a vastly elastic vapour, whose strength when fit to work the engine, is known by its forcing open a valve at e, charged with a sufficient weight, this elastic steam is then let into the barrel a b c d, by turning a cock at F, where by its elastic force it raises the piston G, which drives the air before it thro' a proper cock at top,

after this, that the piston may by its weight descend, a little cold water from a fountain f g n is let in at the bottom by turning a cock at k, which condenses the hot steam in the barrel into 3000 times less space than it was in before the jet of cold water, this makes a sufficient vacuum for the piston to descend in, the piston G, and lever H I being thus put in motion, do accordingly raise and depress the piston K in the barrel of the foregoing pump &c. L M on the other side, which by the pipe M W, draws the water from the depth W.

There are now several forms of fire engines with late improvements, but they all work on these principles, though their forms are much more compleat.

By an experiment made on a fire engine, that made 16 strokes in a minute, content of the steam barrel 113 gallons, the boiler required to be supplied with water at the rate of 5 pints per minute, so $113 \times 16 \times 8 = 14464$ pints of steam per minute, therefore, as 5 : 14464 :: 1 : 2893 :: the rarity of water to that of steam, that is, 1 of water will be 2893 of steam whose force is equal the pressure of the atmosphere, but this number is settled at 3000, as aforesaid, i. e. the 3000 parts of steam is reduced into 1 part of water by the jet of cold water; hence, we may compute the force of steam, as is done of air in the last question; thus, when 3000 is confined in $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. &c. of 3000 it will be as strong as 2, 3, 4, &c. atmosphere's, from these questions also, we have the rarities of water, air and steam, as 1,860 and 3000.

Note. The forementioned cocks F and k, are made to open and shut as need requires, by the working of the engine.

Question 192. As there are 3 sorts of water pumps in use, the first (fig. 167) called the sucking pump, is sufficiently describ'd in the theory &c. it cannot be amiss here to explain the other two sorts?

First, a forcing pump of the common sort, is thus constructed (fig. 216) A B is the barrel standing in the water at W, G C is the piston, and C a solid piece without any hole or valve, because no water is to pass thro' it, this piece should be leathered very fit so that in working neither air nor water may pass between it and the barrel at a distance as D, below is fixed a valve in the barrel, between this and the lowest descent of the piston, as at H, there goes off a pipe, in which as at E is fixed a valve and diaphragma like that at D, now the piston being drawn up from C towards A, rarifies the air above D, and so the water rises into the space C D, then when the piston is forced down, the water finding no passage thro' C or D, is forced into the pipe at

H, and thro' its valve E into the ciſtern at F, (which may be ſet at any diſtance above the pump) and there run off by a ſpout.

Secondly, a lifting pump, is a forcing pump of another form, (fig. 217) F B is a barrel fixt in a frame I L M K which is alſo fixed immoveable with the lower end in the water, E Q H O is a frame with two ſtrong iron rods moveable thro' holes I, K, L, M at the upper and lower parts of the pump in Q H, to the bottom of this frame is fixed an inverted piſton B D, with its bucket and valve at top D, upon F the top of the barrel there goes off a pipe F H, either fixed to the barrel, or moveable by a ball and ſocket, in either caſe it muſt be fitted tight that no air get into the barrel, in this pipe as at C is fixed a valve. Now when the piſton frame is thruſt down into the water, the piſton D deſcends, and the water below will ruſh up thro' the valve D, and get above the piſton, where upon the frames being lifted up the piſton forces the water thro' the valve C up into the ciſtern H, there to run out.

Note. This ſort of a pump is ſet ſo far into the water as that the piſton may play below the ſurface of it.

The 37 following queſtions contain the full practice of Gunnery.

Queſtion 193. If the elevation of a gun, (viz. the angle which its axis makes with the horizon) be 30° and the amplitude 2000 feet, to what height will the ball riſe in the air, alſo, what muſt be the elevation when the amplitude is the greateſt poſſible? See article 364, and 365.

As ſine 60° (co-elevation) is to ſine 30° (the elevation) ſo is 500 (the fourth of 2000) to 287,5 feet, the height of the ball's path, and the required elevation is 45°. To this elevation guns are ſet when fired on trial, and the amplitude meaſured in feet, and marked on the gun, which is a ſtandard to compute other ranges by, and becauſe theſe ranges vary with the product of the ſine and co-ſine of the elevations, it follows that any two elevations equally diſtant from 45°, will have the ſame amplitude; that is 30°, and 60°, alſo 40°, and 50°, &c. will give the ſame amplitude.

Note. In all theſe queſtions about gunnery the force or charge of powder is ſuppoſed to be the ſame, except it be ſaid to the contrary.

Queſtion 194. When the impetus is 4000, what muſt be the two elevations, C A and D A (fig. 218) to ſtrike an object B, at 6969 feet diſtance on the plane of the horizon? See art. 369.

As twice the impetus 8000, or greateſt amplitude is to radius (ſine 90°) ſo is and other amplitude 6969 to 60° 32′ the double of its

elevation, ſo half of 60° 32′ = 30° 16′ = ∠ D A B, of the path A F B, the leſſer elevation, whoſe comp. is 59° 44′ = ∠C A B or the path A E B, to the greater elevation.

Queſtion 195. If the elevation of a piece be 30° 16′, what muſt be the impetus, to hit an object on the plane of the horizon at 6969 feet diſtance (by the laſt queſtion) as s 60° 32′ (twice 30° 16′ : s 90° :: 6969 : 8000 half whereot is 4000 feet the required impetus.

Queſtion 196. If a ball continues 12 ſeconds in the air when projected under an angle of 30°, what will be the time of its flight when the elevation is 60° ?

(By art. 373) As s 30° any elevation is to 12 ſeconds the time of the balls flight ſo is s 60° any other elevation to 20,8 ſecond the time of flight in that elevation.

Queſtion 197. If a ball continue 12 ſeconds in the air when the elevation is 45°, what will the impetus be? See art 373.

Here 144 the ſquare of 12, multiply'd by 16 the deſcent of gravity, gives 2304 feet for the greateſt amplitude, half of which is 1152 feet, anſwer.

Queſtion 198. What is the time of flight, when the elevation is 32°, and amplitude 5280 feet? See art. 373.

As radius is to tangent 32°, the elevation, ſo is 5280 feet the amplitude to 3299, whoſe ſquare root is 57,44, = 4 times the required time in ſeconds, ſo $\frac{1}{4}$ of 57,44 = 14,36 ſeconds anſwer. Theſe two queſtions are of great uſe at ſea, and in adjuſting the fuſee &c.

Queſtion 199. If the elevation of a piece be 45°, and the impetus 1800 feet, and it ſtrike object whoſe horizontal diſtance is 3000 feet, what is the time of the flight? See art. 368.

1. As (57600) 16 times 3600 the double impetus, or greateſt amplitude, is to 1 the natural tangent of the given elevation 45° ſo is 9000000 the ſquare 3000 the horizontal diſtance to 156,25, whoſe ſquare root is 12,5 ſeconds anſwer.

Queſtion 200. With what velocity doth the ball leave the gun, when the impetus is 6400? See art. 374.

1. The ſquare root of 32 times the greateſt amplitude 12800 is 640 feet anſwer, that is, if the ball move uniformly, it will paſs over 640 feet in a ſecond of time.

Queſtion 201. If E y (fig. 218) the height of the projection be 287$\frac{1}{2}$ feet the amplitude A B 2000 feet, with what force will the ball from A ſtrike an object m, whoſe horizontal diſtance is A H 800 feet and

altitude m H (above the horizon) 276 feet, the balls weight being 20 pounds?

1. Draw m n parallel to the horizon A B, then its evident the ball would describe the path m E n, with that velocity that it strikes m with, therefore the velocity fit to describe m E n will be the answer, so, as m a (=H y) 200 is to radius so is twice E a 23 (2 E y — 2 m H) to tangent 6° 30′ the elevation. Then (by art. 369) as sine 13° (twice this elevation) is to its amplitude m n 400 so is radius to 1780, the greatest amplitude, so (by the last quest.) $\sqrt{1780 \times 32} = 238,6$ feet, the required velocity, then $238,6 \times 20$ lb $= 4772$ lb the force with which the ball strikes the object, and leaves the gun with the same force. See question 138.

Question 202. If the elevation (angle B A C fig. 218) be 30° and the amplitude A B 4000 feet, where must the piece be placed to hit an object m, whose perpendicular height H m above A B the level of the piece 400 feet? See art. 398.

1. As tangent 30° elevation, is to 400, objects height, so is 4000, amplitude to 2807017 this taken from 4000000, half (4000) the amplitude squared, leaves 1192983, whose square root 1092 added to half the amplitude 2000 gives 3092 feet = A v. If the ball were to strike the object n, after it has performed half its flight, but if it be to hit an object m, before half its path is pass'd over, then 3092 taken from A B 4000, leaves A H = 908 feet for the answer.

Question 203. Let things be as in the last question, but suppose the object to be m 400 feet below A B the level of the piece? Fig. 168.

1. Having got 2807017, as in the last quest. add it to 4000000 the square of 2000 (half A B), and the square root of that sum added to 2000 the said half amplitude gives 4609 = A L, that m is below L being the object.

Question 204 If the impetus be 4000 feet, what must be the elevation C A B (fig. 218) to hit an object n, whose horizontal distance A v is 5600 feet and height v n 812 feet?

(By art. 399) From 8000 twice the impetus take 1624 twice 812 the objects height, multiply 6376 the remainder by 8000 twice the impetus and from 51008000 that product take 31360000 the square of A v 5600, and there leaves 19642000 whose square root 4432, added to, and taken from 8000 twice the impetus, gives 12432 and 3568, then as 5600 = A v, is to rad. so is 12432 to tangent 65° 45′, the greater elevation and so is 3568 to tangent 32° 30′ the lesser elevation required.

Question 205. Let things stand as in the last question, but let L m (fig. 168) be 812 feet, viz. the object m to be 812 feet below the level of the piece, and A L its horizontal distance 5600 feet &c. as before?

1. To 1624 twice L m 812 add 8000 twice the impetus, multiply 9624 that sum by 8000 twice the impetus, and from 76992000 the product take 31860000 the square of A L 5600, then 6755 the sq. root of 45632000 the remainder added to and taken from 8000 the greatest amplitude gives 14755 and 1245; lastly, as 5600 : radius :: 14755 : tangent 69° 12 ½′ the greater, and so is 1245 to tangent 12° 37 ½ the lesser elevations required.

Question 206. If A L (fig. 168) the horizontal distance of an object m, be 5600 feet, and its angle of depression L A m 8° 15′ and the elevation (angle L A C) of the piece 12° 37 ½′, what must be the impetus so as to hit the object? See art. 399.

As 1,476 four times the sum of the natural tang. 0,224 and 0,145, of 12° 37 ½ ∠ L A C and 8° 15′ ∠ L A m, (but if the object were elevated you must take 4 times the difference of these natural tangents) is to 1,049, the square of 0,224 added to unity so is 5600 to 4000 feet answer.

Question 207. If a gun be planted on the top of a tower, what must be its elevation with the least impetus possible, to strike an object whose horizontal distance is 5600 feet, and angle of depression 8° 15′? See art. 391, and 399.

1. From 90° take the object's depression 8° 15′, half of the remainder 81° 45′ is 40° 52 ½′ the answer, but if the object were elevated 8° 15′, then 90° + 8° 15′ = 98° 15′, half of which is 49° 7 ½′ would be the answer.

Question 208. Things being as in the last question, what is the impetus?

1. As radius is to tangent, elevation 40° 52 ½′ so is 5600 to 4850, half whereof is 2425 the required least impetus, art. 399.

Question 209. If the elevation (B A C fig. 168) be 32° 30′ and its impetus 4000 feet, and the inclination of a plane (angle B A m) 8° 15′, required A m, the amplitude of this projection? See art. 400.

As the square of the secant of 32° 30′ the elevation, is to the secant of 8° 15′, the inclination, so is 4 times the impetus 16000 to a fourth number, which multiplied by the sum and difference of the tangents of 32° 30′ and 8° 15′ the elevation and inclination, gives 8992 feet = A m, if the plane is depressed, but 5658 feet = A m when m is above the horizon.

Question 210. If the inclination of a plane be 8° 15′, what must be the elevation of a piece with a given impetus, to throw the ball the farthest possible on the said plane?

This is the same with question 207, and therefore half the sum and difference of 90° and 8° 15′, is 49° 7 ½′, for the elevation of the piece when the plane is elevated, and 40° 52 ½′ for the said elevation when the plane is depressed.

Question 211. If by 16 lb of gun-powder the impetus be 4000 feet what powder must be taken that the impetus may be 5000 feet?

As 4000 feet is to 16 lb so is 5000 feet to 20 lb answer. But in great charges of powder there is a considerable part of the powder blown out unfired, so that the velocity of the ball in such cases cannot be exactly in the subduplicate ratio of the quantity of powder, therefore this method serves only as a guess. See question 177.

Question 212. Given the distances CS = 8900 NS = 80000 each in feet, (fig. 219) and the angle NSC = 30° on the plane of the horizon, at s a gun is planted, whose elevation is 45° and impetus 40000 so that it might throw its ball to N, at C another gun is planted whose impetus is 90000 feet, what must be the elevation of this gun C, so as both guns be fired at the same instant of time, the balls may hit each other in the air, and the ball from s, be put out of its path by that from C?

Because the two guns are fired at the same time, and the balls are to meet, therefore the lines of flight must be equal, so by art. 382 the heights FG and DE of the two projections will be equal; whence, (by art 378) as 300 the square root of 90000 C's impetus is to 200 the square root 40000 S's impetus so is sine 45° S's elevation to sine 28° 12′ C's elevation required.

Question 213. If s represent the south point of the horizon and N the north point thereof (last question) to what point of the compass must the gun C be directed?

Because DE the height of the path of the gun's, is (by art. 365) = ¼ Ns the greatest amplitude = 20000, which is also (per last quest.) = FG the height of C's path, therefore it will be as tangent 28° 12′ C's elevation is to radius so is 20000 the height of its path, to 37500 which doubled gives 75000 = ½ CQ = CG or GQ, for C's half amplitude. Now as the paths intersect each other in A, where the balls meet, so the amplitudes cross each other in B, the horizontal point directly under A, and because the times of flight are equal, we have (per art. 383) as SB : CB :: SN : CQ, so by plane trigonometry, as CB, or as CQ 150000 : sine ∠ NSC 30° :: SB or SN

80000 : sine $\angle$ B C S, 15° 30′ which added to $\angle$ N S C 30, gives $\angle$ Q B S = N B C = 45° 30′ or south 45° 30′ easterly for the direction of the gun C.

Question 214. Where will the two balls mentioned in question 212 fall, supposing them to be of the same metal, non-elastic, the weight of the ball from C to that of the ball from s as 5 to 3 (C to B) and that when the ball B crosses the path of the ball C, C strikes it with its whole force, the centers of the two balls being then in one right line parallel to the horizon? Fig. 220.

Because when the balls meet in the air their centers are equally distant from the horizon, it follows that what alteration is made by the stroke will only be in their velocities and directions. Whence, in this case the balls may be looked upon as moving in their horizontal ranges S N and C Q, and because the times of flight between meeting in B and falling at N and Q, are equal, therefore, B N and B Q will be as their velocities, and because these velocities shew N and Q, the point of fall if the balls did not interrupt each other by their meeting, so the velocities after the stroke will point out the required places of fall. Now because C strikes B perpendicularly, therefore, B Q $\times$ C = C's momentum against B, but B strikes C obliquely in the direction S N, so from N upon Q C, let fall the $\perp$ N A, then (by theo. 157) B A will express B's velocity against C, so B A $\times$ B = B's moment against C, then by trigonometry, as sine $\angle$ C B S 134° 30′ is to C S 8900 so is sine $\angle$ B C S 30° to B C 6275, and so is sine $\angle$ B C S 15° 30′ to B S 3450, then N S 8000 — B S 3450 = N B 4550, and C Q 150000 — C B 6275 = B Q 143725, again, as radius : N B 4550 :: sine $\angle$ A N B (co-sine $\angle$ N B C 45° 30′) 44° 30′ : A B 3200, then because the balls tend to meet, we'll have (by theorem 160) $\frac{BQ \times C - BN \times B}{C + B} = \frac{143725 \times 5 - 3200 \times 3}{5 + 3} = 88628$, the common velocity after the stroke, which is the distance moved by C, in the direction B Q suppose to R, viz. B R = 88628 feet) before it fall which would also be the place of B's fall, were it not that B is affected by the aforesaid oblique motion, draw N u parallel to C Q making N P = A B, and P u = B R, now B N, B's force being (by theo. 157) resolved into the two forces B A = P N and B P = A N, whereof A N is not altered by the force B R of C, but the other force N P is thereby made = N u, whence B after the stroke will move in the line B u, and come to u in the same time that C comes to R, (see art. 259) therefore, the ball from C will fall at R, which is south 45° 30′ east-

erly and 95903 (C B + B R) feet distant from the gun C, and the ball from s will at the same time fall at u which is also south 45° 30′ easterly and 91828 feet (N P + P u) distant from N.

Question 215. Suppose the weight of the ball B (last question) to be 500, and that of the ball C 1, where will the balls fall? By reasoning as before we'll have $\frac{143725 \times 1 : -3200 \times 500}{1 + 500} = \frac{143725 - 1600000}{501}$ $= \frac{-1456275}{501} = -2906,5$ feet, for the common velocity or distance moved after the stroke, which shews that C is reflected back, and would be followed by B, were it not for B's oblique stroke, whence the directions will be the same as before, viz. south 45° 30′ easterly, and 6275 (C B) — 2906,5 = C d 3369,5 feet, d being the place where the ball from C falls, as also P N 3200 — 2906,5 = N e 293,5 feet, e the place of fall of the ball from the gun s.

Question 216. Let the weights and velocities of the balls B and C be as in the last question, but suppose they strike each obliquely viz. when the line y z parallel to the horizon joins their centers which line y z, with P E the line or plane which the balls touch when they meet on which from N and Q let fall the perpendiculars N P and Q D, which ⊥s are as the velocities with which (see art. 259) the balls directly meet each other, then by trigonometry, as radius : N B 4550 :: sine ∠ N B P 45° : N P 3220, and as radius : B Q 143725 :: sine ∠ D B Q 89° 30′ : Q D 143710 then (by theo. 160) $\frac{DQ \times C - NP \times B}{C \times B} =$ $\frac{143710 \times 1 : -3220 \times 500}{1 + 500} = -2926$ or $= +2926$ (if B be positive and C taken negative) for the direct velocity of both balls after the stroke, and because the balls move from B towards Q and N, and the velocities B D and B P being parallel are not altered by the stroke, these velocities must be on the side P of y z, but if the balls had moved from Q and N towards B the velocities B D and B P must have been set on the other side of y z upon B E, so produce Q D till R D = 2926, and make P u = 2926, join B R and B u, so after the stroke the ball B moves in B u and falls at u 299 feet (P N — P u) south east of N, in the same time that C moving in B R falls at R 146636 feet (Q D + Q R) north west from Q.

Question 217. Suppose the balls B and C (last question) to be perfectly elastic, where then would the bodies fall?

Let $Q =$ the weight 1 $V = 143710$ the velocity of C, and $q = 500$ the weight, $v = 3220$ the velocity B, before the ſtroke, then (by art. 258) $V - \frac{2qV - 2qv}{Q+q} = \frac{QV + qV - 2qV - 2qv}{Q+q} = \frac{QV - qV - 2qv}{Q+q} = -149563$ the velocity of C after the ſtroke and (by the ſame article 258) $\frac{2QV + 2qv}{Q+q} - v = -2633$ for the velocity of B after the ſtroke, and becauſe the balls being elaſtic does not alter their directions, therefore make $DR = 149563$ feet and $Pu = 2633$ feet, ſo is R, and u the places of fall as before.

Queſtion 218. If the diameter of a cannon's bore be 8 inches, what muſt be the diameter of it's ball?

That the bullets may not be ſo big to burſt the piece, nor too little ſo as part of the powder may fly out between the ball and gun, it is thought fit to make the balls diameter about $\frac{19}{20}$ parts of the guns diameter, therefore, as 20 : 19 : : 8 : 7,6 inches anſwer.

Queſtion 219. If the weight of a cannon's ball be 48 lb, and the weight of the cannon 8000 lb, what muſt be the weight of another cannon whoſe ball is 36 lb?

As 48 lb : 8000 lb : : 36 lb : 6000 lb anſwer.

Theſe pieces are made vaſtly heavy in order to be ſufficiently ſtrong ſo as not to burſt by diſcharging &c. The weight of a ball being $33\frac{1}{3}$ lb. the thickneſs of metal at the breech of the cannon ſhould be 6 inches and at the mouth 2 inches &c. for others, the ordinary charge of cannon is to have the weight of the powder equal to half the weight of the ball, their lengths ſhould be ſuch, as that the whole charge of powder may be on fire juſt as the ball quits the piece, for if it be made too long, the weight of air to be driven out before the ball hinders its force, if too ſhort, the powder goes out more of it unfired, the greateſt random of a piece is about 10 times as far as the ball will go point blank, viz. when the piece is laid parallel to the horizon. Cannons are diſtinguiſhed by the diameters of the balls they carry, and are in length from 4 to 12 feet.

Queſtion 220. If a cannon weighing 6400 lb, give a ball of 24 lb weight an uniform velocity of 640 feet per ſecond at the breech of the piece, how much will the cannon recoil in a ſecond, if free to move?

The momentum of the cannon and ball are equal becauſe the powder acts equally in all directions (ſee queſt. 177) ſo (by theo. 154) $\frac{640 \times 24}{6400} = 2{,}4$ feet per ſecond the velocity with which the cannon begins to recoil.

Question 221. Let things be the ſame as in the laſt queſtion but ſuppoſe the cannon to be fixed and its length 12 feet, required a preſſing weight equal to the force of the powder? See art. 252, 253 and 254.

Firſt, as 640 the uniform velocity of the ball is to 1 ſecond, ſo is 24 (twice 12 the cannon's length) to $\frac{24}{640}=\frac{1}{26\frac{2}{3}}$ parts of a ſecond the time of the balls paſſing thro' the cannon at that rate, now ſince the powder conſtantly acts on the ball whilſt in the gun it will therefore drive it along with an accelerated velocity, which velocity in the ſame time of paſſing thro' 12 feet, produces an uniform one of 24 feet as above. But in accelerated velocities the ſpaces paſſed over, are as ſquares of the times, therefore, as $\overline{\frac{1}{26\frac{2}{3}}\text{ſeconds}}\Big|^2$: $\square$ 1 ſecond or as $\square$ 1 : $\square\, 26\frac{2}{3}$ $(711\frac{1}{9})$ ſo is 12 feet to $8533\frac{1}{3}$ feet, which the ball would be carried thro' in one ſecond by the accelerating force of the powder; now, the weights of bodies being as the accelerated forces and theſe as the ſpaces paſs'd thro' in the ſame time, therefore as 16 feet the deſcent of gravity in the firſt ſecond, is to 24 lb, the weight it gives the ball, ſo is $8533\frac{1}{3}$ feet, the ſpace paſſed by the powder in one ſecond, to 12800 lb the whole force of the powder; but if the gun be free to recoil, then this force 12800 lb, is part of it ſpent in giving the gun a velocity of 2,4 feet (ſee the laſt queſtion) per ſecond; but the whole force produces a velocity of 640 feet per ſecond, therefore, as 640 : 12800 : : 2,4 : 48 lb, the weight ſpent in giving velocity to the gun, ſo (12800 — 48) 12752 lb is ſpent in preſſing on the gun and ball giving it a velocity of (640 — 2,4) 637,6 feet per ſecond.

Question 222. If a ball or globe, of caſt iron falling from reſt in vacuo, (viz. in a non-reſiſting medium) do in the firſt ſecond of time acquire an uniform velocity of 32,2 feet per ſecond, what will this velocity be if the fall be in open air? See the next queſtion and theo.

As 7000 the ball's abſolute gravity is to (7000 — 1,2) 6998,8 its ſpecific gravity in the fluid viz. air, ſo is 32,2 to 32,19 feet anſwer.

Question 223. What is the greateſt velocity a ball of caſt iron 6 inches, or half a foot diameter (A) can poſſibly acquire by falling in the air?

1. The denſity of caſt iron being to that of air as 70000 to 12, which is nearly as 5833 to 1 (d to D) we'll have (by article 327) $\frac{4}{3}$ A $\times$: $\frac{d-D}{D}$: $= \frac{4}{3}$ of $\frac{1}{2}$ a foot $\times \frac{5833-1}{1} = \frac{4}{6} \times 5832 = 3888$ feet, the

height fallen in vacuo to acquire the greatest velocity, that is, $\frac{4A}{3D}\times:$ $d-D:=e$, but (by art. 321) $e=\frac{vv}{4s}=\frac{vv}{4\times 16,1}$, therefore, $\frac{4A}{3D}$ $\times: d-D:=e=\frac{vv}{4s}$, whence, as (by art. 328) $v=4\surd:\frac{d-D}{D}$ $sA:=4\surd:\frac{5833-1}{3}\times 16,1\times\frac{1}{2}:=500$ feet, that is, the greatest velocity this ball can ever acquire by falling in the air is that which would carry it over a space of 500 feet nearly, in every second of time, otherwise if $v=\surd\, 2qe$; then per last quest. as $d:d-D::2s:$ $\frac{2sd-2sD}{d}=q$ so $vv=2qe=\frac{4se}{d}\times:d-D:$ (and by writing $\frac{4A}{3D}\times:d-D$ for its equal e) then $vv=2qe=\frac{4Aqs}{3D}\times:d$ $-D$; but if we take $e=\frac{4}{3}A\frac{d}{D}$, then $vv=2qe=\frac{4se}{d}\times:d-D:$ $=\frac{16}{3}as\times:\frac{d-D}{D}:=\frac{16As}{3D}\times:d-D$ as before, hence, if the e mentioned in the theorems to art. 401, be taken $=\frac{4}{3}A\times:\frac{d-D}{D}$, then q must be $=2s$, $=$ twice 16,1, but if $e=\frac{4dA}{3D}$ then q must be $=\frac{2sd-2sD}{d}$, and in these theorems where q is not engaged, e must be taken $=\frac{4dA}{3D}$, $=$ (in this case) 3888,6.

Question 224. If a ball of cast iron half a foot diameter, be thrown directly upwards into the air, with a velocity (g) fit to carry it over 640 feet in the first second of time, how high will it go, and what will be the time of its ascent?

Here per last quest. $e=3888,6$, $g=640$, and $q=32,19$, so (by theo. 7, art. 401) $m=2,3025\,e\times$ log. of $1+\frac{gg}{2qe}:=2,3025\times$ $e\times,4208=3811$ feet for the required height, then for the time, $t=\frac{1}{q}\times$ arch of a circle whose radius is $\surd\,2qe$, and tangent $=g$, thus, as $\surd 2qe=\surd:2\times 32,19\times 3888,6=498,8$, is to 640 (g)

so is, rad. 1, to 1,283, which answers to the natural tangent of 52,066 degrees, then because 57,3° is the radius of a circle whose periphery is 360°, it will be as 57,3 is to 52,066 so is 498,8 to 443,3 the said arch which divided by $q = 32,19$ quotes 13,7 seconds, for the time of the ball's ascending.

Question 225. If a ball of cast iron half a foot diameter be let fall in the air from a height of 3811 feet, what time will it be in falling, and what velocity per second will it acquire by that fall?

Here $m = 3811$, and by quest. 222 and 223, $q = 32,19$ and $e = 3888,6$, then take $\frac{m}{2,3025\,e}$ as a log. and put $n = 1$, divided by its natural number, and we'll have $v = \sqrt{:2qe \times :1-n:}$ and $t = 2,3025$ log. $\frac{1+\sqrt{:1-n:}}{1-\sqrt{:1-n:}} : \times \sqrt{\frac{e}{2q}}$, thus, $\frac{m}{2,3025\,e} = ,4214$ which looked for in the table of logs. its natural number is found $= 2,64$, then $\frac{1}{2,64} = ,378 = n$, so $1 - n = ,622$; therefore, $v = \sqrt{:2 \times 32,19 \times 3889,6 \times ,622:} = 360$ feet, for the distance that would be discribed in one second of time with the velocity acquired by falling, also, $t = \sqrt{\frac{e}{2q}} : \times 2,3025$ log. : $\frac{1+\sqrt{,622}}{1-\sqrt{,622}} = 15,5$ seconds, for the time of falling.

Question 226. From what height in the air must a ball of cast iron half a foot diameter fall, to acquire a velocity of 400 feet per second?

Here $v = 400$, and by the foregoing questions, $e = 3888,6$ and $q = 32,16$, then, from the last question, $m = 2,3025\,e$.log. $\frac{2qe}{2qe - vv} = 2,3025 \times 3888,6 \times ,4418 = 4047$ feet, answer.

If $v =$ the greatest velocity, or $vv = 2qe$, then the divisor $2qe - vv = 0$, so $\frac{2qe}{2qe - vv} = \frac{2qe}{0}$, in which case m, is infinite, that is, the ball by descending in the fluid can never acquire its greatest velocity, tho' the higher or longer time it descends the nearer it comes to it, hence, (and by theo. 166) a falling body can never acquire an uniform velocity neither in a resisting, nor in a non-resisting medium, tho' in either case it will in time come so near it, as that it may be taken as such, for in vacuo, the spaces fallen thro' in each second of time separately, are as the odd numbers 1, 3, 5, 7, 9, 11, 13, 15, 17, &c.

where its plain that the farther these numbers are continued the less they'll differ, or come nearer to equal spaces in equal times, &c.

Question 227. With what velocity must a ball of oak wood $\frac{1227}{4000}$ feet diameter, be thrown into a pool of water 2 feet deep, that it may go to the bottom, and there have 3 feet velocity per second ?

The density of oak being to that of water as 920 to 920, which is as 1 to 1, (d to D) we'll (by quest. 223) have $e = \frac{4Ad}{3D} = \frac{4}{3} \times \frac{1}{1} \times \frac{1227}{4000} = \frac{4908}{12000} = 0,409$, and by the quest. $m = 2$ and $v = 3$, now if we take $n =$ a natural number whose log. is $\frac{m}{2,3025 \times 2e}$ we'll have $g = nv = 36,57$ feet per second for the required velocity ; also, the ball's falling through the fluid is $t = \frac{2e}{vg} \times : g - v : = \frac{0,818}{109,71} \times 35,57 = 0,27$ seconds.

From these two equations $g = nv$ and $t = \frac{2e}{vg} \times : g - v :$ we have $m = 2,3025 \times 2e \times \log. 1 + \frac{tg}{2e} = \frac{2e}{g} \times : n - 1 :$.

Question 228. If a globe of oak wood $\frac{1227}{4000}$ feet diameter be projected into stagnant water, with a velocity of 300 feet per second how far will it have moved in 5 seconds of time ?

Here $t = 5$, $g = 300$, and by the last quest. $e = 0,409$, whence $m = 2e \times 2,3025 : \log 1 + \frac{tg}{2e} : = ,818 \times 2,3025 \times 3,2635 = 6,217$ feet answer.

In these two questions, for varieties sake, I have supposed the densities of oak and water to be equal, in which case q is $= 0$, and so the ball may be considered as moving by its innate force only ; for it cannot be affected in this case, by gravity, for (by quest. 222) q being as the velocity generated by gravity, it follows that in $m = 2,3025$ e log. $\frac{gg \pm 2qe}{vv \pm 2qe}$: (see theo. 7 art. 401) q must be negative if the ball descend, and positive if it ascend, but $q = 0$ if it do neither, or then the densities of the ball and fluid are equal, in the other two cases the density of the ball is meant to exceed that of the fluid, and thus taking q − or +, and having given g, e and m, you may find v the velocity per second, of the ball at any height m, either descending or ascending, and taking $v = 0$, we have the theorem in question 224, &c. you

may find any one of these letters, if all the rest are known, but if the density of the fluid exceed that of the ball, you may take $m = 2{,}3025 e \times \log. \frac{gg + 2qe}{vv + 2qe}$: when the ball is made to descend in the fluid, and $m = 2{,}3025\, e \log. \frac{gg - 2qe}{vv - 2qe}$: when it ascends therein. See Simpson's Essays.

Question 229. With what velocity must a ball of oak wood $\frac{1127}{4000}$ parts of a foot diameter, be thrown into a pool of water 2 feet deep, that it may go to the bottom, and there have 3 feet velocity per second? (v)

The density of oak being to that of water as 920 to 1000, which is as 23 to 25 (d to D) we'll (by question 223) have $e = \frac{4\,A\,d}{3\,D} = \frac{4}{3} \times \frac{23}{25} \times \frac{1127}{4000} = 0{,}37628$, and (by question 222) as $23 : 32{,}2 :: (25 - 23)\ 2 : 2{,}8 = q$, so by the question, (m being $= 2$) we'll have $\log.$ of $\frac{gg + 2qe}{vv + 2qe} = \frac{m}{2{,}3025\, e} = 2{,}310689$, whose natural number by the log. tables is $= 204{,}45 = \frac{gg + 2qe}{vv + 2qe} = \frac{gg + 2{,}087}{11{,}087}$ therefore, $g = \surd : 204{,}45 \times 11{,}087, - 2{,}087 : = 47{,}6$ feet per second answer.

If the required velocity were such, as just to make the ball go to the bottom of the water, then $v = 0$, and so $204{,}45 = 1 + \frac{gg}{2qe}$ therefore $g = \surd : 204{,}45 - 1 : \times 2{,}087 : = 20{,}5$ feet per second, answer.

Question 230. If a ball of cast iron 6 inches diameter be projected in the air with a velocity of 64 feet per second, what must be the angle of elevation, to give it the greatest horizontal amplitude? See art. 401.

Note. In the 5 first theorems in art. 401. m denotes the height fallen in vacuo to acquire the projectile velocity, or impetus $= m$, so (by art. 374) here $m = \frac{\square\, 64}{64} = 64$ feet, and (by question 223) $e = 3888$, whence $c = \surd : \frac{96e - 235m}{192e - 512m} : =, 7072$, the natural sine of $45^\circ\, 01'$, whose complement to 90° is $44^\circ\, 59'$ the required elevation. Now putting $t = 0{,}9995$, and $s = 1{,}414$ the natural tan-

gent and natural ſecant of $44^{\circ}\ 59'$ the elevation, we'll have ($m = 64$ as before) $a = \frac{4\,t\,m}{s\,s} - : \frac{32\,t\,t}{s^4} + \frac{16\,t^4}{s^5} : \times \frac{m\,m}{3\,e} = 126{,}8$ the anſwer, this amplitude and elevation in a non-reſiſting medium is 128 feet, and 45°, ſo the difference is very ſmall. Here it may be obſerved that the elevation, varies with m and e, &c. Theſe 37 laſt queſtions, are more than the practice of gunnery can ever require, but they exerciſe other curious parts of mathematics, and ſo are thought worth a place here; in gunnery there are ſeveral other obſtacles to be minded, ſuch as the make of the gun and ball, if they be exactly round, the ball to be of the ſame metal, &c. See the foregoing queſtions.

In theſe queſtions the ball is ſuppoſed to have its velocity from the force of gun-powder, but other forces may produce this velocity. As, firſt, a ſpring, or bow. 2. By water &c. See queſtion 160. 3. By air ſee queſtion 190; for, if two tubes or gun barrels, A I and F I (fig. 166) be joined together at I, and the air in A I be by a ſyringe, &c. thruſt into the ſpace H I, and then by opening a valve at I where a ball is lodged, the ſaid confined air will puſh the ball out at F with great force, this is the nature of the air or wind gun. 4. By ſteam (ſee queſtion 191) whoſe vaſt force is there computed, it is proved by trial, that if a piſtol barrel cloſe ſtopped with a few drops of cold water in it, have the end where the water is, put into the fire, and when it begins to ſimmer out at the touch hole, the air in the barrel is then exhauſted, then ſtop the touch hole cloſe up and in a little time the water will be converted into ſteam, and will blow out the cork or ball at the other end with as great violence and noiſe as a charge of gun-powder, &c.

The 30 following queſtions explain frictions ſpouting fluids, whirling bodies, waves, &c. &c.

Queſtion 231. Which of the mechanic powers has moſt friction?

1. The ſmoother bodies are (except they be poliſhed ſo fine as to ſtick together which can happen to few bodies) and the leſs ſpace they have to rub on, the ſmaller is the friction, whence in the leaver, if the prop is ſharp pointed, the friction is little or nothing.

2. In the wheel, the friction upon the axis is as the weight upon it, the diameter of the axis and the velocity of the axis periphery, (ſee queſt. 90) this ſort of friction is but ſmall.

3. A rope of 1 inch diameter, whoſe tenſion is 5lb, going over a pulley 3 inches diameter requires a force of 1 lb to bend it, and the

force of bending a rope, is as the square of it's diameter and tension directly, and the diameter of the pulley inversely, (see quest. 128) &c. and 15 lb over a single pulley will draw up but 14lb weight, but there is a great difference in ropes arising from the temper of the weather, the difference of their stiffness, &c.

4. In the wedge, the power to overcome, must be at least, as twice the base to the height. (See question 70)

5. In the screw there is most friction, for it will sustain the weight in any position when the force that raises the weight is taken away, whence, to raise the weight there must be at least a double force applied to that intended in quest. 66.

6. In a fluid running thro a tube, its plain, the parts rubbed against, are the sides of the tube, whence the friction is as the diameter of the tube and velocity of the fluid, or as the periphery and velocity. But the friction is greater in respect to the quantity of the fluid in small tubes, than in large ones, and that inversely as their diameters, but the absolute qnantity of friction in tubes is very small, except the tubes be very long, and the velocity very great ; in rivers, the velocity is greatest where the water runs deepest, and least at the sides where it is shallow, it is common to take half the sum of these two velocities for the mean velocity of the stream. See quest 156.

7. If any tube, the area of whose section is a, be divided into several small equal ones whose number is n, the area of any one of these small ones will be $\frac{a}{n}$, and the friction being as the diameter it will be as $\sqrt{\frac{a}{n}}$, or as $\frac{1}{\sqrt{n}}$ (by taking $a=1$) and so the friction of them all will be as $\frac{n}{\sqrt{n}}$ which is as $\sqrt{n}$, the increase of friction by such division.

8. The resistance of a plane moving thro' a fluid is (by art. 317) as the square of the velocity, or (by art. 323) = the weight of a column of the fluid whose base is the plane, and height $\frac{vv}{4s}$, but in a globe $=\frac{vv}{8s}$ article 326.

9. In all bodies viz. woods and metals, the friction is nearly the same if they be oiled or greased, in wood acting against wood, grease makes the motion $\frac{2}{7}$ easier, metals of the same sort acting against each other have more friction than metals of a different kind, the softer and

rougher bodies are the greater the friction, these experiments have been made (see Mr. Emerson's mechanics) to determine friction, viz. a cubic piece of soft wood 8 lb weight, moving upon a smooth plane of soft wood at the rate of 3 feet per second, the friction is $\frac{1}{3}$ of its weight, but if the wood be rough it is near $\frac{1}{2}$ the weight, other soft wood upon soft wood very smooth, the friction is about $\frac{1}{4}$ the weight soft wood upon hard, or hard upon soft $\frac{1}{5}$ or $\frac{1}{6}$, the weight hard wood upon hard wood, $\frac{1}{7}$ or $\frac{1}{8}$ the weight, polished steel on steel or pewter $\frac{1}{4}$ the weight on brass $\frac{1}{6}$ the weight, on copper or lead, $\frac{1}{5}$ the weight, &c. from which we learn, that there can be no general rule laid down to determine the quantity of friction in all cases.

10. The friction (all things else being the same) increases with the weight nearly in the same proportion, as also with the velocity in most cases; a greater surface also causes more friction with the same weight and velocity, in some cases, but if the body move on clay, soft earth &c. the friction may be increased by the rubbing parts being too small these things are all very plain if we consider friction as a power acting against the motion of the engine &c. for the only cause of friction is the roughness of the rubbing parts which rough particles may be considered as so many obstacles over which the body is to be drawn.

11. Hence, if by increasing the power to overcome the friction the weight on the rubbing parts be also increas'd, then there must be something more added to the said power, &c.

12. Also, if by any means the velocity of the rubbing parts can be made less, the friction will be lessened nearly in the same ratio, &c.

13. From these articles you may compute the friction of any engine &c. by estimating the friction of every part by its self, and then the sum of all those frictions will be that of the engine, &c.

Question 232. If in weighing 5lb, it be found that 1 grain weight will give the scales a cast, what weight will give them cast when they weight 500 lb? See art. 10, last question.

As 5 lb is to 1 grain so is 500 lb to 100 grains answer, nearly.

Question 233. If a cylinder C A A (fig. 192) of A A forty inches diameter require a weight w of 20 lb to make it turn in form of an axle-tree round its diameters, what must be the diameters of two pins or gudgeons, a, and C, that turning round on them (in the same time) the friction or force of turning may be only 3lb? See art. 12, quest. 231.

As 20 lb is to 40 inches so is 3 lb to 6 inches answer. And if you lay the axle-tree of one wheel upon the periphery of another wheel, the friction will still be lessened, such wheels are called friction wheels.

Question 234. If the power P (fig. 193) being 2 lb weight, hold the weight w in equilib. and it be known that the friction be one third of the power, what weight must be added to the power to overcome the friction, or give motion to the weight w?

Here its plain $\frac{1}{3}$ of (P) 2 lb must be added to the power P, but then this additional weight causes a greater pressure on the pullies, and therefore (see art. 11, question 231) requires $\frac{1}{3}$ of its weight to be added, and that again for the like reason will require $\frac{1}{3}$ of its weight and so on, ad infinitum, whence the weights to be added will be $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, $\frac{2}{81}$, &c. the sum of which decreasing geometrical series is (by ex. 47 in algebra) = 1 lb the answer.

Question 235. If in a jet d'eau A D d F (fig. 222) the reservoir A B be constantly kept full of water, whose height L I, above d, the spout of the conduct pipe C D d, is 5 feet 1 inch, its found by experiment that the water spouts to the height d F of 5 feet, to what height would it spout if the water's surface L A in the reservoir were L I 10 feet?

1. The water would rise to the height G, so that d G would be = L I, were it not for the air's resistance, and from thence fall down in the streams F H and F E, &c. now (by quest. 224) the deffect G F is as the square of L I or d G, the height of the water's surface in the reservoir, that is, as 25 (sq. 5) is to 1 so is 100 (sq. 10) to 4 inches, therefore, 10 feet — 4 inches = 9 feet 8 inches the answer. But small jets fall short more than in this ratio, being more retarded by the air's resistance, and the greatest jets never rise 300 feet high for the velocity of the water at d, is then so great that it is dissipated into small drops like rain, by the air's resistance, and for the same reason, water falling from great heights as from the clouds &c. descends in smaller drops the higher it falls.

But to have a set to rise to the greatest height, these things are to be observed. 1. The adjutage (viz. the hole at d, thro' which the water spouts) should be in a thin plate of metal, and should be larger as the reservoir is higher and the larger it is the higher the water will spout (see quest. 224) if there is plenty of water to supply it, the conduct pipe ought to turn in a curve as D E d, and not with an elbow. 2. The diameter of the adjutage to be nearly as the square root of the height of the reservoir, and if the velocity in the pipe of conduct is to be the same at all heights of the reservoir, that the friction may not increase so much, then the square of the diameter of the pipe of conduct, must be as the cube of that of the adjutage. Otherwise, the diameter of the conduct pipe should, at least be 5 or 6 times as big

as the diameter of the adjutage, and if the height of the reſervoir be 50 feet, the diameter of the conduct pipe 6 inches, that of the adjutage ſhould be 1 inch, &c. Laſtly, the ſpout ſhould not be exactly perpendicular in order to hinder the water falling on the adjutage again. When water is carried a great way through pipes, the friction of the pipes will leſſen the velocity of the water at d, and the jet will not riſe ſo high.

Queſtion 236. To make an artificial fountain? Fig. 223.

Let A B I be a veſſel filled with water in at a pipe A B going near the bottom B, now its plain, if this veſſel be turned bottom upwards, there can no water come out at A, but will all run down a pipe A D and there ſpout out in a ſtream D H, and fall upon another like veſſel E F G, and fill it thro' another like pipe E F, which done the fountain may be turn'd upon the end B, and the water will ſpout out at H, and fill the veſſel A B I, by running in at the pipe A B as before, and thus by turning the fountain, it may be kept conſtantly playing.

Note. If with your finger you ſtop the orifice A of the pipe A B, the fountain will ceaſe to play becauſe then no air paſſes into the veſſel A B I and by taking off the finger it will ſpout againſt D as before, &c. An artificial fountain is very eaſily made thus, fill a ſtrong bottle half full of water, into which put a tube to reach near the bottom of the bottle, then ſtop the top of the bottle cloſe, ſo that no air paſs between it and the tube, this done, blow through the tube what air you can, ſo will the air in the bottle be condenſed, and the water ſpout out at the tube (ſee queſt. 190) any of theſe fountains playing in the ſun-ſhine, and a black cloth placed behind them, the ſtream will ſhew all the colours in the rain-bow.

Queſtion 237. If the diameter of the rim of a ſpinning wheel be 24, the diameter of the whorle that carries the feathers 4, and that of the bobbin or twill 2, what is the difference between her twining and taking up?

Theſe things being carried about by the rubbing of the wheels band, it follows, that while the rim makes 1 revolution, the feathers makes ($24 \div 4$) 6, and the twill ($24 \div 2$) 12 revolutions, hence, the quantity taken up is ($12 - 6$) 6.

Note. The leſs a thread is twined (provided it have ſo much twine as to hinder the fibres from drawing out) the ſtronger it is, and will bear more weight. From hence it appears, that the twill and feathers together twine the yarn, and if their revolutions are equal the wheel takes none up, if the feathers ſtand ſtill ſhe twines none, and the greater the rim, the faſter ſhe works.

Question 238. If when the plane of the sail of a ship makes an ∠ with the keel of 1 point, or $11^{\circ}\ 15'$, the leeway is two points, what would the leeway be if the said angle were 3 points, or $33^{\circ}\ 45'$? Fig. 224.

Let S A be the sail ⊥ to which draw S K to represent the force of the wind against the sail, and ⊥ to S D the position of the keel, draw D K, then (by theo. 157) the force S K is resolved into the two forces, S D, the direct force, and D K, the leeway force, let D E and D S be as the resistances the ship has a head and a side with equal velocities, and suppose S C the way of the ship. Then (by art. 317) As resistance a head with velocity S D is to resistance a head with velocity D E so is □ S D to □ D C, and, as resistance a head with velocity D E is to resistance a side with velocity D E so is D E to S D, and as resistance a side with velocity D E is to resistance a side with velocity D C so is □ D E to □ D C, from these three proportions we get, as resistance a head with velocity S D is to resistance a side with velocity D C, so is □ S D × □ D E × D E to □ D E × □ D C × S D so is S D × D E to □ D C, but the resistances are as the forces producing them, therefore, as S D : D K : : S D × D E : □ D C, = D E × D K hence D C is a mean proportional between D E and D K, let radius = 1, tangent ∠ D S K = co-tangent ∠ D S A = t, then (by theo. 47) as 1 : t : : S D : t × S D = D K, and as S D : D C : : 1 : tangent ∠ D S C = $\frac{DC}{SD}$, but $DC = \sqrt{DE \times DK}$, and D K = t × S D, therefore tangent ∠ $DSC = \sqrt{\frac{t \times DE}{SD}}$, hence as resistance a side is to resistance a head with the same velocity, so is radius × co-tangent ∠ D S A to tang. square of ∠ D S C the leeway. Also, in the same ship, D E and S D are constant, so it will be as √ tangent of $78^{\circ}\ 45'$ is to tangent of $22^{\circ}\ 30'$ so is √co-tangent $33^{\circ}\ 45' = 56^{\circ}\ 15'$ to tangent of $12^{\circ}\ 37'$ the leeway required.

Thus by the logs. the tangent of $78^{\circ}\ 45' =$ (comp. $11^{\circ}\ 15'$) 10,7013 so √ tangent $78^{\circ}\ 45' = 10,7013 \div 2 = 5,3506$ take

and tangent 2 points, or $22^{\circ}\ 30' = 9,6172$ } from
and √ tangent $56^{\circ}\ 15' = 10,1751 \div 2 = 5,0875$ }

answer tangent of $12^{\circ}\ 37' = 9,3541$ leaves.

Note. Here the hull and riggin are not considered, but the error from that is but small, and on point of excess.

Question 239. If the wind in direction W S (fig. 225) fall on S A the sail of a ship, making no leeway, what are the ratio's of the

forces, with which the ship is compelled to turn (the sail being at her head) and in direction of her keel S D, as also to windward.

Draw S C $\perp$ S A, and C D $\perp$ S D, then (by art. 318) the force acting upon the sail in direction S C, is as the square of the incident $\angle$ W S A, but (by theo. 158) the forces in direction S C and S D are as S C : S D, or as radius 1 : S $\angle$ S C D, therefore, as radius 1 : $\square$ S $\angle$ W S A (force in direction S C) : : S $\angle$ S C D $=$ S, $\angle$ A S D : $\square$ S $\angle$ W S A $\times$ S $\angle$ A S D $=$ force in direction S D, and the force to turn the ship about, is that in direction D C $\perp$ the keel S D, so taking its opposite $\angle$ C S D, instead of $\angle$ S C D, we'll have $\square$ S, $\angle$ W S A $\times$ S $\angle$ D S C, or $\square$ S, $\angle$ W S A $\times$ co-sine $\angle$ A S D, $=$ force in direction D C, draw S P $\perp$ W S, and D G $\perp$ S P, then (by art. 318) the forces in directions S D and D G (that to windward) are as $\square$ S D : $\square$ G D, or as $\square$ radius 1 : $\square$ S, $\angle$ D S G, $=$ $\square$ co-sine $\angle$ W S D, therefore, as $\square$ radius 1 : $\square$ S, W S A $\times$ S, A S D (force in direction S D) : : $\square$ c $-$ S, W S D : $\square$ S, W S A $\times$ $\square$ co-sine W S D $\times$ S, A S D $=$ force in direction G D, whence, if A the area of the sail, and v the velocity of the wind continue the same, (see art. 320) the forces of the ship forward, to turn her, and to windward are respectively as v v A $\times$ $\square$ S, W S A $\times$ S, A S D and v v A $\times$ $\square$ S, W S A $\times$ co-sine A S D, and v v A $\times$ $\square$ S, W S A $\times$ $\square$ co-sine W S D $\times$ S, A S D, or as S A S D and co-sine A S D, and $\square$ co-sine W S D $\times$ S, A S D, as required.

Question 240. Things being as in the last quest. let it be required to find the velocities instead of the forces? See art. 317.

The square of the velocity of the ship, in any direction is as the force of the wind upon the sail in that direction, or as (its $=$) the resistance the ship meets with in the water, consequently, the square root of any of the expressions for the force, will be as the velocity in that direction, thus, v $\times$ S, W S A $\times$ $\surd$: A $\times$ S, A S D : and v $\times$ S, W S A $\times$ $\surd$: A $\times$ co-sine A S D : and v $\times$ S, W S A $\times$ co-sine W S D $\times$ $\surd$: A $\times$ S. A S D : are respectively as the velocities, forward, to turn and to windward.

Note. If the $\angle$ W S A, be still the same, and S D P C be a half circle upon any line S C, then the force in any direction S D of the keel will be to the velocity in that direction as the cord S D is to its square root $\surd$ S D, for the $\angle$ S D C is still $= 90^\circ$.

Question 241. If the wind blow at right angles to the keel of a ship, what angle must the plane of her sails make with the keel, that she may sail the swiftest possible? See quest 239.

In this cafe W S (fig. 225) is $\perp$ D S, fo the required $\angle$ A S D, is the comp. of $\angle$ W S A, let e = fine $\angle$ A S D, then $\square$ S, W S A $\times$ S, A S D = $1 - ee \times e$ (radius = 1) = $e - eee$, which being a maximum, you'll find $e = \sqrt{\frac{1}{3}} = ,57735$, the fine of 35° 16′ anfwer. In this pofition of the wind, $\angle$ W S D = 90°, its co-fine is = 0, fo $\square$ S, W S A $\times$ $\square$ co-fine W S D (0) $\times$ S, A S D = 0, the force to windward, and $\square$ S, W S A $\times$ co-fine A S D = cube S, W S A, or cube co-fine A S D, which is greateft when W S A = 90° or A S D = 0, that is when the wind falls perperdicularly on the fail, the fhip turns fwifteft.

Queftion 242. How muft S A (fig. 226) the fail of a fhip whofe center of gravity is G, be inclined to the plane of the horizon H O, that the perpendicular force C D of the wind againft the fail S A may neither raife nor deprefs the head F of the fhip, but keep her fteady?

From C the center of gravity of the fail upon H O the horizon or keel of the fhip, let fall the perpendicular C B then if D C be the force of the wind againft the fail, S A then D B, is the force that generates the fhip's progreffive motion and B C the force lifting the fhip upwards now the force D B (= b C) acting at C, in direction D B, endeavours to turn the fhip, round an axis paffing thro' G, with a force as C B $\times$ B D, viz. the abfolute force B D $\times$ diftance C B, and this is the force depreffing her head; alfo the force raifing her head is = B C $\times$ B G, viz. = the force B C in direction B C endeavouring to turn the fhip round an axis thro' G, the contrary way, $\times$ diftance B G (fee art. 280) hence the force to raife her head, is to the force to fink it as B C $\times$ B G to C B $\times$ B D, that is B G to B D, therefore when the point D falls in G, the fhip's head is neither raifed nor depreffed by the wind, fo to anfwer this queftion, the fail S A muft be fo fet as that a line joining the centers of gravity of the fail and fhip, may be at right angles to the plain of the faid fail, but this point G muft be at the meeting of a line H O parallel to the horizon or axis or keel of the fhip, and to pafs thro' the center of preffure or refiftance which the fhip has by the water in her motion, and a line drawn perpendicular the horizon thro' the center of gravity of the fection of the fhip and water.

Queftion 243. If S A (fig. 226) be an artificial kite, or a fail in that form, with a line and ball e faftened to the end A, to keep up the other end S to the wind, blowing in direction W C parallel to the horizon H O, now if this fail and ball be 10 ftone or 140 lb weight, and its area 1000 feet, and the wind blow at the rate of 40 feet per fecond, and the angle of incidence W C be 30°, what angle H D C, with the horizon H O, will the cord C E D, faftened one end to C the fails center of gravity and the other end at D make?

1. Draw C G ⊥ S A, and G E ⊥ H O, so is G C the direction of the force of the wind against the sail, and the force of the wind to keep it up is (by art. 318) as square incident ∠ S C W, or C G E, for because W C is parallel H O, ∠ W C G = ∠ C G H, whence C G E = comp. C G H = comp. W C G = S C W, now because E G is ⊥ horizon C G ⊥ sail, and E C the direction of the cord, therefore C G, G E and E C, are respectively as the force of the wind, weight of the sail and force pulling at the cord. Let v = 40, velocity of the wind, S = S, incidence 30°, A = area 1000, then (by quest. 145) ,00113 S S v v A = ,00113 × ,25 × 1600 × 1000 = 452 lb = force of the wind C G, and G E = 140 lb the weight of the sail, by which and the included ∠ C G E = 30° the ∠s G C E, and G E C are found = 15° and 135°, so ∠ H D C = 45, the answer. Here its plain if C G = G E, the wind can only hold the sail in equilib. if C G be less than G E it cannot move the sail, &c. and the greater C G is in respect of G E, the higher it will rise in the air, and the length of the cord C D makes no alteration in the ∠ H D E.

Question 244. If a plate of iron S A (fig. 226) be fastened with a cord C D to C its center of gravity and D the bottom of a stream of water running at the rate of 10 feet per second, with what force will the cord be stretched, supposing the weight of the plate = 140 lb its area 17,4 feet, and the angle of incidence S C W = 30°, and, the plate put up in the stream as a kite in the air.

By quest. 145, we'll have 0,978 v v S S A = ,978 × 100 × ,25 × 17,4 = 425 nearly, the force of the stream C G, and by this, and the last quest. G E = 140, ∠ C G E = 30°, so the ∠ of altitude H D E = 45. And then as S, ∠ G C E 15° is to G E 140 lb, so is S, ∠ C G E 30° to E C nearly 270 lb the answer, all found per last quest. radius being = 1, and then S = 0,5 the natural sine of 30°.

Question 245. When the wind in direction W S (fig. 225) makes an ∠ W S D with the keel S D of a ship of 60°, what ∠ A S D must the plane of the sail S A make with the keel, that the ship may make the most way a head possible?

By quest. 241, we are to divide the ∠ W S D into two such parts or ∠'s, W S A and A S D as that the square of the sine of W S A, multiplied sine of A S D may be a maximum; let S = sine W S D, a = sine W S A, e = sine A S D, c = co-sine W S D, radius = r, then, to give a clear solution, let A N P (fig. 134) be the arch to be divided whose sine is P G = S, and co-sine C G = c, into the two arches A N and N P, whose sines are N z = a, and P O = e, co-sines C z and C O,

now because ∠ POI = ∠ IGC = ∠s at B, and z, each = 90° there-fore, comp. OIP = comp. CIG = OPI = ICG, so the Δs VOP GIC, BOC, and zNC are similar, therefore as CN : Cz :: CO : $\frac{CO \times Cz}{CN}$ = CB, and as CN : Nz :: PO : VO = GB = $\frac{Nz \times PO}{CN}$, whence CB − GB = $\frac{CO \times Cz - Nz \times PO}{CN}$ = CG,

That is,	1	$:\sqrt{:rr-aa} \times \sqrt{rr-ee}: - ae = re$, for CO $= \sqrt{:rr-ee}:$ and Cz $= \sqrt{:rr-aa}:$
by theo. 44	2	$e\sqrt{rr-aa} + a\sqrt{rr-ee} = rS$
2 squared	3	$aa + ee - \frac{2aaee}{rr} + \frac{2ae}{rr} \times \sqrt{rr-aa} \times \sqrt{rr-ee} = SS.$
from 2 and 3	4	$SS = aa + 2zae + ee$, taking $z = \frac{c}{v}$, or $z = c$, if radius $r = 1$
whence	5	$e = \sqrt{SS - aa + zzaa} - za$, by compleating the square &c.
5 × aa	6	$aae = \sqrt{:a^4SS - a^6 + zza^6:} - za^3$ = a maximum by the question.

And, therefore $\frac{4aSS - 6a^3 + 6z^2a^3}{2:a^4SS - a^6 + zza^6|^{\frac{1}{2}}} - 3z = 0$, which equation solved will give the value of a required.

If in this last equation we write $1 - SS$ for zz, and $\sqrt{1-SS}$ for z (its equal) we'll have $\frac{4aSS - 6a^3 + 6a^3 - 6SSa^3}{2\sqrt{:a^4SS - a^6 + a^6 - SSa^6:}} - \sqrt{9-9SS}$ $= \frac{4aSS - 6SSa^3}{2\sqrt{:SSa^4 - SSa^6:}} - \sqrt{9-9SS} = \frac{4S - 6Saa}{2\sqrt{:aa-a^4:}} - \sqrt{9-9SS} = 0$,

which by transposing $\sqrt{9-9SS}$, squaring each side and multiplying by $4aa - 4a^4$, gives, $SS \times : 16 - 48aa + 36a^4 = 36 \times : SSa^4 - a^4 + a^2 - S^2a^2$, whence, $36a^4 - 36aa - 12SSaa = -16SS$, and $a^4 - \frac{36 - 12SS}{36}aa = -\frac{16SS}{36}$ or $a^4 : -1 - \frac{1}{3}SS : \times aa = -\frac{4}{9}SS$, which by compleating the square &c. and putting $1 + \frac{1}{3}SS = n$, we'll have $a = \sqrt{\frac{n}{2} \pm \sqrt{:\frac{nn}{4} - \frac{4SS}{9}:}} = 0{,}62041$ the natural sine of 38° 22′, so 60° − 38° 22′ = 21° 38′ answer.

Note. It appears that the ſine of the difference of $38^\circ 22'$ ($\angle$ W S A) and $21^\circ 38'$ ($\angle$ A S D) is $= \frac{1}{2}$ of the ſine of 60° ($\angle$ W S D) their ſum, and will ſtill hold ſo, let $\angle$ W S D be what it will, and by theſe ſines you'll find that tangent $\angle$ W S A is always double tangent $\angle$ A S D by which ſuch queſtions may readily be ſolved when S is given.

Queſtion 246. Let things be the ſame as in the laſt queſtion, but ſuppoſe the velocity of the wind to be 9 (V) and that of the ſail, or ſhip a head 3 (v)?

Let S W (fig. 205) be the velocity of the wind, and S b that of the ſail, then its plain, while the ſail with theſe two velocities from S to b, the angles W S A and W S D, becomes the $\angle$s W b a and W b D (the $\angle$ A S H being ſtill equal angle S b A, becauſe the ſail moves parallel to itſelf, draw L S $\perp$ A S A, alſo L R and W Q, each $\perp$ S D) then by the laſt queſtion, we'll have $\square$ S W b a $\times$ S S b a $=$ a maximum, as will otherwiſe appear thus, draw W e $\perp$ to the ſail A b a produced, then ſince W S is the direction and velocity of the wind on the ſail A S A, it is plain we (ſee theo. 158) is that force of the wind which drives the ſail from b to S, which (by art. 318) is as $\square$ W e, or as $\square$ incident $\angle$ W b e, ſo (ſee queſtion 239) $\square$ S $\angle$ W b e $\times$ S $\angle$ S b e $=$ (as above) force in direction b S, now $V = 9$, being $=$ S W and $v = 3 =$ S b, let S $=$ ſine and $z =$ co-ſine of $\angle$ W S D (60°) alſo put $y =$ S a, $e =$ a b, then S a and L a being tangent of the $\angle$s S b a and L b a, to the equal radius b a, we'll (by the note to the laſt queſtion) have L a $= 2$ S a $= 2y$, ſo L S $= 3y$, alſo $\frac{e}{y} = t =$ tangent $\angle$ a S b, (if the radius be S b $= 1$) then by ſimilar $\triangle$s, as $v : y :: 3y : \frac{3yy}{v} =$ S R, ſo $\frac{3yy}{v} - v = \frac{3yy - vv}{v} =$ b R $=$ S R $-$ S b, alſo as $v : e :: 3y : \frac{3ey}{v} =$ L R, but by trigonometry Q W $=$ S V, and b Q $= zV - v$, whence again, as S V : $zV - v :: \frac{3ey}{v}$: (b R) $\frac{3yy - vv}{v}$, therefore, $3yy - vv = : \frac{zV - v}{SV} : \times 3ey$, (or becauſe $yy + ee = vv$) $2yy - ee = : \frac{zV - v}{SV} : 3ey$, which divided by yy, &c. gives $\frac{ee}{yy} + : \frac{zV - v}{SV} : \times \frac{3e}{y} = 2$, which by com-

pleating the ſquare &c. gives $\frac{e}{y} = \surd : 2 + \frac{9}{4} \times \overline{\frac{zV - v}{SV}}\Big|^2 : - \frac{3}{2} \times : \frac{zV - v}{SV} := t = 1{,}155$, the natural tangent of $49^\circ\ 07'$, whoſe complement is $40^\circ\ 53'$ for the required $\angle$ S b a $= \angle$ H S A, ſo as that the effect may be a maximum. If the ſail be at reſt, or $v = 0$, then $t =$

$$\surd : 2 + \frac{9}{4} \times \overline{\frac{zV - v}{SV}}^2 : - \frac{3}{2} \times : \frac{zV - v}{SV} := \surd : 2 + \frac{9zzVV}{4SSVV} : - \frac{3zV}{2SV} = \surd : 2 + \frac{9zz}{4SS} : - \frac{3z}{2S} = \surd : 2 + \frac{9 \times 0{,}25}{4 \times 0{,}75} : - \frac{3 \times 0{,}5}{2 \times \surd 0{,}75}$$

$= {,}7917 =$ the natural tangent of $38^\circ\ 22'$, whoſe comp. is $51^\circ\ 38'$ $= \angle$ S b a $= \angle$ H S A, the ſame as would be found by the laſt queſt. for $\angle$ W S D being $= 60^\circ$ the $\angle$ H S W, will be $= 120^\circ$, whoſe natural ſine is $\surd{,}75$ (for $180^\circ - 120^\circ = 60^\circ$) one third part of which is $\frac{1}{3} \surd{,}75 = {,}2887$ the natural ſine of $16^\circ\ 44'$, then $\angle$ H S W $120^\circ - 16^\circ\ 44' = 103^\circ\ 16'$, half of which is $51^\circ\ 38'$ for the $\angle$ H S A, as before.

Queſtion 246. If the wind or any fluid fall perpendicularly on the plane of a ſail, what angle muſt the ſaid plane make with the direction of its motion, that the effect may be the greateſt poſſible, ſuppoſing the velocities (V and v) of the wind and ſail to be equal? Fig. 205.

Here V = W S, v = S b, Let a = ſine $\angle$ W S A (= rad. 1) which the direction of the ſtream makes with the plane, and e = ſine of the required H S A $= \angle$ S b a, then becauſe A A || A a, the $\angle$ W d e $= \angle$ W S A, then (by the laſt queſt.) the force in the perpendicular direction as is expreſſed by $\square$ w e, ſo the force f in direction b S, will be $\square\, we \times \frac{Sa}{Sb}$, but by plane trigonometry, $ev = Sa = eE$ and $\frac{ev}{a} = Sd$, alſo, as $1 : V - \frac{ev}{a} :: a : Va - ev = ew$, ſo $\square\, ew \times \frac{Sa}{Sb} = \overline{Va - ve}^2 \times \frac{ev}{v} = eVVaa - 2Vvaee + vve^3 = f$ (= vv by queſt. 240) which by the queſtion muſt be a maximum, ſo $VVaa - 4Vvae + 3eevv = 0$, therefore $ee - \frac{4Va}{3v} e = - \frac{VVaa}{3vv}$, whence $e = \frac{2Va}{3v} - \surd : \frac{4VVaa}{9vv} - \frac{VVaa}{3vv} : = \frac{Va}{3v} = \frac{1}{3} = 0{,}3333$ the natur-

al fine of 19° 29′ the anfwer, from this laft equation we have, As $\frac{2}{3}$ the velocity of the fluid ($\frac{1}{3}$ V) is to the velocity of the plane (v) : : the fine of the ∠ made by the plane and its direction (e) to the fine of the angle made by the plane and direction of the ftream (a) when the effect of fuch a machine is the greateft poffible, but this proportion is impoffible if $\frac{aV}{3v} = e$, be more than unity, for e the natural fine of any ∠, cannot exceed radius (1) or unity, if a and e are each $=$ radius (1) or the fail at right angles to the wind and goes directly before it, then $v = \frac{1}{3}V$. Hence, if the wind can produce a velocity in a fhip &c. greater than $\frac{1}{3}$ of its own velocity, its plain the fhip may run fwifter upon an oblique courfe than when fhe fails directly before the wind.

Queftion 246. What angle muft the plane of a fail (of a wind-mill &c.) make with the direction of its motion that it may turn or move with the greateft freedom poffible, the velocity (v) of the fail being $\frac{1}{4}$ of the wind's velocity ($\frac{1}{4}$ V) and the wind blowing perpendicularly on the direction of the fail? See queft. 144.

Let things be the fame as in the laft queft. but one, then S being $=$ radius $= 1$, the co-fine z will be $= 0$, and fo $t = \sqrt{:2 + \frac{9}{4} \times \overline{\frac{zV-v}{SV}}\Big|^2 : - \frac{3}{2} \times \frac{zV-v}{SV}} = \sqrt{:2 + \frac{9vv}{4VV} : + \frac{3v}{2V}} = \sqrt{2 + \frac{9}{64}} + \frac{3}{8} = \sqrt{2{,}1406} + 0{,}375 = 1{,}855$, anfwering to the natural tangent of 61° 40′ the anfwer. If $v = 0$ then $t = \sqrt{2}$, which anfwers exact- with queft. 144, now the velocity of the fail of a wind-mill being as its diftance from the axis of motion, if we fuppofe $v =$ the fails velocity at 1 foot from the faid axis, then the velocity at 2, 3, 4, &c. feet diftance, will be 2 v, 3 v, 4 v, &c. and that the effect may be the greateft, the fail muft be fo twifted as every where to make an angle with the direction of its motion, whofe tangent $= \sqrt{:2 + \frac{9vv}{4VV} : + \frac{3v}{2V}}$.

Queftion 246. If CSB, be a lever, S the prop, $SC = a = 6$, $SB = b = 30$, the weight at $C = w = 500$, what weight or power P placed at B, will give w the greateft momentum poffible? Fig. 158.

Note. Here are 4 queftions of this number 246, in a miftake, but the number is regulated after queftion 294.

1. Let $n =$ the weight that will ballance w, then per lever $bn = aw$, or $n = \frac{aw}{b}$, let $n + q = P$, $2s =$ velocity acquired by gravity in the first second of time equal 32 feet v equal the velocity gain'd or acquired by w in the same time, then as $a : v :: b : \frac{bv}{a} =$ velocity of P, therefore, $wv + \frac{vbP}{a} = \frac{avw + vbP}{a} =$ the sum of the motions of w and P, and must be $= 2Sq$, the motion of q, because by this motion they are both generated, i. e. $avw + vbP = 2Sqa = $ (because $q = P - n$) $2Sa \times : P - n : =$ (because $n = \frac{aw}{b}$) $2Sa \times : P \frac{aw}{b}$: whence $v = \frac{2Sa \times : bP - aw :}{b \times : bP + aw :}$, whence, $wv = \frac{2Sa \times : bPw - aww :}{b \times : bP + aw :} =$ momentum of w, which by the quest. must be a maximum, and therefore (art. 223, w variable) we get $bbPP = 2Pbaw + aaww$, which solved gives $w = \frac{-1 + \sqrt{2}}{a} \times bP = 0{,}414 \frac{bP}{a}$, so $P = 2{,}417 \frac{aw}{b} = \frac{2{,}417 \times 6 \times 500}{30} = 241{,}7$ answer. See question 253.

2. It is plain by the figure, that S B may be the radius of a wheel, and S C the radius of its axle-tree, the process will be the same, and any 3 of the 4 letters a, b, P, w, being given the 4th may be had from the last equation, and when $a = b$ it is a pulley, and then $P = 2{,}417 \times 500 = 1208{,}5$.

3. By writing $\frac{-2 + \sqrt{2}}{a} bP$ in $v = \frac{2Sa \times : bP - aw :}{b \times : bP + aw :}$ for its equal w, we get $v = \frac{2Sa}{b} \times : \sqrt{2} - 1 = 13{,}248 \frac{a}{b} = 2{,}6496$ feet, the velocity acquired by w in the first second of time, and the velocity acquired by P in the same time is $= \frac{b}{a} \times 13{,}248 \frac{a}{b} = 13{,}248$ feet. And in case of a pulley, $a = b$, then 13,248 is the velocity acquired by each weight in the first second of time, when the momentum, or force of each weight is the greatest.

4. In quest. 253, we have $w = \frac{bP}{2a}$, which written in the aforesaid

value of v, instead of w, we'll get $v = \frac{2Sa}{b} \times : \frac{bP - \frac{1}{2}bP}{bP + \frac{1}{2}bP} =$ $\frac{2Sa}{3b} = 2\frac{2}{15}$ feet, the velocity acquired by w in the first second of time, and $\frac{b}{a} \times \frac{2Sa}{3b} = \frac{2S}{3} = 10\frac{2}{3}$, the velocity acquired by P in the same time, when w is raised by P in the least time, and in case of a pulley, or $b = a$, $10\frac{2}{3}$ feet in the first second of time is the velocity acquired by each weight. Here, If $S = 32$, the uniform velocity of gravity then $\frac{2}{3}$ S, becomes $\frac{1}{3}$ S, the same as $\frac{1}{3} v = e$, quest. 150, &c.

Question 247. If a machine is to move a weight of 5000 lb at the rate of 2 feet per second and the power to do this can only move 10 feet per second, what is the weight of the power?

1. In case of an equilib. (see theo. 183) the momentums of the power and weight must be equal, so to put the machine in motion, something must be added to the power, or its velocity increased, so $5000 \times 2 = 10 \times p$, whence $p = \frac{10000}{10} = 1000$ lb for the equilib. ans.

Question. 258. If a vessel of water &c. receive a shake or stroke, the water will undulate up and down the two opposite sides of the vessel, now if the breadth of this moving water be 18 inches, how many times will it undulate in 1 second?

1. If a beam &c, be horizontally suspended by its middle and set a moving, it will vibrate up and down till it recover a position parallel to the horizon and then rest, so will water in a vessel after the vessel has leaned to one side, and then set upright again, whence it follows, that the motion of waves, or ascent and descent of water up and down the two opposite sides of a vessel, is the same with the swinging of a pendulum whose length is half the breadth of the wave, or undulating water, therefore (see question 117) As 9 inches (the $\frac{1}{2}$ of 18 the given breadth) is to sq. 1 second so is 39.2 inches to 4,35 + whose square root is 2,08 the number of times per second answer.

Question 249. If the breadth of a wave in the sea &c. be 18 inches, what is the velocity of the water in that wave, that is, how many inches will it move per second?

By the last question, when the waves are 39,2 inches broad the water will undulate in one second of time, then because waves like pendulums are moved by gravity, it will be as 39,2 : sq 39,2 so is 18 to the square of the required velocity, that is, $\sqrt{:39{,}2 \times 18:} = 26{,}56$ inches per second answer.

Question 250. If A B (fig. 227) be a circle made about any heavy body whofe center of gravity is at C in the center of this circle, and if about this circle's periphery a ftring be wrap'd and one end S thereof be faftened to a pin, and the body let go, how far will it defcend in the firft fecond of time, by unwinding itfelf about the ftring, the radius A C being 10 feet?

Let O be the center of ofcillation to A, the point of fufpenfion, A B being parallel to the horizon, then (by queftion 122) $AO = d + \frac{0,4\,rr}{d} = r + 0,4\,r$ (becaufe in this cafe $d = r$) $= 1,4\,r = 14$ feet. Now when a body ofcillates, its whole force or weight is in O, (art. 291) and fo when it begins its motion, it begins to fall freely as if it were placed in O, and at the fame time begins to whirl about the center of gravity C, now becaufe A, C, O, are ftill in the fame right line, the body conftantly whirls in the fame manner, and therefore as A O is to A C fo is the velocity of the body freely falling to its velocity when whirling, fo is 14 to 10, whence, as 14 is to 16 (the defcent of gravity in the firft fecond of time) fo is 10 to 11 $\frac{3}{7}$ feet anfwer.

In this queftion the body is fuppofed to be a globe with the ftring about its greateft circle; in any other body the fame is true, by looking upon the body as fufpended at A, at the unwinding of the ftring and finding the center of ofcillation O, to that point A.

Queftion 251. Whether will a cylinder 36 inches long, 20 inches diameter and 15 lb weight defcend fafter, by unwinding a cord from about its middle (or rowl as it falls,) or by having the cord wrap'd lengthways and fo turn end over end as it defcends, and in whether cafe will it ftretch the cord moft? See art. 292.

By the laft queftion, as $\frac{2}{3}$ of 36 is to $\frac{1}{2}$ of 36, viz. as 24 is to 18, or, as 4 is to 3, that is, $\frac{3}{4}$ is as its velocity turning endways and as $\frac{5}{4}$ of 10 is to 10, viz. as 12,5 is to 10, or as 5 is to 4, that is, $\frac{4}{5}$ is as its rowling velocity, therefore, as $\frac{4}{5}$ is to $\frac{3}{4}$ fo is 16 to 15, fo is the rowling velocity to that turning endways in the fame time. Secondly, fince the weight &c. of any body lies in its center of gravity, the body in this cafe may be confidered as fupported at the points A and O, and then (by art. 272) C O and C A, will be as the forces in A and O, and therefore, as A O : C O : : weight of the body : force at A, which is equal the tenfion of the cord S A, and is (per laft queft.) always the fame; whence, $(\frac{24-18}{15})\ \frac{6}{15}$ and $(\frac{12,5-10}{15})\ \frac{2,5}{15}$, that is, as 6 is to 2,5, or as 12 is to 5 fo is the tenfion of the ftring when the body turns endways to its tenfion when the body or cylinder rowls.

Question 252 If a pair of wheels were placed at the top of an inclined plane perfectly ſmooth, whether would they ſlide or whirl down it?

If in the two laſt queſtions, inſtead of ſuppoſing the motions to be $\perp$ we ſuppoſe them to be down an inclined plane, its manifeſt the reſult will be the ſame, and the place of the cord will be ſupplied by the friction of the wheels, if the plane be rough, or by teeth in them if it be ſmooth, whence, as a body does not whirl in freely falling, its plain the wheels will ſlide down this plane, and that the reaſon of carriage &c. wheels whirling, is owing to the friction or roughneſs of the ground, &c.

Queſtion 253. If two weights P and w (50 and 70) one at one end of a rope and the other at the other end thereof over a pulley be let go at liberty, with what force is the rope rais'd?

1. If w act with a force $=$ F, and P with one $=$ f, then its plain that $F - f$ is the force with which w is urged towards the horizon, whence becauſe both bodies (by reaſon of the rope) acquire the ſame velocity the motion generated in w alone, or in that part of the whole expreſſed by $\frac{w}{P+w}$ will be $\frac{Fw - fw}{P+w}$, but if the rope were not, the motion of w, would only be F, therefore the loſs of motion by the action of the rope is $F - \frac{Fw + fw}{P+w} = \frac{FP + fw}{P+w}$, for the force ſtretching the rope, or that ſufficient to cauſe the ſaid loſs of motion, and if $F =$ w and $f = P$, then $\frac{2\,Pw}{P+w} = 58\frac{1}{3}$ the ſaid ſtretching force, which doubled (becauſe this force is the ſame at each end of the rope) gives $\frac{4\,Pw}{P+w} = 116\frac{2}{3}$, the weight on the axis of the pulley while the bodies are in motion.

2. If inſtead of the bodies moving in direction perpendicular to the horizon, one of them (as w) moves along an inclined plane B O (fig. 211) by the perpendicular deſcent of P, the rope over a pulley at the angular point O, and the height B E of this plane be to its length B O, as a to b, then by the property of the inclined plane we'll in a caſe of equilib. have $aw = bP$, and therefore $\frac{aw}{b} =$ force F, by which w tends

to move down the plane O B, which written in $\frac{FP+fw}{P+w}$, for F, and P for f, it becomes $\frac{Pw}{P+w} \times : \frac{a+b}{b}$: for the tension of the rope in this case, where if the angle O B E of inclination be $= 30^\circ$, then as b ; a ; : 2 : 1, therefore $\frac{Pw}{P+w} \times : \frac{a+b}{b} := \frac{Pw}{P+w} \times \frac{3}{2} =$ $43\frac{3}{4}$, the tension of the rope, and doubled is $87\frac{1}{2}$, the weight born by the pulley in the time of motion.

3. Since (quest. 246) $v = \frac{Sa \times : bw - aP :}{b \times : bw + aP :} = 4\frac{1}{19}$ feet, in the first second of time, that the body w would descend in direction ⊥ to the horizon and P ascend the same distance directly upwards, or the contrary, therefore as $4\frac{1}{19}$ feet : □ 1 second : : any perpendicular height : □ time in seconds, of moving thro' that height, and in case the bodies hang at liberty over a pulley, then $a = b$, and so $v = \frac{Sw - SP}{w+P} =$ $2\frac{2}{3}$ feet, in the first second of time &c.

Note. $S = 16$ feet the descent of gravity in the first second.

4. If the weight w is to be drawn up the inclined plane B O (fig. 211) by the power P descending perpendicularly at the other end of the rope over a pulley at O, in the least time possible, let $b = BO$ the plane's length, and $a = EO$ its height, then since as above, the force tending to move w being as $\frac{aw}{b}$, we have $P - \frac{aw}{b} = \frac{bP - aw}{b}$, for the force f by which the bodies are accelerated, and so from $t \propto \sqrt{\frac{Sb}{f}}$ (theo. 151) we have $t \propto$ (b being $= 1$ and $\frac{bP - aw}{b} = f$) $\frac{b}{\sqrt{: bP - aw :}}$, which being made a maximum (b variable) we get $bP = 2aw$, now if b, be as radius 1, then a will be as sine ∠ inclination O B E, then $P = 2aw$, or $\frac{P}{2w} = a = 0{,}3571 =$ sine of that angle &c. If $a = b$, then the weight w, and power P are at liberty over a pulley, and then $P = 2w$, whence, the power must be equal to twice the weight to raise it in the least time possible but (quest. 246) $p = 2{,}417$ w, when the momentum is greatest. Hence, in any combination of pulleys, if it be as velocity of w : velocity of P : : 0,414 P : w (or : : $\frac{2}{5}$ P : w) then will w have the greatest momentum (or be raised in the

least time), also by (quest. 115) $P = 1{,}5\ w$, when the machine performs the greatest effect with the most ease or freedom, but if the motion of the machine be uniform then (quest. 150) $P = \frac{4}{9}\ w$, when it performs the greatest effect.

Question 255. If the breadth of a house be 24 feet, what must be the height of the roof above the eaves, that water &c. may be the least time possible in running down it?

Let I H (fig. 146) = 12 feet, half the breadth of the room, and A I the height of the pike, A H being the slope of the roof, put e = sine ∠ I H A, the inclination of the roof with the horizon, which is also = sine ∠ I B H, because ∠ B H A = 90°. Let a = 12 = I H, and d = 16 feet, the descent of gravity, then (by theo. 172) $\sqrt{A\,B}$ will be as the time of falling down A H, therefore by trigonometry, $\sqrt{:1-ee:}$ $: a :: \text{radius } 1 : \dfrac{a}{\sqrt{:1-ee:}} = A\,H$, and as $e : (A\,H)\ \dfrac{a}{\sqrt{:1-ee:}} :: \text{radius } 1 : \dfrac{a}{e\sqrt{:1-ee:}} = A\,B$, but as $\sqrt{c} : 1$ second $:: \sqrt{:A\,B:}$ $\sqrt{:\dfrac{a}{ee\sqrt{1-ee}}:}$: time of falling thro' A H, which by the question must be a minimum, so (by theo. 148) we have $e = \sqrt{\frac{1}{2}}$ = natural sine of 45° whence A I must be = I H, = 12 this is also evident (by theo. 172) for since the times of falling down A B and A H are equal these times are least when A B is least in respect of I H, and that will be when I H is $= \frac{1}{2}$ A B.

Question 256. What must be the inclination of a plane with the horizon, that a heavy body sliding, or rolling down it, may strike an object with the greatest force possible, the object being perpendicular to the horizon? Fig. 168.

Let e = sine ∠ C A B, which the plane C A makes with the horizon A B, radius = 1, then $\sqrt{:1-ee}$ = sine ∠ A G H let A P be the obstacle perpendicular to A B, now (by theo. 157) if G A express the force of the heavy body, then H A, will be it's force against the obstacle P A, but (by theo. 170) the velocity, or force G A, is as $\sqrt{G\,H}$, or as $\sqrt{e}$; therefore, by trigonometry, as radius $1 : \sqrt{e}\ (A\,G) :$ $\sqrt{:-ee:}$ (sine ∠ A G H) : $\sqrt{e} \times \sqrt{:1-ee:} = \sqrt{:e-eee:}$; = the force in direction H A, now if this expression be a maximum, then (by theo. 148) $e - 3eee = 0$, or $e = \sqrt{\frac{1}{3}}$ = natural sine of 35° 16′ for the ∠ G A H, as required.

Question 257. A beam B a (fig. 156) is to be supported in a given position (∠ a B P) by a prop C m, of a given length, insisting on a horizontal beam B A supported at its ends A and B, what must be the

position of the prop C m, so that the beam A B on which it stands may be the least strain'd, the beam being of equal thickness ?

Suppose B n perpendicular C m, then (by theo. 191) the stress or pressure upon the prop C m is as $\frac{1}{Bn}$ and therefore the force tending to break A B in m, will be as $\frac{1}{Bm}$, but (by art. 298) the strength of B A in m, is as $\frac{1}{Bm \times Am}$ (strength being inversely as stress), whence as $\frac{1}{Bm \times Am}$ is to $\frac{1}{Bm}$ so is 1 to A m, but (by the quest) this ratio 1 to A m viz. A m is to be a minimum, consequently B m must be a maximum, but B m is to sine $\angle$ B C m as C m is to sine $\angle$ C B m, whence B m is greatest when sine $\angle$ B C m is so, and that is when it is $= 90^{\circ}$, or the prop C m is $\perp$ to B a.

Question 258. What is the side (a) of a cube of wood, which swimming in fresh water is 1 inch (e) above the water's surface, but in sea-water 2 inches (z) dry, or above the surface ?

Here $aa \times \overline{a-e} =$ the wet part in fresh water, and $aa \times \overline{a-z}$ the wet part in salt water, and (by art. 338) $100\,aa \times \overline{a-e} = 103\,aa \times \overline{a-z}$ (100 being to 103 as the density of fresh water to that of salt water), which reduced gives $3a = 103z - 100e = 206 - 100 = 106$, so $a = 106 \div 3 = 35\frac{1}{3}$ inches answer.

Question 259. If an upright vessel constantly filled with water 9 inches deep, have a hole at the bottom which runs at the rate of 3 gallons per second, what length must a tube be, fixt to the hole and of the same bore therewith, that the water voided at the tube's bottom may be 4 gallons per second. Because the diameters of the hole and tube are equal, the quantity of water voided will be as its velocity at the hole and tube (viz. at the top and bottom of the tube, which velocities being as the $\square$ root of the heights fallen thro', it will be as 3 gall. is to 3 (square root of 9 inches the water's surface above the hole) so is 4 gall. to 4 (the square root of 16 inches the said surface above the tube's lower end) so $16 - 9 = 7$ inches the tubes length answer.

Hence it appears that a vessel will be sooner emptied thro' a pipe at the bottom, than thro' a hole there &c.

Question 260. If a cannon ball in latitude 53° be projected directly upwards to the height of 2640 feet, where will it fall ? Fig. 229.

1. The time of the balls aſcent and deſcent being equal it will be as 16,1 feet : □ 1 ſecond : : 2640 feet : 163,98, whoſe ſquare root doubled, is 25,6 ſeconds the time ball is in motion, now in that time the cannon will be carried thro' a ſpace of 23666 feet, by the rotation of the earth about its axis, thus as radius is to co-ſine of any latitude ſo is the ſpace moved by any point at the equator to the ſpace moved by a point in that latitude in the ſame time.

2. Let E T B, be an arch of the earth in latitude 53° q the earth's center, then if the ball is projected at E, the point E in 25,6 ſeconds will be carried to B, over the arch E B = 23666 feet, now the ball (carried by a motion compounded of the earth rotation and its projectile motion) will deſcribe an ellipſis whoſe focus is at q, and bodies in motion deſcribing equal areas in equal times, the elliptic area E A tq = circular area E T B q. or area t A E t = area t q B, but becauſe T A is very ſmall in reſpect of E q = t q, the area t A E t, may be taken for that of a parabola, ſo let b = E q = 21000000 feet, the earth's radius, d = 23666 = E B, h = T A = 2640 feet, a = t B the required diſtance, then $\frac{1}{2}$ b a = $\frac{2}{3}$ h × : d — a :, ſo 3 b a = 4 h d — 4 h a, whence $a = \frac{4hd}{3b+4h}$, nearly $= \frac{4hd}{3b} = 3{,}967$ feet that the ball will fall weſterly from the gun, becauſe the earth moves eaſterly.

3. But if the ball were ſuſpended in the air directly above the gun &c. at the height of 2640 feet, and there let fall, (ſuppoſing it to fall 16,1 feet in the firſt ſecond of time, and the earth to be a globe &c. as before) it would ſtrike the earth at $\frac{1}{2}$ of 23666 feet weſt of the gun &c. becauſe in this caſe the ball would not be affected by the earth's motion as when projected from the earth for then the earth's motion is communicated to it before it be projected, and ſo conſequently it muſt move by the joint effect of theſe two motions or velocities.

The 32 following queſtions ſhews the nature of circular motion.

Queſtion 261. Whether will a ſling be more ſtrain'd, in whirling a ſtone about at the rate of 2, or 5 feet per ſecond?

Let C A = C Q = $\frac{1}{2}$ a = the length of the ſling (fig. 228) v = 2 and u = 5, and one end of the ſtring faſtened to the center C, with with the ſtone at A the other end of it, then if the ſtone (being at reſt) were ſtruck by any power or force it would move in a right line as A H, were it not for the ſtring which pulls it out of the ſaid line or tangent, a diſtance H E = A d, and ſo cauſes it to move thro' the

arch A E, instead of the distance A H, and thus the stone constantly endeavours to fly off from the center which is called the centrifugal force, with this force it acts on the center C, and this is called the centripetal force, these two together are called the central forces of the revolving body, also A H the space which the body A moves thro' in a constant particle of time by an uniform velocity is called the projectile force, or velocity of the body, and if this A H be supposed very small then the arch A E or its chord are each equal A H, join Q E, so will the △ Q E A be a right one and by similar △s, as A Q (a) : A E : : A E : $\frac{\square A E}{a}$ = A d, the central force, whence, if A E = the projectile force = u = 5, or = v = 2, then we have as $\frac{uu}{a} : \frac{vv}{a}$, that is, as u u : v v, or as 25 : 4 : : the stress on the string when the stone moves at 5 feet, to its stress when the velocity of the stone is at the rate of 2 feet in the same time, thus the greater the central force or the more the string is strain'd, the faster the stone is whirl'd about.

Question 262. What must be the velocity, and time of one revolution (called the periodical time) of a body moving round a circle when its central force is double to its gravity, the circle's diameter being (a) 60 feet?

Let P = the periodic time, v = the velocity and f = the central force, c = 3,1416, then (per last quest.) as A E : 1 : : c a : $\frac{ca}{AE}$ = P, the time of describing the whole circle, the motion being uniform, therefore P = $\frac{3{,}1416\,a}{AE} = \frac{ca}{v}$, whence v = $\frac{ca}{P}$, so v v = $\frac{cca a}{PP}$,

but f = $\frac{vv}{a}$ (last quest.) whence f = $\frac{ccaa}{PPa} = \frac{cca}{PP}$, or f is as $\frac{a}{PP}$ (because c c is constant) hence f is as $\frac{vv}{a}$ and as $\frac{a}{PP}$, that is, the central force is directly as the square of the velocity and inversely as the square of the periodic time in the same circle, and therefore P P v v = a a, or P v = a, again, as c a : P : : A E : $\frac{AE \times P}{ca}$, or $\frac{vP}{ca}$ = t the time of describing A E, and (per falling bodies) as $\square$ 1 second is to 16,2 feet so is ($\square$ t t) $\frac{vvPP}{ccaa}$ to the distance fallen in the time t, by the

force of gravity, hence as $\frac{vv}{a} : \frac{16,2\ vvPP}{ccaa} :: 1 : \frac{16,2\ PP}{cca} ::$

$\frac{cca}{16,2\ PP} : 1 :: 16,2\ PP : cca :: PP : 0,615\ a ::$ the central force, to the force of gravity, now by the queſtion, $2\ PP = 0,615 \times 60$, ſo $P = \surd\ 18,45 = 4,3$ ſeconds (nearly) for the time of one revolution, and as 4,3 ſeconds : $60 \times 3,1416 :: 1 : v = 43,8$ feet per ſecond anſwer.

Queſtion 263. If a pendulum 3 feet in length be made to deſcribe a conical ſurface, what muſt be the diameter of the cone's baſe deſcribed by the bob, when the pendulum makes each revolution in one ſecond of time?

Let $vA = vB$ (fig. 127) be the length of the pendulum, $AB = a$, the diameter ſought, $vD = h$, the cone's height, c and P, as per laſt queſt. now as the bob, or revolving body, endeavours to leave the center D, by the central force in direction DA, it is compelled towards it by the force of gravity in direction parallel to vD, but by the laſt queſtion, the central force is to that of gravity as $\frac{cca}{16,2PP} : 1$, therefore as $\frac{cca}{16,2PP} : 1 :: \frac{1}{2} a\ (DA) : h\ (vD)$ ergo $\frac{1}{2} a = \frac{ccah}{16,2PP}$, or $16,2\ PP = 2cch$, and (per falling bodies) as □ 1 ſec. : 16,2 feet :: PP : 16,2 PP, the diſtance fallen thro' in the time of one revol. of the pendulum, which call d then from the two laſt equations we have $d = 16,2\ PP = 2cch$, ſo $h = \frac{d}{2cc} = \frac{16,2}{19,72} = 0,82$ feet $= vD$ $\surd$: □ vA − □ vD: $= DA = 2,9$ fere, ſo $AB = a = 5,8$ feet anſwer.

Note. By the queſt. the diſtance d muſt be $= 16,2$ feet, being the diſtance deſcended in the given time one ſecond.

Queſtion 264. If a pendulum be made to deſcribe a conical ſurface, whoſe axis is to its diameter as 9 to 2 (h to a) in what time (P) will the pendulum make one revolution?

From the laſt queſt. we have $16,2\ PP = 2cch$, ſo $P = c \surd : \frac{h}{8,1} : = 3,1416 \surd : 1,1111 : = 3,1416 \times 1,05 = 3,3$ ſeconds nearly, anſwer. Let the diameter (a) of the cone's baſe be what it will, P will be equal 3,3 ſeconds nearly, if the axis h be $= 9$ feet, as is plain by the laſt queſtion.

Question 265. If a pendulum 3 feet long ($\frac{1}{2}$ a) be whirled about in (P) 2 ſeconds of time in a horizontal direction, what is its gravity to its centrifugal force?

1. As □ 1 ſecond : 16,2 feet : : □ 2 ſeconds : 64,8 feet, the force of gravity generated in the time P of one vibration of the pendulum, and (by queſt. 262) $\frac{cca}{PP}$ is = the central force, ſo by this queſtion as $64,8 : (\frac{cca}{PP})\ \frac{6 \times \square\, 3,1416}{4} = 14,78 : : 162 : 37$ nearly, : : gravity : central force, that is, if the pin on which the pendulum hangs, be pulled with a force of 37 in direction of the pendulum, it will be pulled by a force of 162 downwards in direction perpendicular to the horizon, &c.

Question 266. If a pendulum, or cord 2 feet long = a, with its bob, or ball 3 ounces weight = w, be whirled about in half a ſeconds time = P, ſo as to deſcribe a circle whoſe radius is the ſaid cord, with what weight or force = f, will the cord be ſtretched? See queſt. 262.

1. When gravity is 1, the central force is $= \frac{cca}{16,2PP} = \frac{0,615a}{PP}$ = (becauſe a here = radius) $\frac{1,23\,a}{PP}$, therefore, when gravity = w, the central force $f = \frac{1,23\,aw}{PP} = \frac{1,23 \times 2 \times 3}{0,5 \times 0,5} = 29,52$ ounces anſwer.

Question 267. If a cord 2 feet long (a) with the weight of its bob 3 ounces (w,) be whirled about ſo as to deſcribe a conical ſurface in half a ſecond of time (P), with what force (f) will the cord be ſtretched? See queſtion 263.

1. Let h = the axis of the cone, r = the radius of its baſe, then gravity being as h, and the central force as r, the tenſion muſt be as a, the ſide of the cone, or length of the cord, now $16,2\,PP = 2cch$, we have $1,186 \sqrt{h} = P = \frac{1}{2}$, or $2,376 \sqrt{h} = 1$, ſo $h = \frac{1}{5,665376} = 0,1767$, then as $h : w : : a : \frac{aw}{h} = \frac{6}{0,1767} = 33,89$, the tenſion required, in ounces.

2. $\sqrt{} : aa - hh : = 1,93$, whence in this caſe, the ſtreſs on the pin to which the cord is faſtened in theſe 3 directions, viz. of the cord, perpendicular, and parallel to the horizon, are as the numbers (a) 2 (h) 0,1767 (r) 1,93.

3. If a $=$ 39,12 inches $=$ 3,26 feet, the length of a second's pendulum, and we put gravity $1 = \frac{1,23\ a}{P\ P}$ we'll find $P = \surd : 4,0098 :$ $= 2$ seconds, fere $=$ the time the pendulum makes one revolution, or two swings, then $\frac{1,23a}{P\ P} = f = 1$ (when a $=$ 3,26 and $P = 2$) the force of gravity, and hence, if a pendulum is any how moved by gravity alone, the stress on the pin to which the pendulum is fastened, is only that caused by its own weight the same as if it were at rest.

Question 268. If the diameter of the equator or greatest circle of the earth, be 42000000 feet (a) in what time must the earth make one revolution about its axis, that all bodies there may lose their weights, and be as liable to fall off as to abide thereon?

This must be when the forces with which bodies cleave to, and recede from the periphery of that circle are equal, viz. when gravity is $=$ centrifugal force, so (by quest. 262) $P\ P = 0,615$ a, therefore $P = \sqrt{0.615\ a} = 5083''$ or $= 84'\ 43''$ answer.

Question 269. What is the centrifugal force at the equator to the power of gravity there allowing the diameter of that circle to be 42000000 feet (a) and the earth to turn round its axis in 24 hours or 68400 seconds (P)?

1. (By quest 262) As $P\ P : 0,615\ a :: 289 : 1$ nearly, $::$ gravity : centrifugal force answer. But in other latitudes, see the following questions.

Question 270. If at the equator the gravity be to the centrifugal force as 289 to 1, what will it be in any other latitude suppose in the latitude of 60° (fig. 228)? See question 124.

Since the time of revolution of a body at the equator A Q, and any parallel of latitude b B, is equal, it will be (by quest. 262) as centrifugal force at A Q : that at b B $:: \frac{A\ Q}{P\ P} : \frac{b B}{PP} ::$ A Q : b B, let a $= \frac{1}{2}$ A Q, c $= \frac{1}{2}$ b B, and $f = \frac{1}{289} =$ centrifugal force at A Q, then by this proportion we'll have $\frac{c\ f}{a} =$ centrifugal force at B b, but in any latitude b B, because the direction of gravity is always towards C, the earth's center, therefore the centrifugal force is not (as at the equator) opposite to the whole gravity, for produce a B the direction of the centrifugal force to D, and B C the direction of gra-

vity to F, from D upon C F let fall the ⊥ D F, then if B D be the whole centrifugal (f) force, we have B F for that part of it which acts directly against gravity, and by similar △s, as B F : B D : : A B : B C $=$ E C, therefore as a : c : : $\frac{cf}{a}$: $\frac{ccf}{aa}$ $=$ F the centrifugal force at B b opposite to gravity, whence if a $=$ radius $=$ 1, then c $=$ 0,5 $=$ co-sine latitude 60°, then as a a : c c : : f : F, that is, as 1 : 0,25 : : $\frac{1}{289}$: $\frac{1}{289 \times 4}$: : centrifugal force A Q, to centrifugal force at B b, which opposes gravity there. Hence, as (c c) the square of the co-sine of any latitude is to 289 so is the centrifugal force, to the gravity there.

Question 271. If the force of gravity at the equator be 1, what will it be (G) at the poles, it being proved by observation that a second pendulum at the equator, is 39,1 inches long, and at London, 39,2 inches? Fig. 228.

Produce C F to G, and draw G D perpendicular D a, then if B D be the whole centrifugal force at the equator, and B G the whole decrease of gravity caused there by it, and since (by the last question) in the latitude B, the centrifugal force which acts directly against gravity is F B, the line F B will also express the decrease of gravity in the latitude B, so the difference G F will be the increase of gravity at B, so by similar △s as B G : F G : : □ B G : □ D G : : □ B C : □ a C : : □ radius : □ sine latitude at B, : : decrease at equator : increase at B, now (by question 116,) the gravities at places are as the lengths of pendulums vibrating seconds there, that is, as 392 : 391 : : gravity at London : gravity at the equator, and by the last proportion as 1 (□ radius 1) : ,6131 (□ sine latitude 51° 32′) : : G — 1 (decrease gravity at equator) : ,6131 G — 0,6131 (decrease of gravity at London) to which add the gravity at the equator 1 and we have 0,6131 G + 0,3869 the gravity at London, so as 392 : 391 : : 0,6131 G + 0,3869 : 1, whence G $=$ 1,0043 &c. for the gravity at the poles, N and S, that is, as 229 : 230, or nearer, as 689 : 692 : : gravity at equator : gravity at poles : : least gravity on the earth's surface to the greatest thereon.

Question 272. If any two equal bas'd cones of earth &c. with their vertexes in the earth's center, and bases in its periphery, be in equilib. with each other, what is the equatoral diameter A Q (fig. 228) to the axis N S, allowing the earth to revolve in 24 hours about N S?

The centrifugal force being greatest at A, or Q, and nothing at N, or S, and this force acting at A, directly against gravity whose seat

of action is at B, its plain, that a column of particles C A, must be so much heavier (viz. so much longer if the matter is the same) as to make up that part of gravity lost at A, by the centrifugal force there, which force does not alter the column of particles C N. Whence (and by the last question) as 689 : 692 : : the axis N S : diameter A Q, which proves the earth to be an oblate spheroid flattened at the poles N and S, and rais'd at the equator, or middle A Q, and since A Q is about 8000 miles, it will exceed N S by about 35 miles? See question 274.

Question 273. If the earth were not to revolve about its axis, whether would pendulums gain or lose time, and how much per day in the latitude of 54° 30′, supposing the earth a globe?

If c = the natural co-sine of the latitude 54° 30′, then (by quest. 269) if the gravity at that latitude be 289. the centrifugal force there will be $cc = 0{,}33721249$, so $289 - 0{,}33721249 = 288{,}6627875$, that is, if 289 be the gravity when the earth is at rest, 288,6627 &c. will be the gravity when it revolves in 24 hours in latitude 54° 30′, now (by question 116) the lengths of pendulums being as the gravities and the vibrations as the square roots of these lengths it will be, as $\sqrt{288{,}6627875} : \sqrt{289} : : 86400$ (seconds in 24 hours, earth being in motion) : 86451,83 (seconds when the earth is at rest in 24 hours) so the difference 51,83 seconds is the time gain'd per day.

Question 274. Whether or no is it likely that the earth is of an uniform density throughout?

1. The earth at the creation being in a fluid state, it is reasonable to suppose that the heaviest matter subsided first, according to the laws of gravity, and therefore that the earth is more dense and compact the nearer we go to the center; whence it follows, that the centrifugal force (quest. 269) may be somewhat more than 289 viz. because there are more particles between the equator A, and center C, (fig. 228) than there would be if the earth were of an uniform density as is there supposed, therefore (question 272) 689 : 692, is nearer the ratio of N S to A Q, than that of 229 to 230, and also answers nearer by experiments and this is the form of the earth, as settled by Sir Isaac Newton.

Question 275. Whether or no doth bodies near the earth's surface gravitate in directions, that pass thro' the earth's center?

1. Its evident, that if a heavy body be let fall any where near the earth's surface, it will strike the horizon perpendicularly, now the horizon is a plane or tangent line, touching the surface in the point where the body strikes it, and therefore the earth being a spheroid, this line

or direction of the body being at right angles to the tangent in the point of contact with the curve, would not pass thro' the center, except at the equator and the poles, for at those places the said tangent is at right angles with the diameters of the generating ellipsis. Hence in any other place, since the earth's center is the center of attraction to which all bodies from its surface would fall (if not interrupted,) it follows that the path is not a right line but a curve to which the line of direction of the body is a tangent, to the point where it would first begin its descent.

Question 276. If the direction of a falling body in latitude 45° were continued thro' the earth, how far from the center would it cut the axis, the equatoral diameter being about 8000 miles, and the axis 7965 miles?

1. Let E be the given place (fig. 229) in latitude 45°, b E the direction of the body at right angles to the horizon H O, which b E produced will meet the axis N S in G, now $\angle$ O H S is $=$ 45° the latitude, so $\angle$ E G H will be $=$ co-latitude, whose natural tangent call t, to the radius 1, let $c = \frac{1}{2}$ N S and $a = \frac{1}{2}$ the equatoral diameter Q A, and $e =$ N F, also, put $\frac{aa}{cc} = zz$, then, by the property of the ellipsis $FE = z\sqrt{:2ce - ee:}$ and $FG = zz \times : c - e$, but by trigonometry, as Radius $1 : t \angle EPF :: FG : FE$, so $tzz \times : c - e : = z\sqrt{: 2ce - ee :}$, or $tz \times : c - e : = \sqrt{: 2ce - ee:}$ which squared is $ttzz : \times cc - 2ce + ee : = 2ce - ee$, by transposition $ttzzcc = 2ce + 2ttzzce - ee - ttzzee = 2Sce - bee$, by writing $S = c + ttzz$ and $-b = -1 - ttzz$, which equation by compleating the square &c. gives $e = \frac{Sc}{b} + \sqrt{: \frac{SScc}{bb} - \frac{ttzzcc}{b} :} =$ N F, but the solution will be easier if we put $e =$ F C, and a, c, z, t, as before, for then we'll have $ttzzee = cc - ee$, so $e = \frac{c}{\sqrt{:ttzz + 1:}} = \frac{3982.5}{\sqrt{:1,009 + 1:}} = \frac{3982,5}{1,419} = 2807 =$ F C, now $FG - FC = zzc - e = 2843 - 2807 = 36$ miles $=$ C G ans.

Question 277. In what latitude will a right line drawn from the point of suspension of a plumb line and continued thro' the earth's center, make the greatest angle possible with the said plumb line, supposing the equator diameter to the axis as 230 to 229?

Let the point E (fig. 229) be the required place, so will E G (being $\perp$ horizon H O) represent the plumb line, join C E, so is $\angle$ C E G

that made by E G and a line E C paſſing thro' the earth's center, let $e = CF$, and make it $\square SC : \square CA :: 1 : 1+n$ (viz. :: $\square\, 1 : \square\, \frac{115}{114{,}5}$) then (by the properties of the ellipſis) $EF = \sqrt{:1+n:\times:1-ee:}$ and $FG = e + ne$, therefore $CE = \sqrt{:\square FE + \square CF:} = \sqrt{:1+n-nee:}$ and $GF = \sqrt{:1+n:\times:1+nee:}$ then by ſimilar Δs, as $GE : FE :: CG : Cy$ (Cy being $\perp EG$) $= \frac{ne\sqrt{1-ee}}{\sqrt{:1+nee}}$, and by trigonometry as $CE : Cy ::$ radius 1 : ſine $\angle CEy = \frac{ne\sqrt{:1-ee:}}{\sqrt{1+nee}\times\sqrt{1+n-nee}}$, which by the queſt. muſt be a maximum, and then e will turn out $= \sqrt{\tfrac{1}{2}} = {,}7071067 = CF$, whence $FG =: 1+n:\times e = .713328$, $FE = {,}7101241$, $CE = 1.0021857$, $Cy = {,}0043878$, $\angle$s $FEG\ 45^\circ\ 7'\ 39''$, $EGF = 44^\circ\ 52'\ 21''$ the co-latitude and $CEG = 15'\ 3''$, the angle of the plumb line, in latitude $45^\circ\ 7'\ 39''$.

Queſtion 278. In what time at the equator would a body by the force of gravity, fall freely from the ſurface of the earth to its center?

Becauſe (ſee queſt. 268) when the earth revolves in $84'\ 43''$ time, the power of gravity is thereby deſtroyed, it follows that while the earth would at that rate make $\frac{1}{4}$ of a revolution, that a heavy body would fall from its ſurface to its center, therefore $84'\ 43'' \div 4 = 21'\ 11''$ anſwer.

Queſtion 279. If the axis of the earth be 7940 miles, and the diameter of the equator 7974 miles, in what time (in latitude $45^\circ\ 14'$) would a heavy body by its own gravity freely deſcend in a right line from the earth's ſurface to its center? Fig. 229.

Let $c = CN$, $a = CA$, $CF = e$, $F = 21'\ 11''$, the time of deſcent at the equator and $t =$ time of deſcent from E the given latitude along E C, which muſt alſo be $= \frac{1}{4}$ the time of revolution in the circle whoſe radius is C E, (ſee the laſt queſt.) now the force being inverſely as the diſtance from the center, it will be as $CA : CE ::$ force with which the body deſcends from E, to that with which it deſcends from A, which forces are as the centripetal forces, and becauſe (by queſt. 262) $\frac{a}{pp} = f$, or $P = \sqrt{\frac{a}{f}}$, it will be as $\sqrt{\frac{CA}{CE}} : \sqrt{\frac{CE}{CA}} ::$ T

: t, but by the property of the ellipsis, $\frac{aa}{cc} \times : cc - ee = \square EF$, so

$\square EF + \square CF = \square CE$, that is, $aa - \frac{aaee + ccee}{cc} = \square CE$,

put $d = \frac{cc + aa}{cc}$ then $CE = \surd : aa - dee$: whence the last proportion becomes, as $\frac{\surd a}{\overline{aa - dee}|^{\frac{1}{2}}} : \frac{\overline{aa - dee}|^{\frac{1}{2}}}{\surd a} :: T : \frac{T : \overline{aa - dee}|^{\frac{1}{2}}}{a}$;

$= t = 20' \; 11''$ answer.

Note. I here take the surface of a spheroid, to be the same with that at the surface of a globe, for the difference is very small.

Question 280. It is found by observation that the moon revolves about the earth in nearly 27,3 days, and that her mean distance from the earth's center is to that from the earth's surface as 60 to 59, now if for the revolving body or stone mentioned in quest. 261, we place the moon, for the center we place the earth, for the centripetal force or sling, the earth's power of Attraction, and for the projectile force the Almighty power of God at the creation, what will be the law of this centripetal force.

1. A body (see quest. 268) revolves at the earth's surface in 5083 seconds, let P = 2362000 the seconds in 27,3 days the moon's periodic time, a = 60, half diameters of the earth her distance from the earth's center, the said body at the surface being 1 of these half diameters from the earth's center, and put f the centripetal force = the nth power of the distance a, viz. $f = a^n =$ (see quest. 262) $\frac{a}{PP}$, then if n be taken $= 0, 1, 2, -1, -2$, &c. we'll have $P = \surd a$, $P = 1$, $P = \frac{1}{\surd a}$, $P = a$, $PP = aaa$, &c. respectively, this last equation is the law of the whole planetary system as constant observations have made out, viz. the squares of the periodic times are as the cubes of the mean distances, of any of the planets revolving about the sun, or of any of their moons about them, thus, in this question, as cube of 1 : square of 5083 : : cube of 60 : square of 2362000, = 27,8 days, hence $f = a^n = a^{-2} = \frac{1}{aa}$, that is, the centripetal force is inversely as the $\square$ of the distance from the center.

Question 281. How high above the earths surface must a body 400lb weight, be raised, to weigh only 300 lb, the earth's radius in that place being 3980 miles (d)?

Let e = the required height above the earth's ſurface, then becauſe (by the laſt queſt.) the power of gravity is inverſely as the ſquare of the diſtance, it will be as 300 : 400, or lower as $3 : 4 :: aa : \overline{a+e}|^2$, ſo $a + e = a \times \surd \frac{3}{4} = 3980 \times \surd \frac{1}{4} = 4596 - 3980 = 616$ miles anſwer.

Queſtion 282. If a ball 4 lb weight at the earth's ſurface be carried up in the air ſo high that it weighs but 3 lb, with what velocity would it begin to fall, and what would its velocity be at the earth's ſurface, gravity at that place being $16\frac{1}{12}$ feet in the firſt ſecond.

Let a = the earth's radius (3980 miles) in feet, c = 24266880 feet, in 4596 miles the height of the ball (by the laſt queſtion) above the earth's center, e = any diſtance at firſt deſcended from that height, now the diſtance deſcended in the firſt ſecond of time being as the gravity or weight at that place, it will be as $4 : 16\frac{1}{12} :: 3 : 12\frac{1}{16}$ feet = d, the diſtance fallen in the firſt ſecond, from the height 616 miles above the horizon, therefore 2 d = the uniform velocity at that place, ſo (by queſt. 280) as $\overline{c-e}|^2 : 2d :: cc : 1 = \frac{2dcc}{\overline{c-e}|^2}$, the velocity generated per ſecond by falling thro' e, and therefore, the velocities acquired by falling thro' e, 2 e, 3 e, 4 e, &c. will be 1, 2, 3, 4 &c. to v, ſo (by theo. 86) $\frac{vv}{2} = \frac{2dce}{c-e}$, and $v = \frac{2\surd dce}{\surd : c-e :}$, now if e be taken = 3252480 feet in 616 miles we'll have $v = 2\surd : \frac{dce}{c-e} :$ = 13310 feet per ſecond anſwer.

Queſtion 283. Things being as in the laſt queſtion, in what time would the ball be in falling to the ground, the ſaid ball, here as in the laſt queſtion, being ſuppoſed to meet with no reſiſtance, but to fall freely in vacuo?

Let v, c, d, and e = as in the laſt queſtion, then as there we have $\frac{2dcc}{\overline{c-e}|^2}$, for the expreſſion generating the velocity v, and the velocity being inverſely as the time, therefore from $v = 2\surd : \frac{dce}{c-e} :$ we'll have $1 \div 2\surd : \frac{ace}{c-e} : = 2\surd : \frac{c-e :}{dce} = \frac{\surd\overline{c-e}}{2\surd dc \times \surd e}$ for the expreſſion generating the time t, which may be put into a ſeries &c.

by theo. 86, or multiplying it by $\frac{\sqrt{c-e}}{\sqrt{c-e}}$, it becomes $\frac{1}{2\sqrt{cd}} \times \frac{c-e}{\sqrt{:ce-ee:}}$, whence, if radius $= 1$, S $=$ the fine, and a $=$ the arch whofe verfed fine is $2e \div c$, we'll have $t = \frac{1}{4}\sqrt{\frac{c}{d}} \times : S + a :$ now $\sqrt{\frac{c}{d}} = 1418$, and $\frac{2e}{c} = 0{,}26795$, anfwering to an arch of 42° 56′ whofe length is 0,7494, and that of its fine 0,6811, hence, $\frac{0{,}7494 + 0{,}6811}{4} \times 1418 = 507$ feconds $= 8' \, 27''$ anfwer.

Queftion 284. If a folid foot of fome fort of metal weigh 1000 lb, (b) at the earth's furface, and a cylinder of that metal 3252480 feet high (e) and bafe 5 feet area (A) be fet upright on the ground, what weight (w) will it weigh, the earth's femi-diameter at that place being 21014400 feet (a)?

1. The weights of bodies being inverfed as the fquares of the diftances from the earth's center, (fee queft. 280) and b A being as the weight at the furface we have as $\overline{a+e}|^2 : bA :: aa : \frac{bAaa}{\overline{a+e}|^2}$, the expreffion generating the weight, now this expreffion $bA \times \frac{aa}{\overline{a+e}|^2}$ compared with what is done in prob. 185, part 1ft. will give $bA \times \frac{ae}{a+e} = w = 8444000000$ lb nearly anfwer.

Queftion 285. Required the weight of a cylinder fet perpendicular to the horizon, whofe bafe is 3 feet area (A) and height (e) infinite, and a folid foot of metal of that cylinder 1000 lb (b) at the earth's furface, the earth's femi-diameter at that place being 21014400 feet (a)?

1. From the laft queftion, we have $w = bA \times \frac{ae}{a+e} = bAa \times \frac{e}{a+e}$, now if e be taken infinitely great, then a in the factor $\frac{e}{a+e}$ may fafely be taken $= 0$, becaufe an infinitely great quantity, cannot be made more by addition or lefs by fubtracting a finite quantity from it, and therefore $\frac{e}{a+e} = \frac{e}{0+e} = \frac{e}{e} = 1$, whence $w = bAa =$ 63043200000 lb anfwer.

Question 286. If a heavy body fall from an infinite height (freely as in vacuo) to the earths surface; what velocity will it have there?

Let a = 21014400 feet the earth's radius, d = $16\frac{1}{12}$ feet, the force of gravity there, viz. the distance freely fallen thro' in the first second of time at the earth's surface) e = the infinite distance of the body above the horizon, then (see quest. 282) as $\overline{a+e}|^2 : 2d :: aa : \frac{2daa}{\overline{a+e}|^2}$, which (see the last quest.) gives the velocity $v = 2\sqrt{:da} \times : \sqrt{:\frac{e}{a+e}} : 2\sqrt{da} = 36768$ feet per second answer. Hence the velocity of any body, acquired by falling to the earth's surface can never exceed 36768 feet per second, be the body ever so heavy, fall ever so far and with all the freedom possible.

Question 287. Where must a tub of 20 inches diameter, be placed to hold the most liquor possible, and how many ale gallons will it hold more at that place than at the earth's surface?

1. The figure of the earth, viz. land and water being nearly spherical, and by reason of the earth's largeness, a vessel 20 inches diameter, viz. the area of such a diameter may at the earth's surface, be looked upon as a plane, and so to hold no liquor, but this plane being placed at the earth's center, and liquor poured upon it, there the liquor will rise to the form of a half globe before any of it run off, so (by ex. 294) 928 gallons, such a vessel would hold more at the earth's center than at its surface, answer.

Question 288. Required the ratio of the diameters, and bulks of the sun, earth and moon? Fig. 230.

1. Let E be the earth, m the moon or any other planet, then is the $\angle$ m E d, the moon's apparent semi-diameter, as seen from E, the earth's center, and would be the same if seen from its surface at D, (the moon being in the horizon) because D E || d m, this $\angle$ by observation, at a mean distance of the moon from the earth, is = 1876,5 half seconds, and (by question 280) the distance E m is found = 60, so by plane trigonometry as radius 1 : t $\angle$ d E m 938,25″ :: E m 60 : d m 0,27523, parts of the earths semi-diameter because E m is 60 of such parts, therefore as 1 : 0,27523 :: the earths diameter to the moon's diameter, which is nearly as 109 : 30. Again, if we suppose m to be the sun, the $\angle$ d E m is found by observation, = 16′ 06″ the sun's apparent radius, and (by quest. 280) E m, the distance between the sun and earth will be found ≐ 20000 fere, semi-diameters of the

earth; therefore, as radius 1 : tangent ∠ d E m 16′ 06″ : : E m 20000 : d m 91,47. ſo as 1 : 91,47, or nearly as 109 : 10000 : : the earth's diameter : the ſuns, otherwiſe, let R = 3864, half 11 and r = 3753 half ſeconds the apparent diameters of the ſun and moon, A = 10000 and a = 30, their real diameters D = 20000 and d = 60, their diſtances from the earth, then becauſe any body appears larger the bigger it is, and leſs the further it is diſtant, we'll have as D R : d r : : A : a &c. hence, their diameters are as 10000, 109 and 30, and their bulks as $\overline{10000}|^3$, $\overline{109}|^3$ and $\overline{30}|^3$ anſwer.

Note. ∠ D m E is called the horizontal parallax of the planet, and is the earth's apparent radius as ſeen from the planet at m, this ∠ at the ſun is about 105″ but at the moon at a mean is 60′, and may be thus found, as m E the diſtance between the earth and planet : D E, the earth's radius : : radius 1 : tangent ∠ D m E.

Queſtion 289. Required (Q to q) the quantities of matter in the ſun and moon, their apparent diameters being 32′ 12″ and 31′ 165″ (R and r) and diſtances from the earth D = 20000 and d = 60, and that the heights of the tides on the ocean, at the new and quarter moons are about as 6 to 4? Fig. 230.

1. Any of the planets by their attraction will diſturb the waters or the earth (as alſo on one another if any water &c. be on them) but the moon by its nearneſs and the ſun by its largeneſs affect the waters on the earth the moſt, that by the planets being very ſmall on this earth where the force of attraction is moſt from the moon becauſe of its being ſo near to it, and is always ſeen to anſwer in the tides, in their conſtantly obſerving the moons ſouthing &c. Now at the new moon its evident, the ſun and moon act jointly together, and at 90° diſtant againſt each other, ſo in the former caſe the waters riſe with both their forces (F + f = t,) and in the latter, with the difference of theſe forces (f — F = 4) whence as F : f : : 1 : 5 : : force of ſun : force of moon, now e = D E, t = tangent ∠ d E m = half of 31′ 16½″, and N = 10000 the denſity of the ſun to n, that of the moon, then as t ∠ d E m : radius 1 : : d m : E m, whence d t = d m, ſo $\overline{dm}|^3 \times n = n d d d t t t$ ∝ q, which (ſee queſt. 280) divided by $\overline{d - e}|^2 = \square$ S m, gives

$$\frac{n d d d t t t}{\overline{d - e}|^2} = \frac{n d d d t t t}{dd - 2de + ee} = n t^3 \times : d + 2e + \frac{3ee}{d} + \frac{4eee}{dd}$$

+ &c. for the force with which the moon attracts the earth's ſurface at the point S, from which taking $\frac{n d^3 f^3}{dd} = n d f^3$ the force with

which ſhe affects the center E, there leaves $n\,t\,t\,t \times : 2e + \frac{3ee}{d}$ &c. $\propto f$ which becauſe e is very ſmall in reſpect of d, we may leave $\frac{3ee}{d}$ out and then $e\,n\,t^3 \propto f$, but t is as n, ſo $e\,n\,r\,r\,r \propto f$, and by the ſame way of reaſoning we'll have $e\sqrt{R^3} \propto F$, whence as $e\,N\,R^3 : e\,n\,r^3$, or as $N\,R^3 : n\,r^3 :: F : f$, and therefore, as $N : n :: \frac{F}{R\,R\,R} : \frac{t}{r\,r\,r} :: \frac{1}{\overline{3864}|^3} : \frac{5}{\overline{3753}|^3} :: 10000 : 48911 ::$ denſity ſun : denſity moon, now, by the laſt queſtion, we have as $D\,R : d\,r :: A : a$, whence as $R : r :: \frac{A}{D} : \frac{a}{d}$, therefore as $R^3 : r^3 :: \frac{A^3}{D^3} : \frac{a^3}{d^3}$, ſo as $F : f :: R^3 N : r^3 n : \frac{A^3 N}{D^3} : \frac{a^3 n}{d^3} :: \frac{Q}{D^3} : \frac{q}{d^3}$ (for by art. 338, as $A^3 N : a^3 n :: \overline{10000}|^3 \times 10000 : \overline{30}|^3 \times 48911 :: 10000 : ,0013 ::$ quantity of matter in the ſun : quantity of matter in moon, anſwer.

Note. From hence it appears, the forces with which the planets diſturb the earth are as the cubes of their apparent diameters, multiplied by their denſities, or as the cubes of their true diameters multiplied by their denſities and divided by the cubes of their diſtances, or as $Q \div D\,D\,D$.

Queſtion 290. What are the denſities and quantities of matter in the ſun and earth.

Let $N = 10000$ the denſity of the ſun, to n, that of the earth, $Q = 10000$ the quantity of matter in the ſun to q, that in the earth, $D = 20000$ and $d = 60$, the diſtances of the ſun and moon from the earth, $P = 365,65$ and $p = 27,3$ days their periodic times, if either the diſtances or periodic times be given, (ſee queſt. 280) the other may be found, then becauſe the power of attraction is inverſely as the ſquares of the diſtances, (ſee queſt. 280) from the centers of forces, it will be as $\frac{Q}{D\,D} : \frac{q}{d} : F : f ::$ force of ſun on the earth : force of earth on the moon, the earth being nearly at the center of the moon's orb, and the ſun the center of the earth's (and other planets) motion, but (ſee queſt. 262) as $F : f :: \frac{D}{P\,P} : \frac{d}{P\,P}$, hence, as $F : f :: \frac{Q}{D\,D} : \frac{q}{d\,d} :: \frac{D}{P\,P} : \frac{d}{P\,P}$, therefore as $Q : q :: \frac{D\,D\,D}{P\,P} : \frac{d\,d\,d}{P\,P} :: 10000 : 0,1518$

the anſwer for the quantities of matter, now let A = 10000 and a = 109, the diameters of the ſun and earth, then (by art. 338) as $\frac{Q}{A^3}$: $\frac{q}{a^3}$: : N : n : : 10000 : 39539, the ratio of their denſities; in the ſame manner, the quantities of matter and denſities in the planets Saturn and Jupiter are found, becauſe theſe two planets have moons revolving about them.

Queſtion 291. If a body weigh 10000 lb, at the ſurface of the ſun, what will it weigh at the ſurfaces of the earth and moon?

Becauſe (by queſt. 280) the force of attraction is inverſely as the ſquare of the diſtance, therefore, the quantities of matter in theſe bodies divided by the ſquares of their diameters, the quotients will be as the weights of bodies on their ſurfaces, ſo 10000 ÷ □ 10000, and ,0512 ÷ □ 109 and ,0013 ÷ □ 30 are as 10000 and 431 and 146, that is, the body 10000 lb on the ſun, would weigh 431 lb on the earth, and 146 lb on the moon anſwer. From theſe 4 queſtions it appears, that if the diameter of the ſun be taken = 10000, that of the earth will be 109, and of the moon 30, and their ſolidities are as $\overline{10000}|^3$, $\overline{109}|^3$ and $\overline{30}|^3$, their quantities of matter as 10000 ,0512 and ,0013, their denſities as 10000, 39539, and 48911.

Note. If two meridian altitudes of the ſun, moon, or any ſtar be taken by two perſons at the ſame inſtant of time about $69\frac{1}{2}$ miles aſunder, theſe altitudes will differ about one degree; hence, the earth being nearly ſpherical, and $69\frac{1}{2}$ miles make one degree on its ſurface, it will be as 1° : 69,5 : : 360° : 25020 miles the earth's periphery, and as 22 : 7 : : 25020 : 8000 miles nearly, for the earth's diameter, by which the diameters diſtances &c. of the ſun, earth, and moon may be had in miles from the foregoing ratio's.

Queſtion 292. Required the ratio of the forces (F to f) of the earth and moon to produce tides at each others ſurface, their denſities being as 39539 to 48911 which is nearly as 3 to 4, (N to n) and their diameters ſeen at each others ſurface viz. the moons apparent diameter and horizontal parallax as 16′ to 60′ (r to R)?

1. In queſtion 289, we have e n r r r ∝ f, where e is as the diameter of the body acted on n the denſity, and r the apparent diameter of the acting body, ſo if A = the earth's real diameter to a, that of the moon's, then we'll have A n r r r ∝ f, and a N R R R ∝ f but (queſt. 288) we have R : r : : A : $\frac{rA}{R}$ = a (becauſe D = d) whence

as $F : f :: aNR^3 : Anr^3 :: \frac{rA}{R} NR^3 : Anr^5 :: NRR : nrr ::$ 11 : 1 nearly, i.e. the earth will raiſe the waters 11 times higher at the moon, than the moon can raiſe them at the earth, but its likely there are no waters in the moon becauſe no atmoſphere is ſeen about her, for if any bright ſtar is ſeen near her it's light is not darkened which could not be if ſhe had an atmoſphere like our earth.

Queſtion 293. How many miles from the earth's ſurface is the center of gravity of the earth and moon, (ſee prob. 197 and queſt. 291.

1. Their diſtance being 60 of the earth's ſemi-diameter, and quantities of matter as ,0512 to ,0013, or nearly as 40 to 1, (1) we'll have as $43 + 1 : 1 :: 60 : \frac{60}{44}$ which multiply'd by 4000 the miles in one ſemidiameter gives 5854 miles, and $5854 - 4000 = 1854 =$ L S (fig. 230) E, being the earth and m the moon.

2. Now ſince the earth and moon act on each other by attraction they muſt be always in equilib. upon this point L, which point muſt be the center of the moon's orb. (and not E the earth's center) and ſo is that point about which the earth and moon reſolves to maintain their equilib. therefore L is that point which deſcribes the earth's orb about the ſun, whence the earth is ſometimes in her orb a a, and ſometimes out of it, but ſtill near it becauſe L S is but ſmall in reſpect of E m, but the center of the moon muſt deſcribe a much more irregular curve than that of the earth, becauſe it is at greater unequal diſtances from the ſun, for the ſame reaſons (not the ſun) but a point about 0,8 of the ſun's ſemi-diameter diſtant from its ſurface is the center of the ſolar ſyſtem about which the ſun and all the planets revolve for if the ſun and planets in their orbs were all in one right line, this point would be their center of gravity, now the ſun being always very near this point, and its diſtance from it continually varying by the different poſitions of the planets, its motion about it muſt be very irregular whilſt that of the planets being far diſtant from it, will be nearly uniform and circular.

F I N I S.

DEFINITIONS AND EXPLANATIONS,

of the Terms used in this WORK.

NOTE. P. signifies problem, to be found in part 1st and 2d. A. article in part 2d. Ex example in the first 8 sections of part 3d. And Qu. question, in section 9th of the 3d part.

ABSCISSA, A. 183.
Action, and *Reaction*, A. 234.
Air, and its properties, A. 347 Qu 178.
Amplitude, A 364.
Angle, A. 120. of incidence and reflection, A. 260.
—— *of Traction*, the angle made by the direction of a power with an inclined plane.
—— *of inclination*, the angle an inclined plane makes with the horizon.
—— *Acute*, *obtuse*, *oblique*, P. 120.
Arches, P. 37, gothic, &c.
Axis, A. 240. In geometry the same as perpendicular.

B.

Balance, Qu. 56.
Barometer, Qu. 178.
Bellows, Qu. 174.
Base of a figure, denotes the line or side on which the perpend. falls.
Body, A. 235.
Bridge, and *butments*, P. 37.
Bodies immersed in fluids, P. 199.

C.

Catenaria, P. 35.
Celerity, A. 230.
Center, the point in the midst of a circle, sphere, &c.
—— *of motion*, magnitude, gravity, percussion, oscillation, A. 240, and 241.
Central &c. forces, Qu. 261.
Chain, P. 229.
Circle, and its parts P. 53, and 54, as also P. 179, the periphery of every circle is supposed to be divided into 360 equal parts called degrees, every degree into 60 equal parts called minutes, each minute into 60 other equal parts called seconds, and so on for thirds fourths, &c.

Chord and *Cord*, is any right line O T O, Fig. 43, drawn within a circle leſs than its diameter C A.

Cone, a ſolid formed by the revolution of the baſe and hypothenuſe, of a right angled triangle about its perpendicular, ſuch as a round ſpire ſteeple, P. 155.

Conic ſections, P. 188.

D.

Denſity, A. 236.

Diameter is any right line C A drawn thro' the center of a circle, dividing it into two equal parts. Fig. 43.

Diagonal is a right line drawn in any fig. connecting its oppoſite angles, dividing regular figures equally and irregular figures unequally, ſo A D and A C are diagonals in Fig. 15 &c.

E.

Equations, ſimple, quadratic, adfected, their ſolutions, P. 171.

Equilibrium, A. 247,

F.

Force, any thing that acts on a body to put it in motion.

Friction, A. 242.

Fruſtum, (A B E F, fig. 127) is the lower part of a cone or pyramid, when the top part is cut off, or the middle part of a globe, &c. when two equal and oppoſite ſegments are cut off, this fruſtum is called a middle zone.

G.

Gravity, the weight of bodies relative, ſpecific, abſolute, A. 238.

H.

Hoofs, of all kinds, A. 208.

Hydroſtatics, A. 312.

Hydraulics, A. 313.

I.

Irregular figures, or *polygons* are ſuch as conſiſt of unequal ſides and angles.

Irregular ſolids, ſuch as uneven timber, craggy ſtone, &c. whoſe dimenſions cannot be taken.

Impetus, is the force wherewith one body ſtrikes another.

L.

Leaver, or *lever*, a bar or pole to raiſe weights.

Level, ſemicircle, theodolite, protractor, plane table circumferenter, &c. The inſtruments uſed in ſurveying meaſuring heights, levelling, &c. P. 129. alſo compt. ſupplement Ar. Co. uſe of Gunter's ſcale logarithms &c. definitions and inſtruments uſed in trigon. P. 120.

Logarithms, conſtructed, P. 174.

M.

Mechanics, A 224.

Mechanic powers are 6 *viz.* lever, ballance, wheel, ſcrew, pulley, wedge.

Motion, *momentum*, &c. P. 192.

Moving force, any active force or power that moves a body.

O.

Ordinate, A. 183.

Oscillation, swinging of a pendulum or other pendulous body.

Oval, or *ellipsis*, an imperfect circle, &c. or the plane of the section arising by cutting the cone oblique to its axis for it and its diameters, as also the hyperbola, &c. see P. 30, 32 and 180.

P.

Parallelogram, or long square, is a figure with two equal sides and two equal ends, when the sides and ends are at right angles the parallelogram is right angled, as doors, tables, &c. when the ends and sides bevel, then the parallelogram is oblique.

Perpendicular, is the shortest right line that can be drawn from any point or angle to the opposite side, and cuts it square-wise or at right angles,

Parallel lines are such as are equi-distant in all their parts and if infinitely extended would never either meet or converge.

Point, is that which hath no parts, or the beginning of all magnitude, by the motion of which a line is generated or form'd.

—— *a line* is understood to have length, but neither breadth nor thickness, the motion of a point in a constant direction, traces out a streight line,

—— in a variable direction a curve, crooked, or mixt line.

Periphery, or *circumference*, implies the circuit or compass of any figure or thing.

Perimeter, the sum of the sides or outmost line of any body.

Parameter, or *latus rectum* P. 181.

Parabola, A. 185, P. 31.

Property, equation or nature of any figure or curve, A. 186.

If a semi-parabola be turned about its axis the inscribed space is a parabolic conoid, and if the whole parabolic curve be turned about its greatest ordinate a parabolic spindle is generated.

—— The same reasoning holds in respect to hyperbolic curves. Also if a semi-ellipsis be turned about the transverse axis an egg-like solid is formed, called a right or prolate spheroid, but if half the elliptic curve be turned about its conjugate axis a turnip-like solid is generated called an oblate spheroid.

Pyramids, are regular tapering solids, whose bases may be triangular square or polygonial, ending in a point perpendicular to the center of the base, as a spire, steeple, &c.

Prisms, are regular solids, clothed with equal parallelograms, (and may be triangular, square or multangular,) the ends being equal polygons. A round prism is called a cylinder, such is a rolling stone, a square one is a cube comprehending, six equal square plains as a die.

—— One bounded by 4 equal parallelograms, and two equal ends is called a parallelopipedon.

Prismoid, the frustum of a pyramid whose bases are parallel but disproportional, if one base be an ellipsis and the other a circle its called a cylinderoid.
Pressure, P. 199.
Pendulums, &c. P. 195.
Percussion, the striking of one body against another.
Piston, pump weather glasses &c P. 199.
Prop, any article that bears up or supports a heavy body, that under a lever is called a fulcrum.

Q.

Quantities, proportional progressive analogies &c. P. 173.
Quadrant, a quarter or fourth part of a circle as A E D. Fig. 43.

R.

Radius, that extent of the compasses with which any circles periphery is described or half the circles diameter.
Rhombus, or oblique angled square, a figure of 4 equal sides, making oblique angles with one another. Fig. 23.
Resistance, of fluids. P. 199.
Rhomboides, an oblique angled parallelogram. Fig. 21.
Regular polygons, in general are such figures as consist of an equal number of sides regularly and equally disposed about a center, resolveable into so many equal triangles as the polygon hath sides.
—— As any number of equal sides and equal angles make a regular polygon, so some number of regular polygons of the same kind constitute a regular solid body, of which there can be only 5, for 3 plane angles (at best) must be taken (see P. 154) to make a solid angle, the sum of which must be less than 4 right angles, otherwise their angular points being joined (P. 156.) together will all lye in one plain restitution or substitution, A 43.
Random, range, P. 200.

S.

Sector, a part cut from the center of a circle, oval, &c, and may be greater or less than a semi-circle as S A Q B (fig. 153) is a sector but the part B G A Q is a segment,
Slice, second segment, &c. P. 186.
Solidity, or solid content, shews how many equal cubes of a known dimension any solid is composed of, as for example, if a solid body be 15 feet in solidity, whatever its form be such body may be cut into 15 pieces, each 1 foot solid, or pieces that will make 15 cubes a side of each 1 foot.
Segment, a part cut off &c.
Series, in order or succession, &c.
—— infinite, sums &c. P. 110, 172.
Similar, alike or homogenial, are all such figures or bodies whose angles are respectively equal and sides proportional.
Sine, tangent, secant, &c, P. 54.
Sphere, a hollow globe, formed by turning a semicircle about its diameter.

Springy, or elastic and non-elastic bodies, A 257, 258.
Superficial content, area, flat content or surface, shews how many squares of a given dimension, or pieces that will make up so many squares a plane or curve surface may be cut into, or is composed of; convex surface shews how many such squares will clothe or cover the outside of any solid.
Stress and strength, P. 198.

T.

Triangles of all kinds P. 120, 184.
Trapezia, or quadrangle, any figure of 4 unequal sides and angles.
Trapezoid, is when two opposite sides of a trapezia are parallel.

V.

Velocity, laws of motion, uniform accelerative &c. P. 192.
Valve, see pump, P. 199.
Vibration, the swinging of a pendulum back and forward.

Several more terms you'll find explained in the parts of *Algebra*, *Trigonometry*, *Geometry*, *Mensuration*, *Mechanics*, &c.

4. The Elements of Trigonometry; containing the Properties, Relations, and Calculations of Sines, Tangents, Secants, *&c.* The Doctrine of the Sphere, and the Principles of plain and spherical Trigonometry, all plainly and clearly demonstated.

5. The Projection of the Sphere, Orthographic, Stereographic, and Gnomonical. Both demonstrating the Principles, and explaining the Practice of these three several Sorts of Projection, 8vo.

The above Five by *W. Emerson.*

7. Philosophiæ Naturalis Principia Mathematica. 4to.

8. Lectiones Opticæ annis, 1669, 1670 & 1671, in Scholis publicis habitæ; & nunc primum ex MSS. in lucem editæ. 4to.

9. The same in *English.* 8vo.

10. Optics; or, A Treatise of the Reflection, Refraction, Inflection, and Colours of Light. The Fourth Edition, 8vo.

11. The same in *Latin,* by *S. Clarke,* 8vo.

12. Universal Arithmetic; or, A Treatise of Arithmetical Composition and Resolution. To which is added, Dr. *Halley*'s Method of finding the Roots of Equations arithmetically. The Second Edition, 8vo.

The above Six by Sir *Isaac Newton.*

13. An abstract of Sir *Isaac Newton*'s Chronology of Antient Kingdoms. By Mr. *Reid,* 8vo.

14. Geometria Organica; sive Descriptio Linearum Curvarum universalis: Auctore *Colino Mac-Laurin,* R. S. S. 4to.

15. Methodus Incrementorum directa & universa, per *Brook Taylor,* 4to.

16. Epistola ad Amicum de Cotesii inventis Curvarum ratione, &c. 4to.

17. Dr *Halley*'s Astronomical Tables with the Precepts, *Latin* and in *English.* 4to.

18. Dr *Desagulier*'s Course of Experimental Philosophy, 2 Vol. 4to.

☞ A few Copies left of the large Paper.

19. Professor *Gravesande*'s Mathematical Elements of Natural Philosophy, confirmed by Experiments. Translated by Dr *Desaguliers,* 2 Vol. 4to. with 127 Copper Plates.

20. —— Explanation of the *Newtonian* Philosophy, 8vo.

21. —— Essay on Perspective, 8vo. with 32 Copper Plates

22. A new Mathematical Dictionary, 8vo. The last Three by *E. Stone.*

23. Dr *Boerhaave*'s Academical Lectures on the Theory of Physic. Being a genuine Translation of his Institutes and Explanatory Comment, collated and adjusted to each other, as they were dictated to his Students at the University of *Leyden,* containing Signs of Health, Constitutions, and Diseases; with the Methods of preserving Health, preventing Distempers, procuring Longevity, and of removing present Diseases. In six Volumes 8vo.

24. —— Aphorifms concerning the Knowledge and Cure of Difeafes, Tranflated from the laft Edition printed in Latin at *Leyden*. With ufeful Obfervations and Explanations. 8vo.

25. —— Materia Medica; or, a Series of Prefcriptions adapted to the Sections of his Aphorifms, concerning the Knowledge and Cure of Difeafes, Tranflated from the Latin Original of the laft genuine Edition of the Author. 8vo or 12mo.

26 —— Experiments concerning Mercury. 8vo.

27 Pharmacopœia Extemporanea; or, The Family Difpenfatory. With Remarks on the Compofitions, and an Explanation of their Virtues. By *Thomas Fuller*, M. D. 8vo.

28. Pharmacopœia Bateana; or, *Bates*'s Difpenfatory. Tranflated from the laft Edition of the Latin Copy. Publifh'd by Mr. *James Shipton*. Containing his choice and felect Recipe's, their Names, Compofitions, Preparations, Virtues, Ufes, and Dofes, as they are applicable to the whole Practice of Phyfic and Surgery. The *Arcana Goddardiana*, and their Recipe's interpofed in their proper Places, compleated with above fix hundred chemical Proceffes, and their Explications at large, various Obfervations thereon, and a Rationale upon each Procefs. To which are added, the fam'd Dr. *Goddard*'s Drops, *Ruffel*'s Powder, *Rabell*'s Stiptick Powder, *Tinctura de Sulphure Metallorum*, and the *Emplaftrum Febrifugum*. By *Will. Salmon*, M. D. The fifth Edition, 8vo.

29. The fame in *Latin* by *Thomas Fuller*, M. D. 12mo.

30. *Samuelis Dalei*, M. L. Pharmacologia, feu Manuductio ad Materiam Medicam: In qua Medicamenta Officinalia fimplicia, hoc eft Mineralia, Vegetabilia, Animalia eorumque partes in Medicinæ Officinis ufitata, in Methodum naturalem digefta fuccincte & accurate defcribuntur. Cum Notis generum Characterifticis, Specierum Synonymis, differentiis & vitibus. Opus Medicis, Philofophis, Pharmacopœis, Chirurgis, &c. utiliffimum. Ad calcem adjicitur Index duplex: Generalis alter, Nominum, &c. alter Anglo-Latinus; in gratiam Tyronum. Tertia Editio, multis emendata & aucta, 4to.

31. Synopfis Univerfæ Medicinæ Practicæ: Sive Doctiffimorum Virorum de Morbis eorumque Caufis ac Remediis Judicia. Praxi & Obfervationibus confirmata & nonnihil aucta. Authore *J. Allen* M.D. In 2 Vols. 8vo.

32. The fame in *Englifh*. In 2 Vols. 8vo.

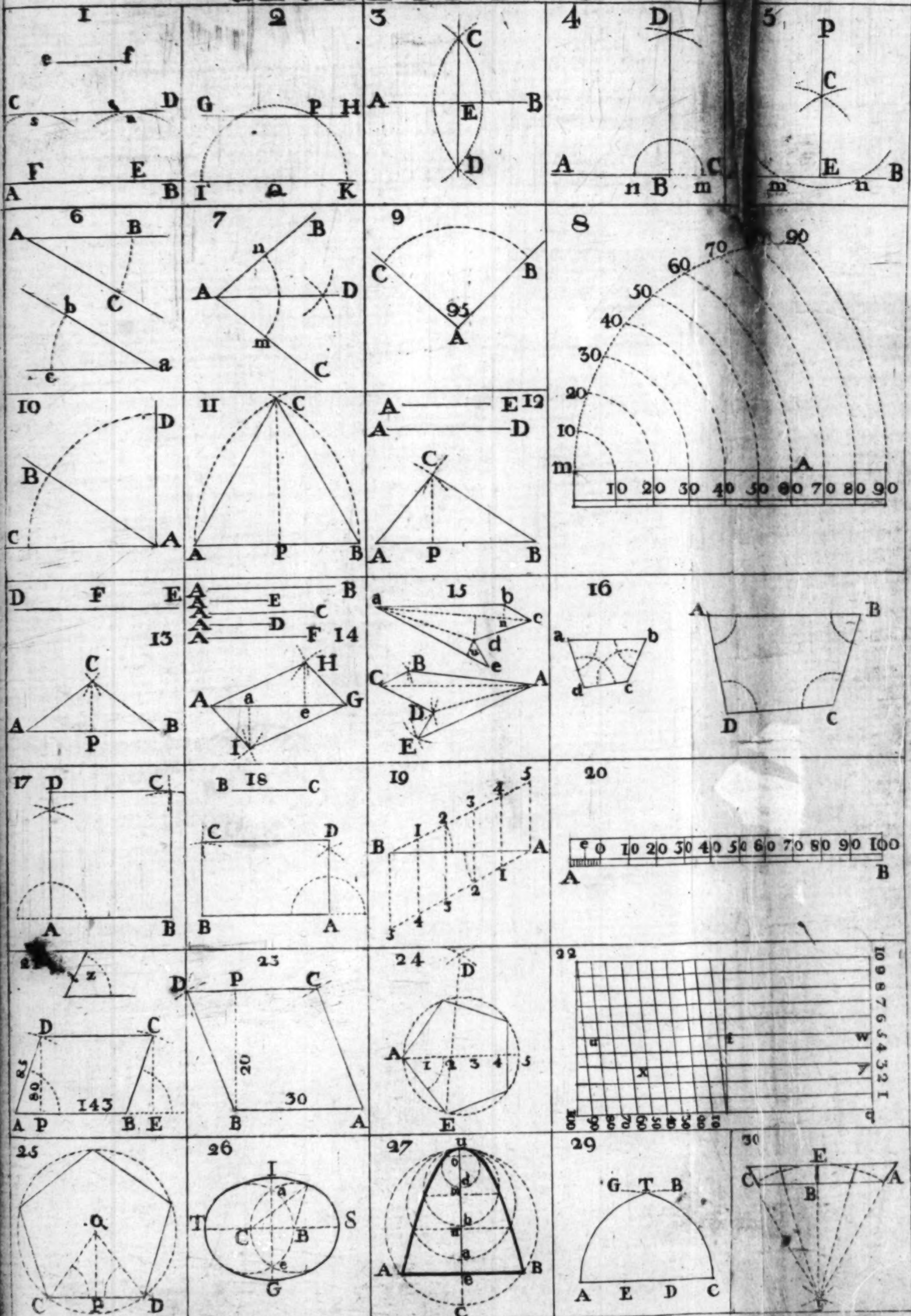
33. Tractatus de Fœtu Nutrito: Or a Difcourfe concerning the Nutrition of the Fœtus in the Womb, demonftrated to be by ways hitherto unknown. In which is likewife difcovered the Ufe of the Gland Thymus, with an Appendix; being fome practical Obfervations on the Food of Children newly born, and the Management of the Milk of Women. By *F. Bellinger*, of the College of Phyficians. 8vo.

34. *Daventer*'s Midwifery. 8vo.

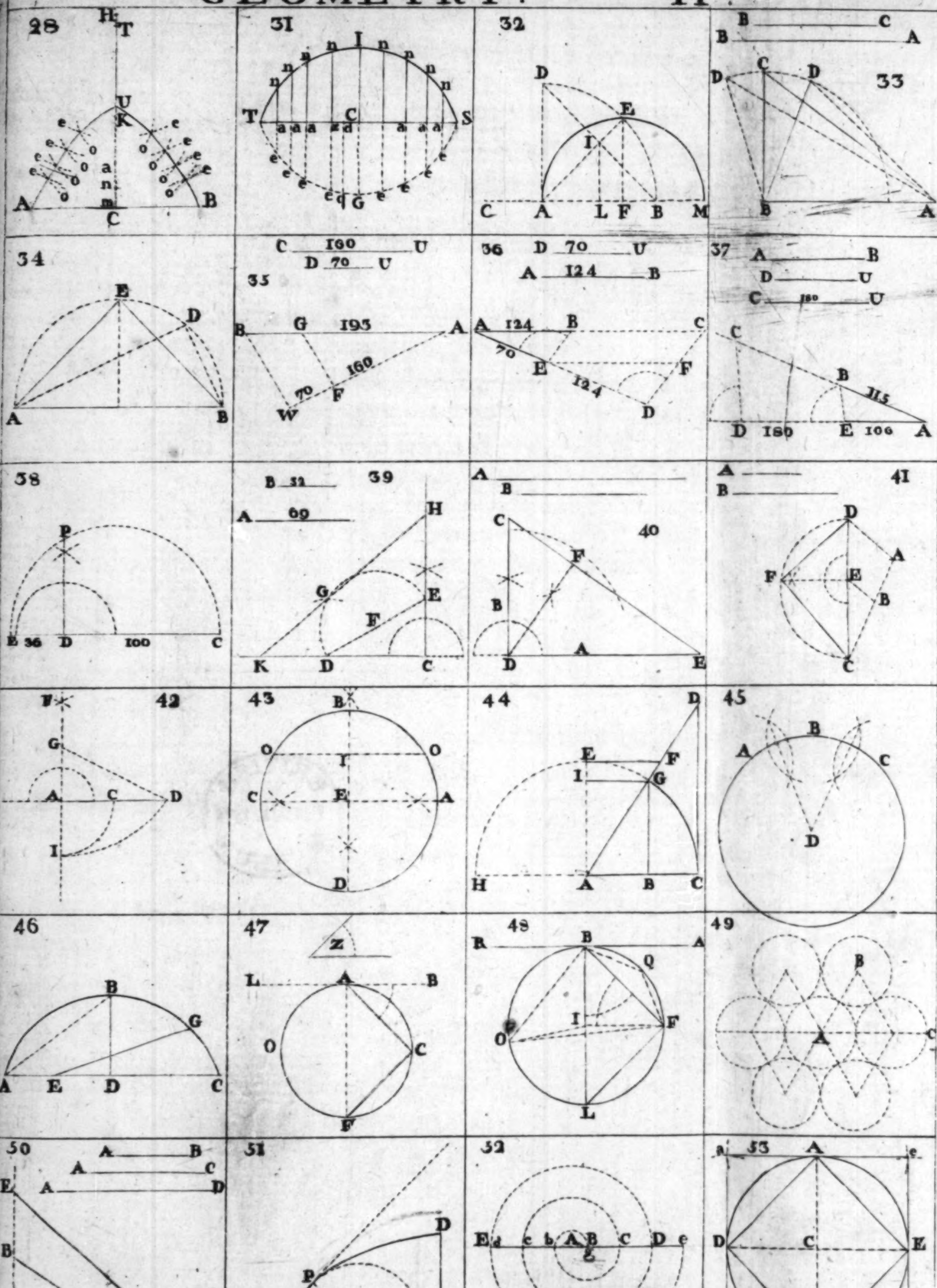
35. *Dionis*'s Surgery. 8vo.

36. *Deider* de Morbis Venereis. 8vo.

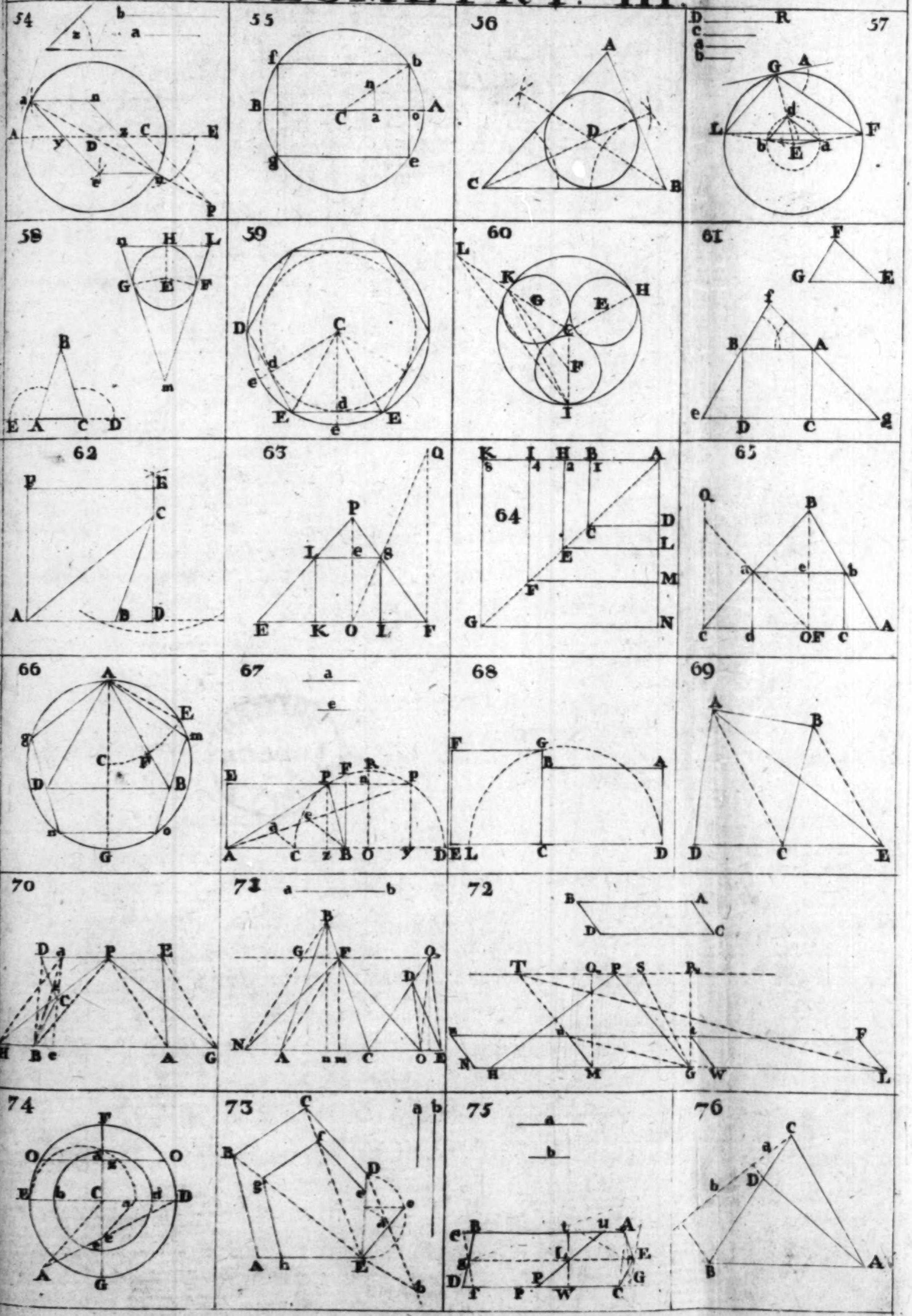
GEOMETRY. I.



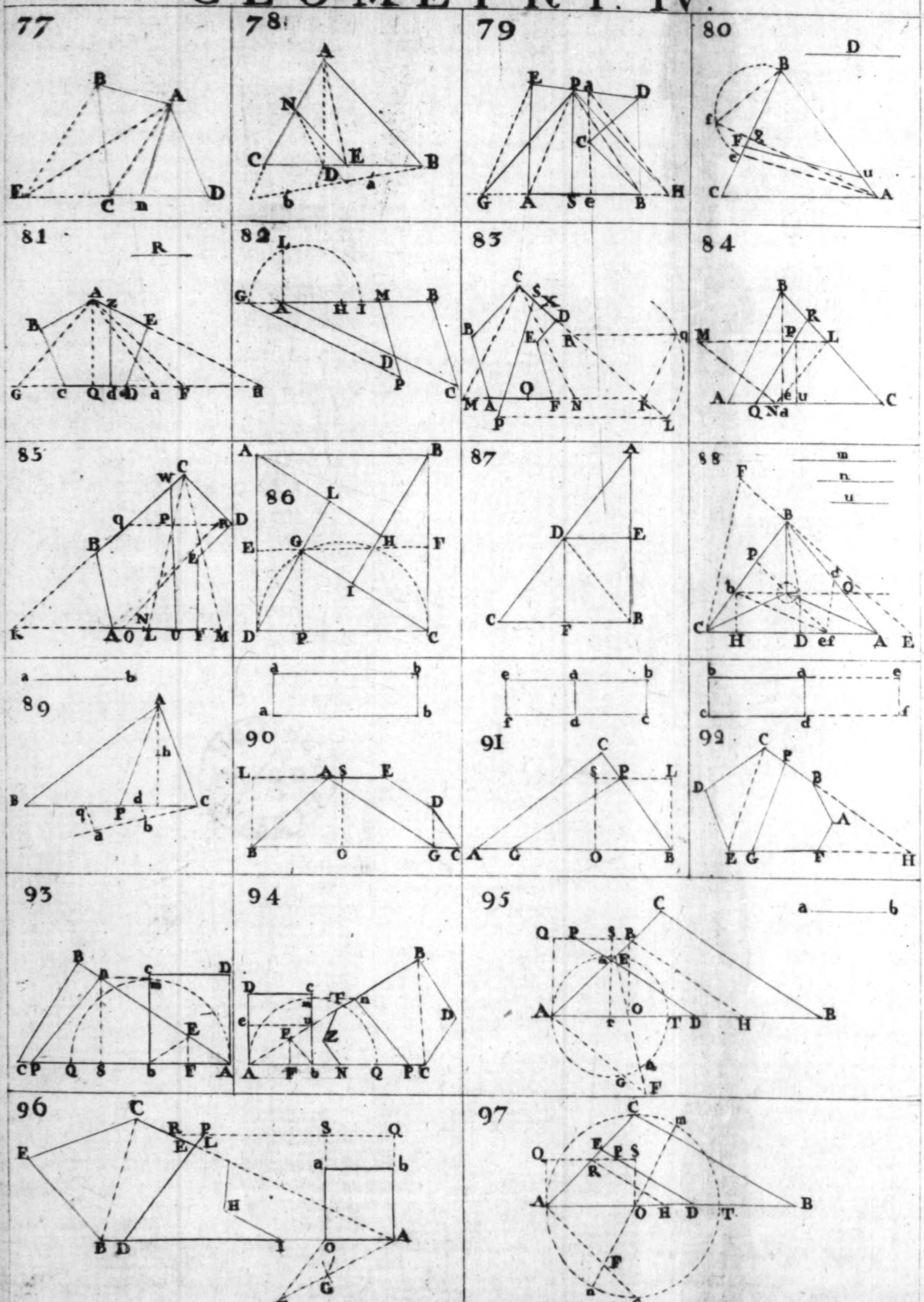
GEOMETRY. II.


28
31
32
33
34
35
C 160 U
D 70 U
B G 193 A
70 F 160
36
D 70 U
A 124 B
124
70
124
37
C 180 U
115
D 180 E 100 A
38
E 36 D 100 C
39
B 32
A 60
40
41
42
43
44
45
46
47
48
49
50
51
52
53

GEOMETRY. III.

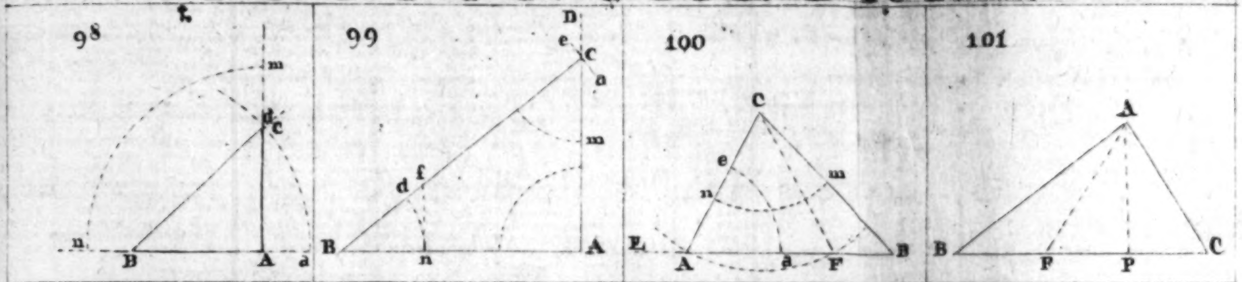


GEOMETRY. IV.



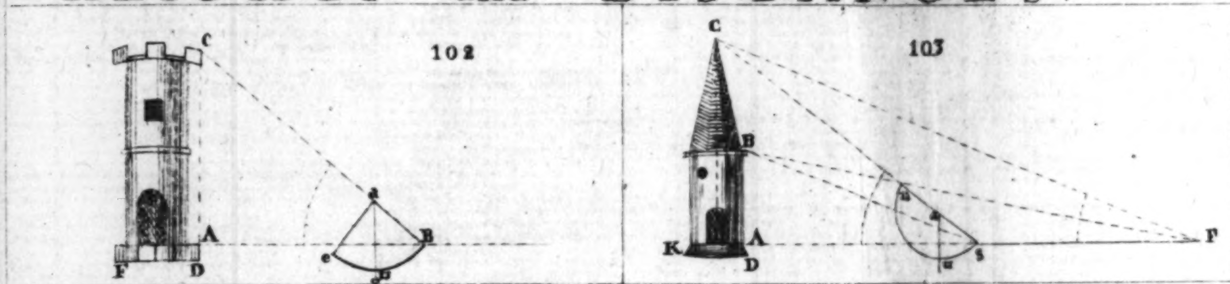
TRIGONOMETRY. V.

98 99 100 101

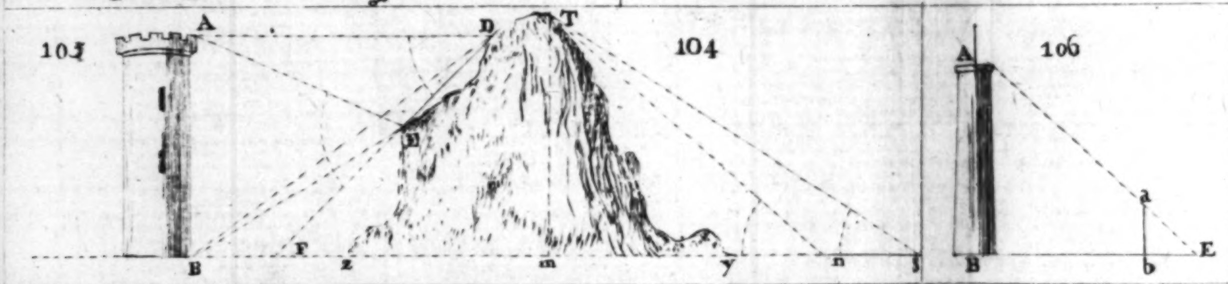


HEIGHTS and DISTANCES.

102 103

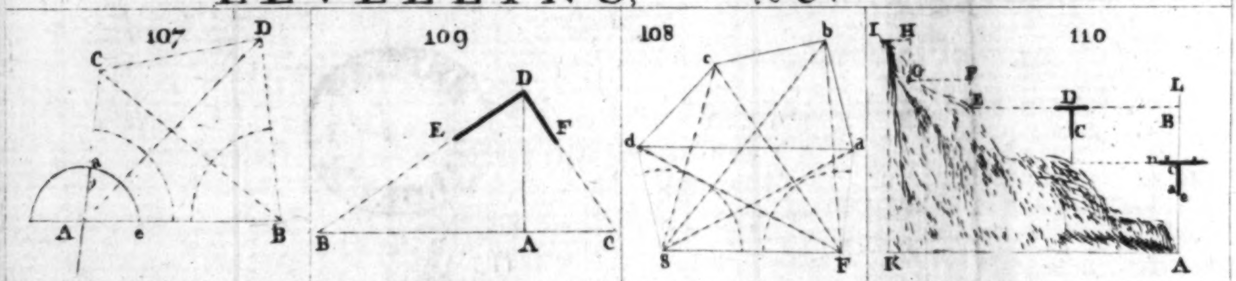


105 104 106

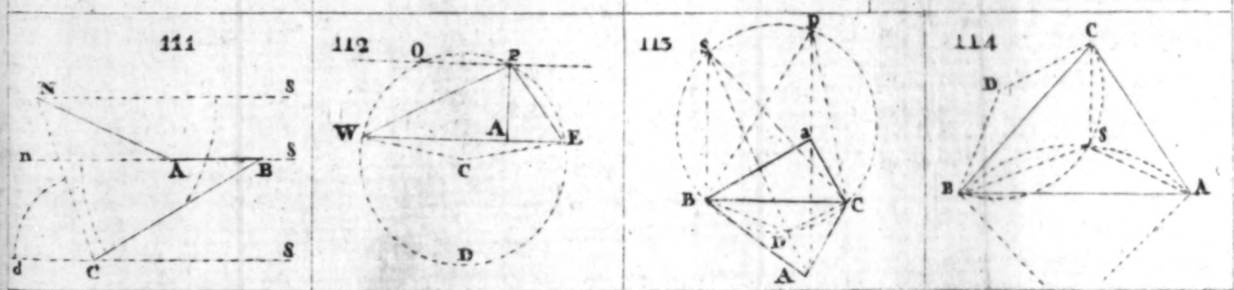


LEVELLING, &c.

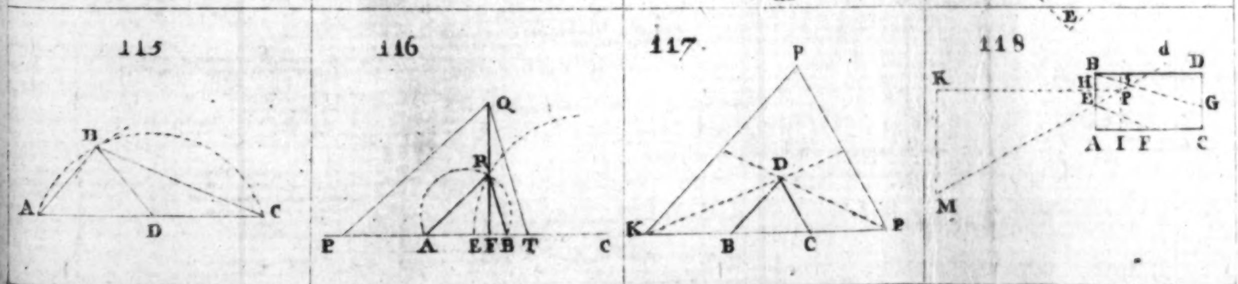
107 109 108 110



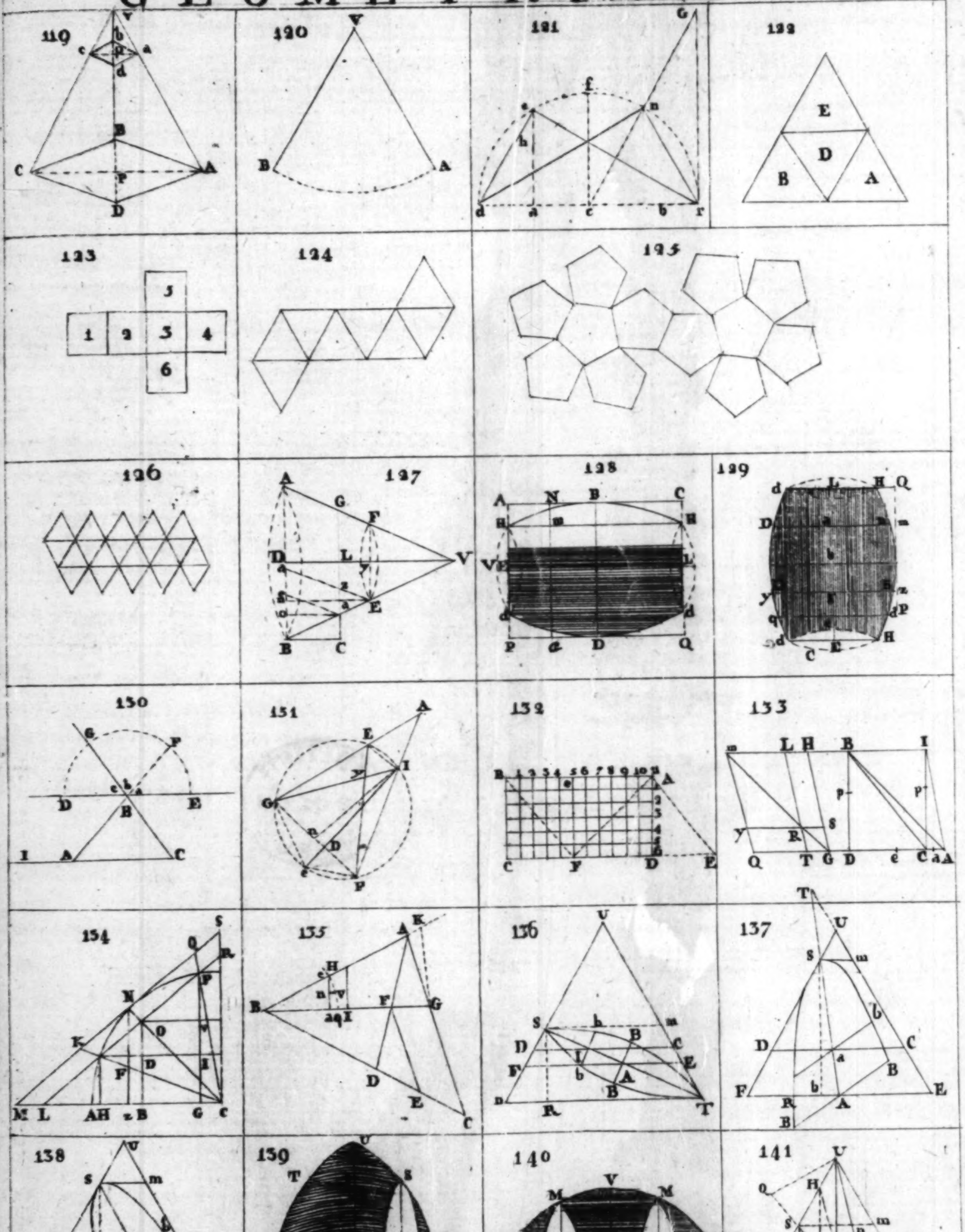
111 112 113 114

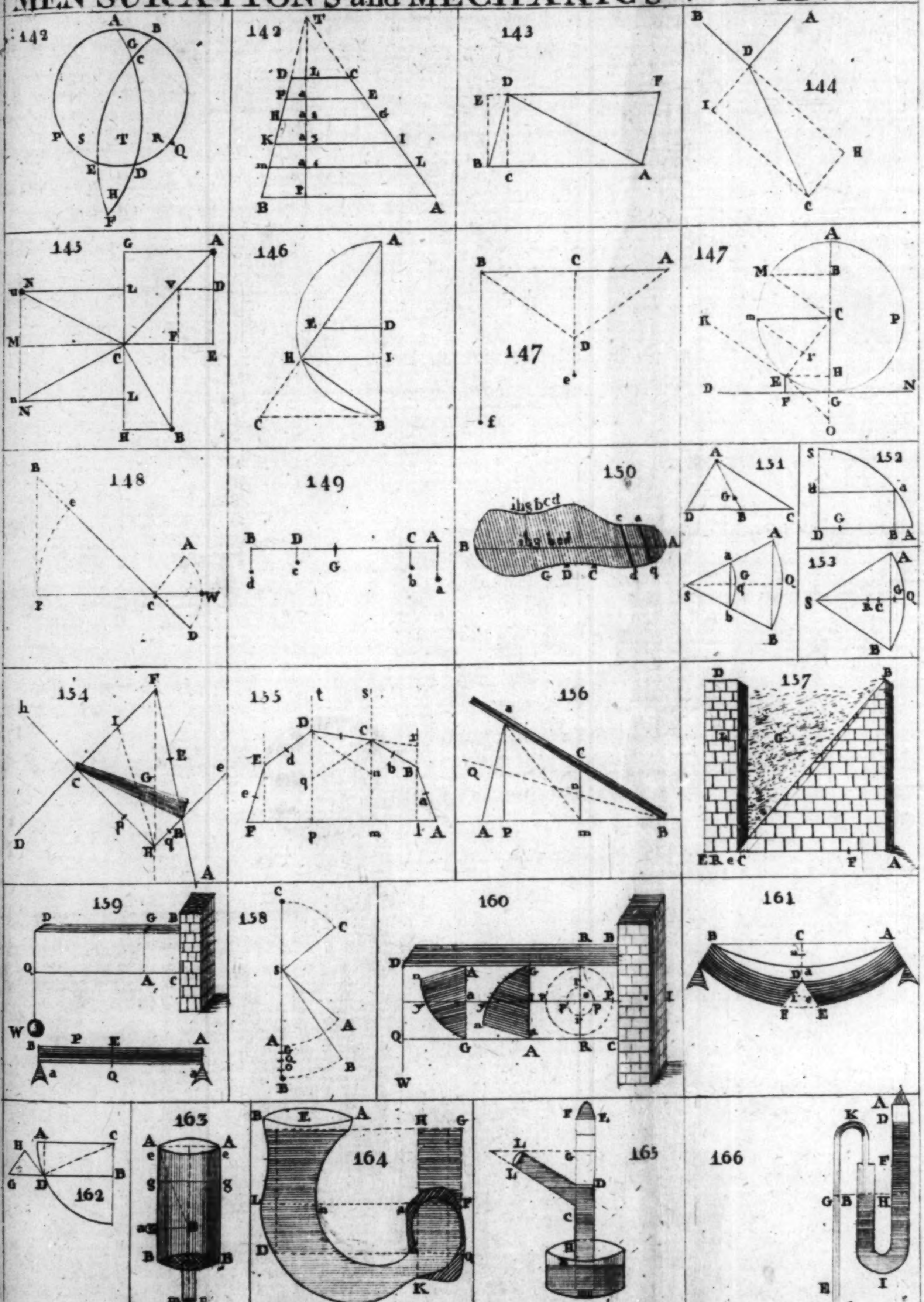


115 116 117 118

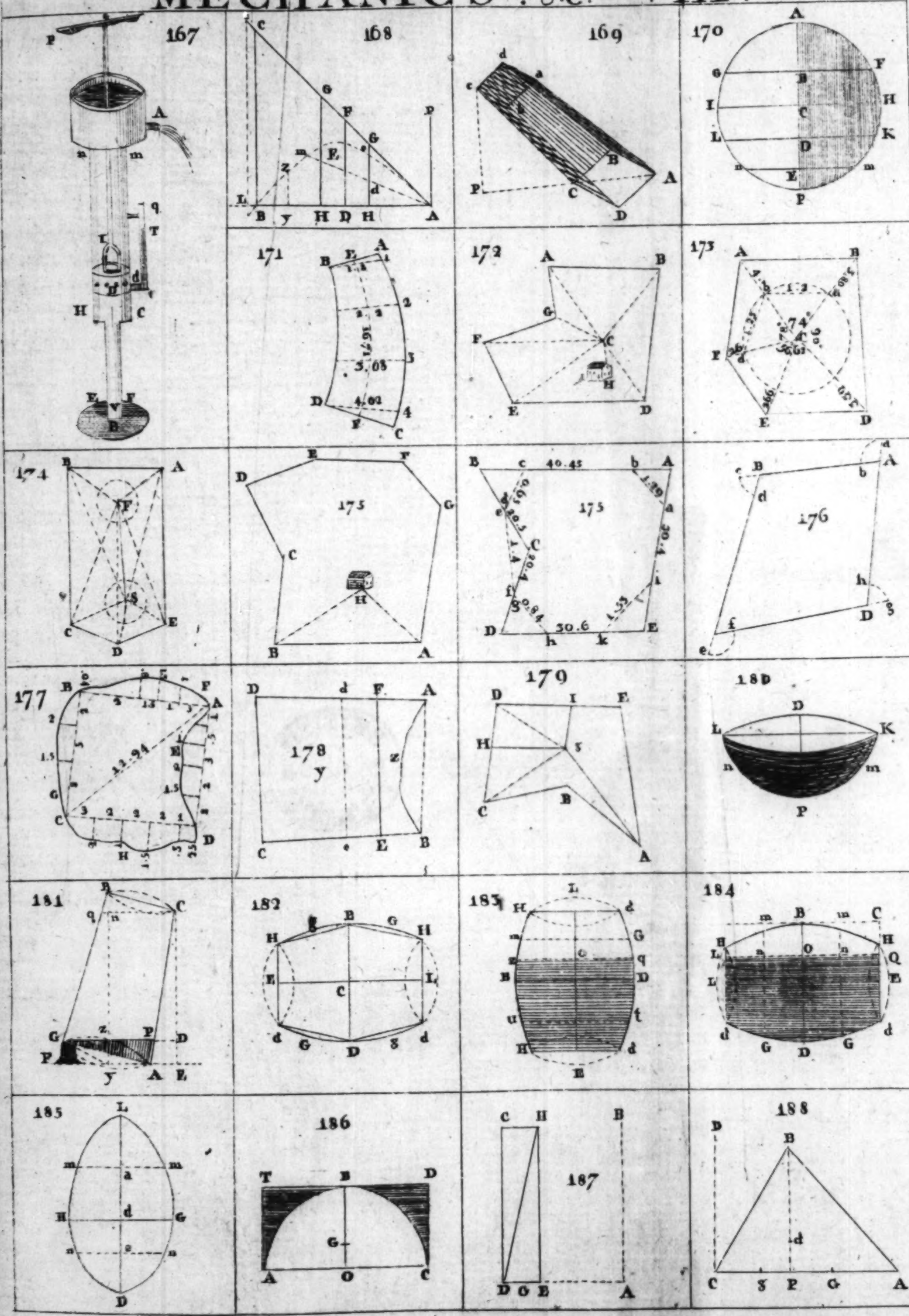


GEOMETRY &c. VI.

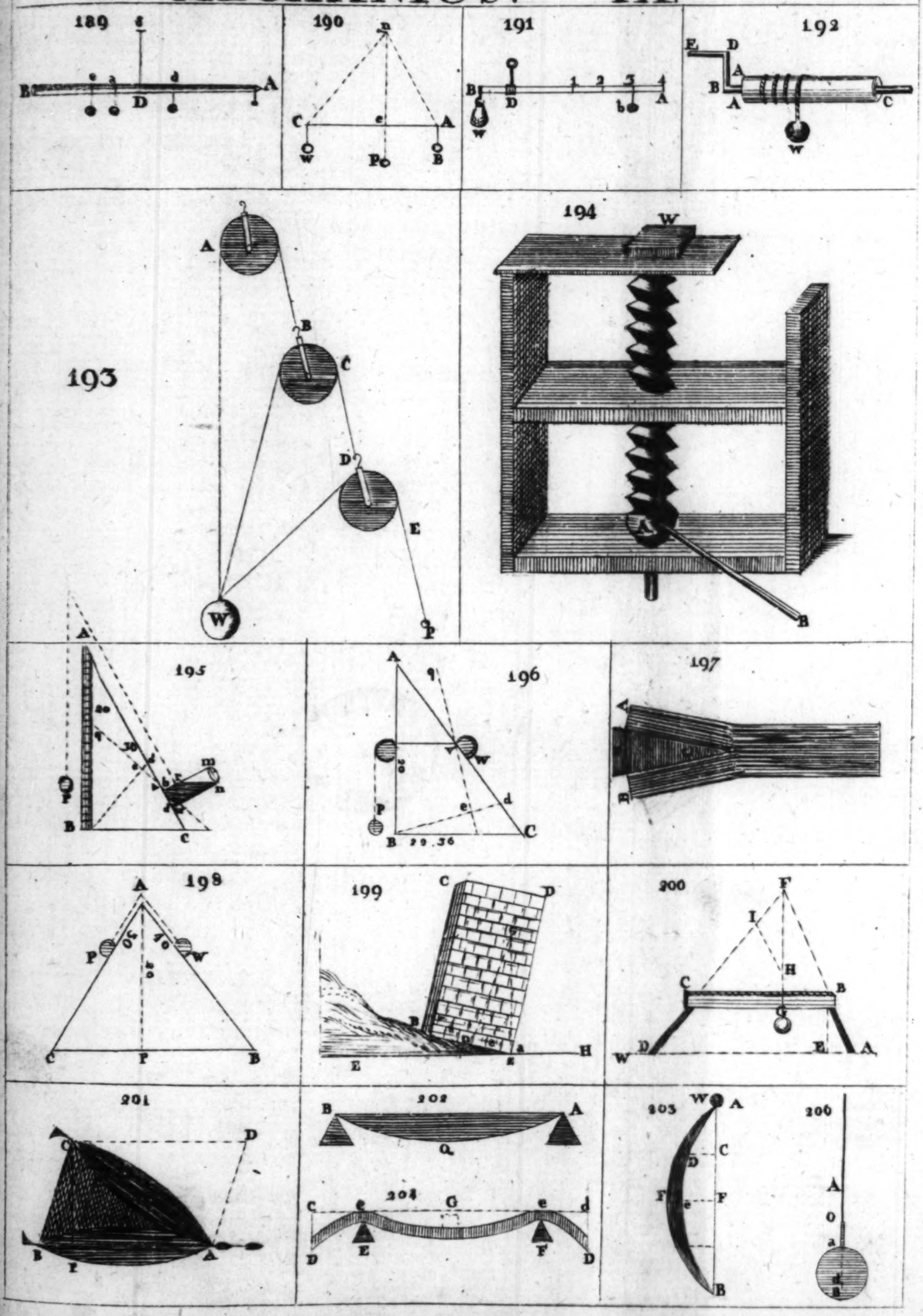



142
142
143
144
145
146
147
147
148
149
150
the bed
the bed
151
152
153
154
155
156
157
159
158
160
161
162
163
164
165
166

MECHANICS. &c. VIII.

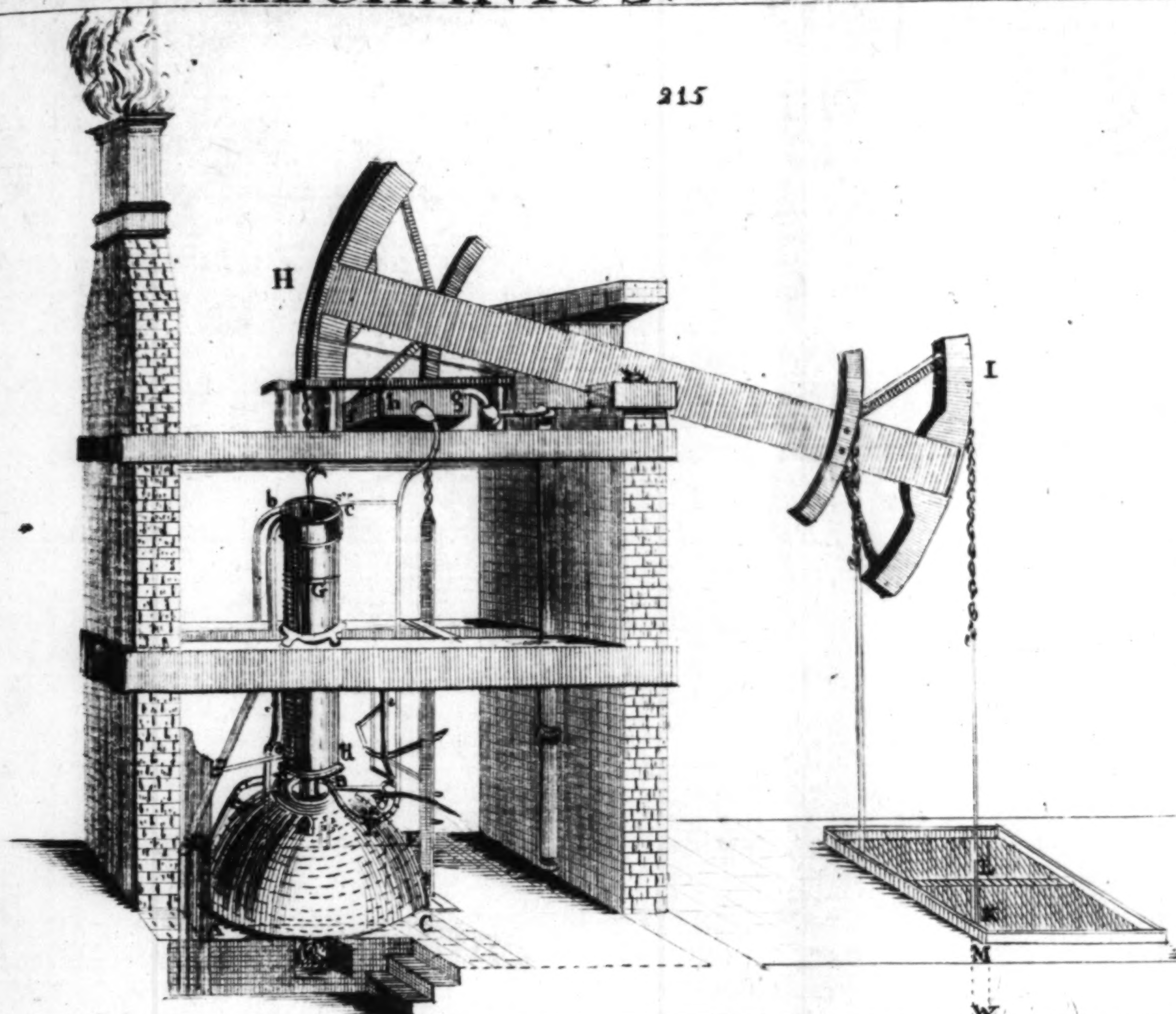


MECHANICS. IX.

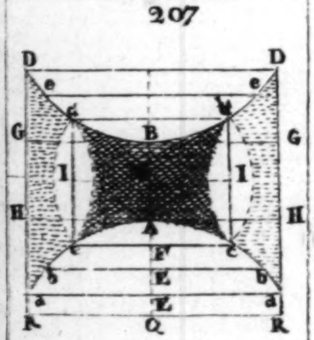


MECHANICS. X.

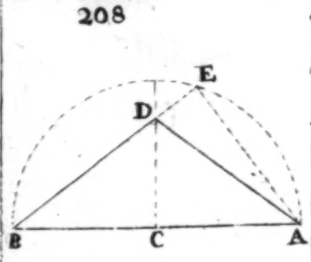
215



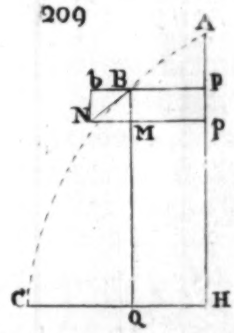
207



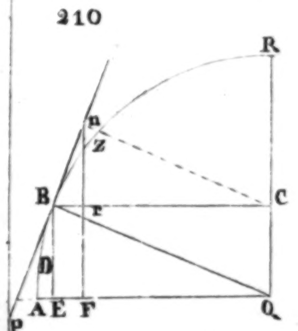
208



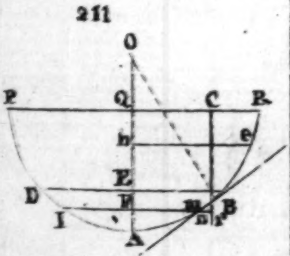
209



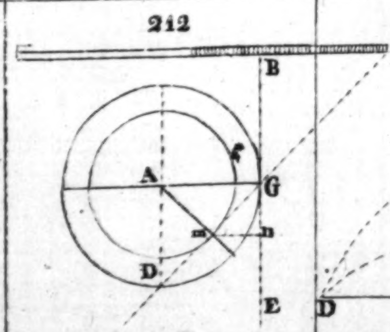
210



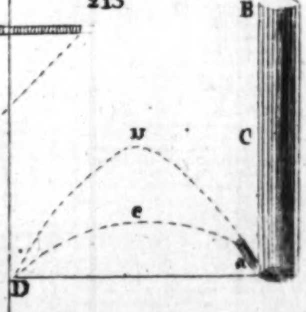
211



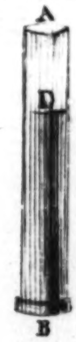
212



213



214



MECHANICS. XI.